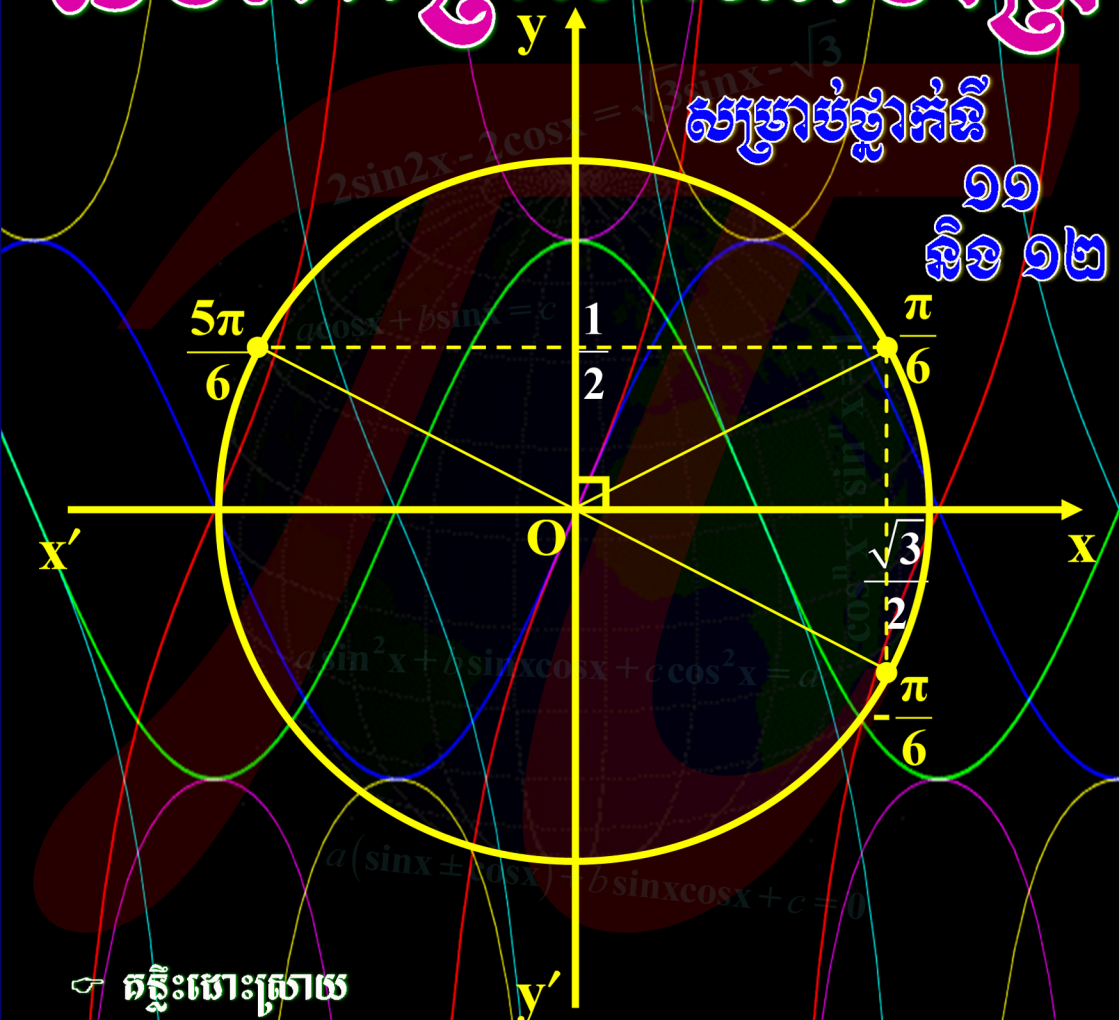
# សមីការត្រីកោណមាត្រ




សម្រាប់ថ្នាក់ទី ១១ និង ១២

- តន្ទឹះដោះស្រាយ
- លំហាត់គំរូ និង លំហាត់អនុវត្តន៍
- កំណែលំហាត់ជ្រើសរើស


ស៊ុខ ពិសិដ្ឋ



ស្របតាមកម្មវិធីសិក្សាថ្មី



រក្សាសិទ្ធិ

# សម័យការត្រីកោណមាត្រ

## ថ្នាក់ទី១១


រៀបរៀង និង ទាយអត្ថបទដោយ:

លោក សុខ ពិសិដ្ឋ (គ្រូបង្រៀនគណិតវិទ្យានៃវិទ្យាល័យកំពង់ត្របែក)

Tel : 088 80 60 387 , 016 510 532

ត្រួតពិនិត្យដោយ:

កញ្ញា សុខ សីម៉ា (គ្រូបង្រៀនគណិតវិទ្យានៃវិទ្យាល័យហ៊ុន សែន ចេក)

Tel : 016 28 69 23

លោក សុខ ឡៀវ (គ្រូបង្រៀនគណិតវិទ្យានៃវិទ្យាល័យព្រះអង្គឌួង)

Tel : 088 88 59 000

លោក ស្រ៊ី សំណាង(គ្រូបង្រៀនគណិតវិទ្យានៃវិទ្យាល័យហ៊ុន រ៉ានី ហ៊ុនសែន ព្រៃពោធ)

Tel : 088 880 51 61

## អារម្ភកថា

សូស្តីលោកគ្រូ អ្នកគ្រូ ប្អូនៗសិស្សានុសិស្ស និងប្រិយមិត្តអ្នកអាន ទាំងអស់ជាទីមេត្រី !

សៀវភៅ **សមីការត្រីកោណមាត្រ** ដែលលោកគ្រូ អ្នកគ្រូ ប្អូនៗសិស្សានុសិស្ស និង ប្រិយមិត្តកំពុងអាននេះ ខ្ញុំបាទរៀបចំឡើង ដើម្បីជាជំនួយដល់ប្អូនៗសិស្សានុសិស្សក្នុងការស្វ័យសិក្សាស្រាវជ្រាវទៅ លើការដោះស្រាយ សមីការត្រីកោណមាត្រ និង ជួយសម្រួលក្នុងការ បង្រៀនសម្រាប់លោកគ្រូ អ្នកគ្រូ ដែលមិនមានពេលវេលាគ្រប់គ្រាន់ ក្នុងការរៀបចំមេរៀនដើម្បីបង្រៀនសិស្ស។ ក្នុងសៀវភៅនេះ រួមមាន ៖

- គន្លឹះដោះស្រាយ
- លំហាត់គំរូ
- កំណែលំហាត់ជ្រើសរើសផ្សេងៗ និង លំហាត់អនុវត្តន៍

ខ្ញុំបាទសូមអភ័យទោស នូវរាល់កំហុសទាំងឡាយដែលកើតឡើង ដោយអចេតនាក្តី ដោយការពិនិត្យមិនបានសព្វគ្រប់ជ្រុងជ្រោយក្តី ឬ ដោយបច្ចេកទេសកុំព្យូទ័រក្តី ខ្ញុំបាទរង់ចាំទទួលនូវមតិរិះគន់បែបស្ថាបនា ពីសំណាក់លោកគ្រូ អ្នកគ្រូ ប្អូនៗសិស្សានុសិស្ស និងប្រិយមិត្តអ្នកអាន ទាំងអស់ដោយក្តី សោមនស្សរីករាយ។

សូមជូនពរ លោកគ្រូ អ្នកគ្រូ ប្អូនៗសិស្សានុសិស្ស និងប្រិយមិត្តអ្នក អានទទួលបានជោគជ័យគ្រប់ភារកិច្ច និង ការសិក្សា។

ព្រៃវែង ថ្ងៃទី ០២ ខែ មករា ឆ្នាំ២០១៤

អ្នករៀបរៀង


សុខ ពិសិដ្ឋ

លេខទូរស័ព្ទ : **016 510 532**

**E-mail:** pisethsok87@yahoo.com

# មាតិកា

ចំណងជើង ទំព័រ

## ផ្នែកទី១ : រូបមន្តត្រីកោណមាត្រ


១. រង្វង់ និង តារាងត្រីកោណមាត្រនៃមុំពិសេស ..... ២

២. រូបមន្តត្រីកោណមាត្រសំខាន់ៗ ..... ៤


## ផ្នែកទី២ : គន្លឹះដោះស្រាយសមីការត្រីកោណមាត្រ


១. សមីការរាង $\cos x = \cos \alpha$ ..... ១១

២. សមីការរាង $\sin x = \sin \alpha$ ..... ១៦

៣. សមីការរាង $\tan x = \tan \alpha$ ..... ២២

៤. សមីការរាង $\cot x = \cot \alpha$ ..... ២៦

៥. សមីការរាង $a\cos x + b\sin x = c$ ..... ៣៣

៦. សមីការដឺក្រេទី២ នៃអនុគមន៍ត្រីកោណមាត្រ ..... ៥១

៧. សមីការរាង $a\sin^2 x + b\sin x\cos x + c\cos^2 x = d$ ..... ៥៦

៨. សមីការរាង $a\left(\sin x \pm \cos x\right) + b\sin x\cos x + c = 0$ ..... ៦២

៩. សមីការ $\sin^m x$ ឈមនឹង $\cos^n x$ ដែល $m$ និង $n$ ជាចំនួនគត់គូ ... ៦៩

១០. សមីការ $\sin^m x$ ឈមនឹង $\cos^n x$ ដែល $m$ និង $n$ ជាចំនួនគត់
សេសមិនស្មើគ្នា ..... ៧៥

១១. សមីការរាង $\cos^n x + \sin^n x = 1$ ..... ៧៨


## ផ្នែកទី៣ : លំហាត់ជ្រើសរើស និង ដំណោះស្រាយ

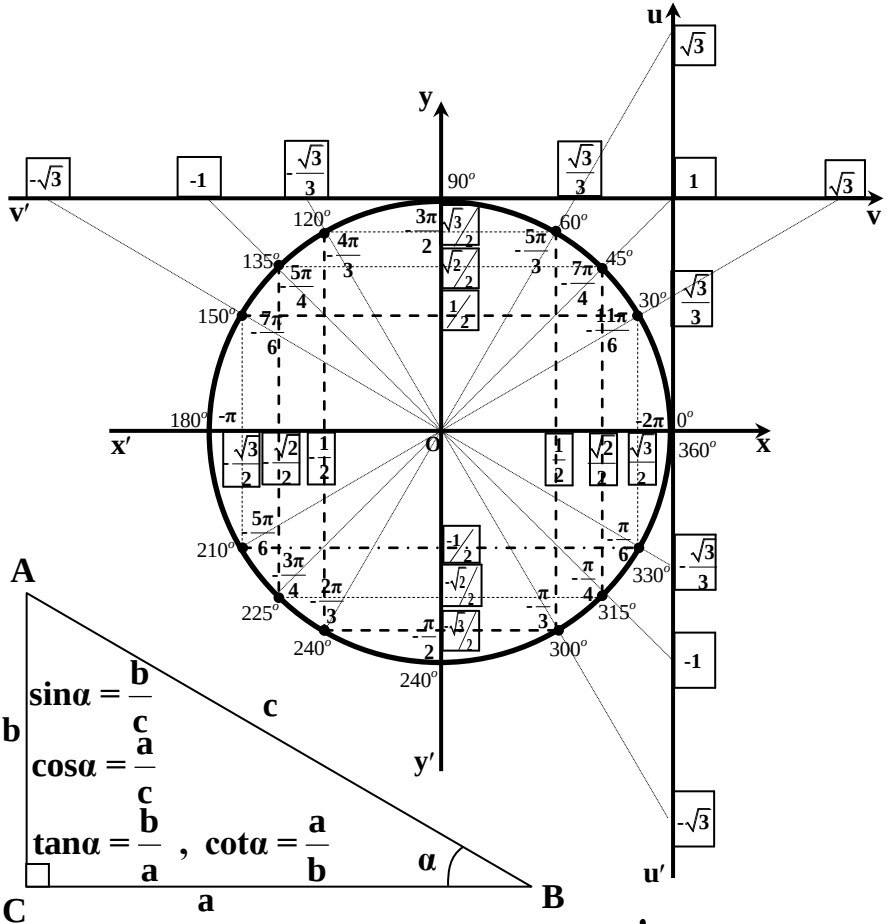

លំហាត់ និង ដំណោះស្រាយ ..... ៨៣


## ផ្នែកទី៤ : លំហាត់អនុវត្តន៍


លំហាត់អនុវត្តន៍ ..... ២០៦


TRIGONOMETRY

# រង្វង់ត្រីកោណមាត្រ




# តារាងផលធៀបត្រីកោណមាត្រនៃមុំពិសេស

| មុំ( $\alpha > 0$ ) | | អនុគមន៍ | | | | មុំ( $\alpha < 0$ ) | |
|---|---|---|---|---|---|---|---|
| Deg | Rad | sinα | cosα | tanα | cotα | Rad | Deg |
| $0^o$ | 0 | 0 | 1 | 0 | ‖ | $-2\pi$ | $-360^0$ |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $30^o$ | $\frac{\pi}{6}$ | $\frac{1}{2}$ | $\frac{\sqrt{3}}{2}$ | $\frac{\sqrt{3}}{3}$ | $\sqrt{3}$ | $-\frac{11\pi}{6}$ | $-330^o$ |
| $45^o$ | $\frac{\pi}{4}$ | $\frac{\sqrt{2}}{2}$ | $\frac{\sqrt{2}}{2}$ | $1$ | $1$ | $-\frac{7\pi}{4}$ | $-315^o$ |
| $60^o$ | $\frac{\pi}{3}$ | $\frac{\sqrt{3}}{2}$ | $\frac{1}{2}$ | $\sqrt{3}$ | $\frac{\sqrt{3}}{3}$ | $-\frac{5\pi}{3}$ | $-300^o$ |
| $90^o$ | $\frac{\pi}{2}$ | $1$ | $0$ | $\|\|$ | $0$ | $-\frac{3\pi}{2}$ | $-270^o$ |
| $120^o$ | $\frac{2\pi}{3}$ | $\frac{\sqrt{3}}{2}$ | $-\frac{1}{2}$ | $-\sqrt{3}$ | $-\frac{\sqrt{3}}{3}$ | $-\frac{4\pi}{3}$ | $-240^o$ |
| $135^o$ | $\frac{3\pi}{4}$ | $\frac{\sqrt{2}}{2}$ | $-\frac{\sqrt{2}}{2}$ | $-1$ | $-1$ | $-\frac{5\pi}{4}$ | $-225^o$ |
| $150^o$ | $\frac{5\pi}{6}$ | $\frac{1}{2}$ | $-\frac{\sqrt{3}}{2}$ | $-\frac{\sqrt{3}}{3}$ | $-\sqrt{3}$ | $-\frac{7\pi}{6}$ | $-210^o$ |
| $180^o$ | $\pi$ | $0$ | $-1$ | $0$ | $\|\|$ | $-\pi$ | $-180^o$ |
| $210^o$ | $\frac{7\pi}{6}$ | $-\frac{1}{2}$ | $-\frac{\sqrt{3}}{2}$ | $\frac{\sqrt{3}}{3}$ | $\sqrt{3}$ | $-\frac{5\pi}{6}$ | $-150^o$ |
| $225^o$ | $\frac{5\pi}{4}$ | $-\frac{\sqrt{2}}{2}$ | $-\frac{\sqrt{2}}{2}$ | $1$ | $1$ | $-\frac{3\pi}{4}$ | $-135^o$ |
| $240^o$ | $\frac{4\pi}{3}$ | $-\frac{\sqrt{3}}{2}$ | $-\frac{1}{2}$ | $\sqrt{3}$ | $\frac{\sqrt{3}}{3}$ | $-\frac{2\pi}{3}$ | $-120^o$ |
| $270^o$ | $\frac{3\pi}{2}$ | $-1$ | $0$ | $\|\|$ | $0$ | $-\frac{\pi}{2}$ | $-90^o$ |
| $300^o$ | $\frac{5\pi}{3}$ | $-\frac{\sqrt{3}}{2}$ | $\frac{1}{2}$ | $-\sqrt{3}$ | $-\frac{\sqrt{3}}{3}$ | $-\frac{\pi}{3}$ | $-60^o$ |
| $315^o$ | $\frac{7\pi}{4}$ | $-\frac{\sqrt{2}}{2}$ | $\frac{\sqrt{2}}{2}$ | $-1$ | $-1$ | $-\frac{\pi}{4}$ | $-45^o$ |
| $330^o$ | $\frac{11\pi}{6}$ | $-\frac{1}{2}$ | $\frac{\sqrt{3}}{2}$ | $-\frac{\sqrt{3}}{3}$ | $-\sqrt{3}$ | $-\frac{\pi}{6}$ | $-30^o$ |
| $360^o$ | $2\pi$ | $0$ | $1$ | $0$ | $\|\|$ | $0$ | $0^o$ |
| Deg | Rad | $\sin\alpha$ | $\cos\alpha$ | $\tan\alpha$ | $\cot\alpha$ | Rad | Deg |
| ម៉ុំ( $\alpha > 0$ ) | | អនុគមន៍ | | | | ម៉ុំ( $\alpha < 0$ ) | |

# រូបមន្តត្រីកោណមាត្រសំខាន់ៗ

## ១. ទំនាក់ទំនងរវាងអនុគមន៍ត្រីកោណមាត្រ ៖ $\cos\theta$ និង $\sin\theta$, $\cos\theta$ និង $\tan\theta$, $\sin\theta$ និង $\cot\theta$, $\tan\theta$ និង $\cot\theta$

$$\cos^2\theta+\sin^2\theta=1$$

$$1+\tan^2\theta=\frac{1}{\cos^2\theta}$$

$$1+\cot^2\theta=\frac{1}{\sin^2\theta}$$

$$\tan\theta\cdot\cot\theta=1 \vee \cot\theta=\frac{1}{\tan\theta}$$

## ២. មុំផ្ទុយគ្នា $\theta$ និង $(-\theta)$

- $\cos(-\theta)=\cos\theta$
- $\sin(-\theta)=-\sin\theta$
- $\tan(-\theta)=-\tan\theta$
- $\cot(-\theta)=-\cot\theta$

## ៣. មុំបំពេញ $\left(\frac{\pi}{2}-\theta\right)$ និង $\theta$

- $\sin\left(\frac{\pi}{2}-\theta\right)=\cos\theta$
- $\cos\left(\frac{\pi}{2}-\theta\right)=\sin\theta$
- $\tan\left(\frac{\pi}{2}-\theta\right)=\cot\theta$
- $\cot\left(\frac{\pi}{2}-\theta\right)=\tan\theta$

## ៤. ម៉ុំបន្ថែម $(\pi-\theta)$ និង $\theta$

- $\sin(\pi-\theta)=\sin\theta$
- $\cos(\pi-\theta)=-\cos\theta$
- $\tan(\pi-\theta)=-\tan\theta$
- $\cot(\pi-\theta)=-\cot\theta$

## ៥. ម៉ុំដែលមានផលសងស្មើ $\pi$

- $\sin(\pi+\theta)=-\sin\theta$
- $\cos(\pi+\theta)=-\cos\theta$
- $\tan(\pi+\theta)=\tan\theta$
- $\cot(\pi+\theta)=\cot\theta$

## ៦. ម៉ុំដែលមានផលសងស្មើ $\frac{\pi}{2}$

- $\sin\left(\frac{\pi}{2}+\theta\right)=\cos\theta$
- $\cos\left(\frac{\pi}{2}+\theta\right)=-\sin\theta$
- $\tan\left(\frac{\pi}{2}+\theta\right)=-\cot\theta$
- $\cot\left(\frac{\pi}{2}+\theta\right)=-\tan\theta$

## ៧. រូបមន្តផលបូកនិងផលដក

- $\cos(\alpha-\beta)=\cos\alpha\cos\beta+\sin\alpha\sin\beta$
- $\cos(\alpha+\beta)=\cos\alpha\cos\beta-\sin\alpha\sin\beta$
- $\sin(\alpha+\beta)=\sin\alpha\cos\beta+\sin\beta\cos\alpha$
- $\sin(\alpha-\beta)=\sin\alpha\cos\beta-\sin\beta\cos\alpha$
- $\tan(\alpha+\beta)=\dfrac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$
- $\tan(\alpha-\beta)=\dfrac{\tan\alpha-\tan\beta}{1+\tan\alpha\tan\beta}$
- $\cot(\alpha+\beta)=\dfrac{\cot\alpha\cot\beta-1}{\cot\alpha+\cot\beta}$
- $\cot(\alpha-\beta)=\dfrac{\cot\alpha\cot\beta+1}{\cot\beta-\cot\alpha}$

## ៨. រូបមន្តមុំពីរ

- $\sin 2\alpha = 2\sin\alpha\cos\alpha$
- $\cos 2\alpha = \cos^2\alpha - \sin^2\alpha = 1 - 2\sin^2\alpha = 2\cos^2\alpha - 1$
- $\tan 2\alpha = \dfrac{2\tan\alpha}{1-\tan^2\alpha}$
- $\cot 2\alpha = \dfrac{\cot^2\alpha - 1}{2\cot\alpha}$

## ៩. រូបមន្តកន្លះមុំ

- $\sin^2\dfrac{\alpha}{2} = \dfrac{1-\cos\alpha}{2}$
- $\cos^2\dfrac{\alpha}{2} = \dfrac{1+\cos\alpha}{2}$
- $\tan^2\dfrac{\alpha}{2} = \dfrac{1-\cos\alpha}{1+\cos\alpha}$
- $\tan\dfrac{\alpha}{2} = \dfrac{\sin\alpha}{1+\cos\alpha}$

## ១១. រូបមន្តកន្សោម $\sin\alpha$ , $\cos\alpha$ , $\tan\alpha$ ជាអនុគមន៍នៃ $t = \tan\dfrac{\alpha}{2}$

- $\cos\alpha = \dfrac{1-t^2}{1+t^2}$
- $\sin\alpha = \dfrac{2t}{1+t^2}$
- $\tan\alpha = \dfrac{2t}{1-t^2}$

## ១២. កន្សោម $\sin 3\alpha$ , $\cos 3\alpha$ , $\tan 3\alpha$

- $\sin 3\alpha = 3\sin\alpha - 4\sin^3\alpha$
- $\cos 3\alpha = 4\cos^3\alpha - 3\cos\alpha$
- $\tan 3\alpha = \dfrac{3\tan\alpha - \tan^3\alpha}{1-3\tan^2\alpha}$

## ១៣. រូបមន្តបំលែងពីផលគុណទៅជាផលបូក

- $\cos\alpha\cos\beta=\frac{1}{2}\left[\cos(\alpha+\beta)+\cos(\alpha-\beta)\right]$
- $\sin\alpha\sin\beta=\frac{1}{2}\left[\cos(\alpha-\beta)-\cos(\alpha+\beta)\right]$
- $\sin\alpha\cos\beta=\frac{1}{2}\left[\sin(\alpha+\beta)+\sin(\alpha-\beta)\right]$
- $\cos\alpha\sin\beta=\frac{1}{2}\left[\sin(\alpha+\beta)-\sin(\alpha-\beta)\right]$

## ១៤.រូបមន្តបំលែងពីផលបូកទៅជាផលគុណ

- $\cos p+\cos q=2\cos\frac{p+q}{2}\cos\frac{p-q}{2}$
- $\cos p-\cos q=-2\sin\frac{p+q}{2}\sin\frac{p-q}{2}$
- $\sin p+\sin q=2\sin\frac{p+q}{2}\cos\frac{p-q}{2}$
- $\sin p-\sin q=2\sin\frac{p-q}{2}\cos\frac{p+q}{2}$
- $\tan p+\tan q=\frac{\sin(p+q)}{\cos p\cos q}$
- $\tan p-\tan q=\frac{\sin(p-q)}{\cos p\cos q}$
- $\cot p+\cot q=\frac{\sin(p+q)}{\sin p\sin q}$
- $\cot p-\cot q=\frac{\sin(q-p)}{\sin p\sin q}$

## ១៥. អនុគមន៍ត្រីកោណមាត្រនៃមុំ $\alpha$ និង $(\alpha+k\pi)$ ដែល $k\in\mathbb{Z}$

- $\sin(\alpha+k\pi)=\begin{cases}\sin\alpha \text{ បើ k គូ}\\ -\sin\alpha \text{ បើ k សេស}\end{cases}$
- $\cos(\alpha+k\pi)=\begin{cases}\cos\alpha \text{ បើ k គូ}\\ -\cos\alpha \text{ បើ k សេស}\end{cases}$
- $\tan(\alpha+k\pi)=\tan\alpha\ ,\ \forall k\in\mathbb{Z}$
- $\cot(\alpha+k\pi)=\cot\alpha\ ,\ \forall k\in\mathbb{Z}$

## ១៦. សមីការត្រីកោណមាត្រ

- $\sin x=\sin\alpha\Rightarrow\begin{bmatrix}x=\alpha+2k\pi\\ x=\pi-\alpha+2k\pi\end{bmatrix},k\in\mathbb{Z}$
- $\cos x=\cos\alpha\Rightarrow x=\pm\alpha+2k\pi\ ,k\in\mathbb{Z}$
- $\tan x=\tan\alpha\Rightarrow x=\alpha+k\pi\ ,k\in\mathbb{Z}$
- $\cot x=\cot\alpha\Rightarrow x=\alpha+k\pi\ ,k\in\mathbb{Z}$

## ចំណាំ:

- $\sin x=0\Rightarrow x=k\pi\ ,k\in\mathbb{Z}$
- $\cos x=0\Rightarrow x=\dfrac{\pi}{2}+k\pi=\dfrac{2k+1}{2}\pi\ ,k\in\mathbb{Z}$
- $\{k\pi\}\cup\{2k\pi\}=\{k\pi\}\ ,k\in\mathbb{Z}$
- $\{k\pi\}\cap\{2k\pi\}=\{2k\pi\}\ ,k\in\mathbb{Z}$
- $\{2k\pi\}\cup\{(2k\pm1)\pi\}=\{k\pi\}\ ,k\in\mathbb{Z}$


TRIGONOMETRY
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# សមីការត្រីកោណមាត្រ

## ១. សមីការរាង $\cos x = \cos\alpha$

| |
|---|
| សមីការ $\cos x = \cos\alpha$ មានចម្លើយ $x = \pm\alpha + 2k\pi$ , $k \in \mathbb{Z}$ |

ឧទាហរណ៍: សមីការ $\cos x = \cos\frac{\pi}{3}$ មានចម្លើយ $x = \frac{\pi}{3} + 2k\pi$ ឬ

$x = -\frac{\pi}{3} + 2k\pi$ ដែល $k \in \mathbb{Z}$ ។

### ដំណោះស្រាយសមីការ $\cos x = a$

- បើ $|a| \le 1$ នោះសមីការ $\cos x = a$ មានឫសក្នុងសំណុំចំនួនពិត ។
  យើងត្រូវបម្លែងឲ្យបាន $a = \cos\alpha$ នោះសមីការទៅជា $\cos x = \cos\alpha$
  យើងបាន $x = \pm\alpha + 2k\pi$ , $k \in \mathbb{Z}$
- បើ $|a| > 1$ នោះសមីការ $\cos x = a$ គ្មានឫសក្នុងសំណុំចំនួនពិត ។

**Note:** បើ $|a| \le 1$ នោះ សមីការ $\cos x = a$ មានចម្លើយ:

$x = \pm\arccos a + 2k\pi, k \in \mathbb{Z}$

☞ $\arccos a$ ( ឬ $\cos^{-1} a$ ) ជាអនុគមន៍ច្រាសនៃ $\cos a$ ។

លំហាត់គំរូទី១: ដោះស្រាយសមីការ $\cos x = \frac{1}{2}$

**ចំណោះស្រាយ**

យើងមានសមីការ $\cos x = \frac{1}{2}$

$$\Leftrightarrow \cos x = \cos\frac{\pi}{3}$$

$$\Rightarrow x = \pm\frac{\pi}{3} + 2k\pi \text{ , } k \in \mathbb{Z}$$

ដូចនេះ សមីការមានឫស $\boxed{x = \frac{\pi}{3} + 2k\pi \text{ , } x = -\frac{\pi}{3} + 2k\pi, k \in \mathbb{Z}}$

លំហាត់គំរូទី២: ដោះស្រាយសមីការ $\cos x = \frac{\sqrt{3}}{2}$

**ចំណោះស្រាយ**

យើងមានសមីការ $\cos x = \frac{\sqrt{3}}{2}$

$$\Leftrightarrow \cos x = \cos\frac{\pi}{6}$$

$$\Rightarrow x = \pm\frac{\pi}{6} + 2k\pi \text{ ,} k \in \mathbb{Z}$$

ដូចនេះ សមីការមានឫស $\boxed{x = \frac{\pi}{6} + 2k\pi \text{ , } x = -\frac{\pi}{6} + 2k\pi \text{ ,} k \in \mathbb{Z}}$

លំហាត់គំរូទី៣: ដោះស្រាយសមីការ $\cos x = -\frac{1}{\sqrt{2}}$

**ចំណោះស្រាយ**

យើងមានសមីការ $\cos x = -\frac{1}{\sqrt{2}}$

$$\Leftrightarrow \cos x = -\frac{\sqrt{2}}{2}$$

$$\Leftrightarrow \cos x = \cos\frac{3\pi}{4}$$

$$\Rightarrow x = \pm\frac{3\pi}{4} + 2k\pi \ , \ k \in \mathbb{Z}$$

ដូចនេះ សមីការមានឫស $\boxed{x = \frac{3\pi}{4} + 2k\pi, x = -\frac{3\pi}{4} + 2k\pi, k \in \mathbb{Z}}$

**សម្គាល់ៈ**

- សមីការ $\cos x = 0$ មានចម្លើយ $x = \frac{\pi}{2} + k\pi = \frac{2k+1}{2}\pi \ k \in \mathbb{Z}$

  ឬ $\cos u(x) = 0 \Leftrightarrow u(x) = \frac{\pi}{2} + k\pi, \ k \in \mathbb{Z}$
- សមីការ $\cos x = 1$ មានចម្លើយ $x = 2k\pi \ , k \in \mathbb{Z}$
- សមីការ $\cos x = -1$ មានចម្លើយ $x = (2k+1)\pi, k \in \mathbb{Z}$
- សមីការ $\cos x = \pm 1 \Leftrightarrow x = k\pi \ , k \in \mathbb{Z}$
- $\cos u(x) = \cos v(x) \Leftrightarrow u(x) = \pm v(x) + 2k\pi, k \in \mathbb{Z}$

  ដែល $u$ និង $v$ ជាអនុគមន៍នៃ $x$ ។

លំហាត់គំរូទី៤: ដោះស្រាយសមីការ $\cos\left(4x - \frac{\pi}{3}\right) = \cos\left(x - \frac{\pi}{6}\right)$

**ចំណោះស្រាយ**

យើងមានសមីការ $\cos\left(4x - \frac{\pi}{3}\right) = \cos\left(x - \frac{\pi}{6}\right)$

$$\Leftrightarrow 4x - \frac{\pi}{3} = \pm\left(x - \frac{\pi}{6}\right) + 2k\pi \ , \ k \in \mathbb{Z}$$

$$\Leftrightarrow \begin{cases} 4x-\frac{\pi}{3}=x-\frac{\pi}{6}+2k\pi \\ 4x-\frac{\pi}{3}=-x+\frac{\pi}{6}+2k\pi \end{cases}$$

$$\Leftrightarrow \begin{cases} 3x=-\frac{\pi}{2}+2k\pi \\ 5x=-\frac{\pi}{6}+2k\pi \end{cases}$$

$$\Rightarrow \begin{cases} x=-\frac{\pi}{6}+\frac{2k\pi}{3} \\ x=-\frac{\pi}{30}=\frac{2k\pi}{5} \end{cases}$$

ដូចនេះ សមីការមានឫស $\boxed{x=-\frac{\pi}{6}+\frac{2k\pi}{3}, x=-\frac{\pi}{30}+\frac{2k\pi}{5}, k\in\mathbb{Z}}$

លំហាត់គំរូទី៥: ដោះស្រាយសមីការ $\cos(3x-\frac{\pi}{6})=1$

**ដំណោះស្រាយ**

យើងមាន $\cos(3x-\frac{\pi}{6})=1$

$$\Leftrightarrow 3x-\frac{\pi}{6}=2k\pi \ , \ k\in\mathbb{Z}$$

$$\Leftrightarrow 3x=\frac{\pi}{6}+2k\pi$$

$$\Rightarrow x=\frac{\pi}{18}+\frac{2k\pi}{3}$$

ដូចនេះ សមីការមានឫស $\boxed{x=\frac{\pi}{18}+\frac{2k\pi}{3} \ , \ k\in\mathbb{Z}}$

លំហាត់គំរូទី៦: ដោះស្រាយសមីការ $\cos\left(3x+\frac{\pi}{3}\right)=\cos\left(x-\frac{\pi}{6}\right)$

**ចំណោះស្រាយ**

យើងមាន $\cos\left(3x+\frac{\pi}{3}\right)=\cos\left(x-\frac{\pi}{6}\right)$

$$\Leftrightarrow\begin{cases}3x+\frac{\pi}{3}=x-\frac{\pi}{6}+2k\pi\\3x+\frac{\pi}{3}=-x+\frac{\pi}{6}+2k\pi\end{cases},k\in\mathbb{Z}\Leftrightarrow\begin{cases}2x=-\frac{\pi}{2}+2k\pi\\4x=-\frac{\pi}{6}+2k\pi\end{cases},k\in\mathbb{Z}$$

ដូចនេះ សមីការមានឫស $\boxed{x=-\frac{\pi}{4}+k\pi,x=-\frac{\pi}{24}+\frac{k\pi}{2},k\in\mathbb{Z}}$

លំហាត់គំរូទី៧: ដោះស្រាយសមីការ $\cos\frac{\pi}{6}\cos x-\sin\frac{\pi}{6}\sin x=\cos\frac{\pi}{4}$

**ចំណោះស្រាយ**

យើងមាន $\cos\frac{\pi}{6}\cos x-\sin\frac{\pi}{6}\sin x=\cos\frac{\pi}{4}$

ដោយ $\cos\frac{\pi}{6}\cos x-\sin\frac{\pi}{6}\sin x=\cos\left(\frac{\pi}{6}+x\right)$

នោះសមីការទៅជា $\cos\left(\frac{\pi}{6}+x\right)=\cos\frac{\pi}{4}$

$$\Leftrightarrow\begin{cases}\frac{\pi}{6}+x=\frac{\pi}{4}+2k\pi\\\frac{\pi}{6}+x=-\frac{\pi}{4}+2k\pi\end{cases},k\in\mathbb{Z}$$

$$\Leftrightarrow\begin{cases}x=\frac{\pi}{4}-\frac{\pi}{6}+2k\pi=\frac{\pi}{12}+2k\pi\\x=-\frac{\pi}{4}-\frac{\pi}{6}+2k\pi=-\frac{5\pi}{12}+2k\pi\end{cases},k\in\mathbb{Z}$$

ដូចនេះ សមីការមានឫស $\boxed{x=\frac{\pi}{12}+2k\pi, x=-\frac{5\pi}{12}+2k\pi, k\in\mathbb{Z}}$

លំហាត់គំរូទី៨: ដោះស្រាយសមីការ $\sin^4 x-\cos^4 x=\frac{1}{2}$

**ចំណោះស្រាយ**

យើងមាន $\sin^4 x-\cos^4 x=\frac{1}{2}$

$\Leftrightarrow \left(\sin^2 x+\cos^2 x\right)\left(\sin^2 x-\cos^2 x\right)=\frac{1}{2}$

$\Leftrightarrow \sin^2 x-\cos^2 x=\frac{1}{2}$

$\Leftrightarrow -\cos 2x=\frac{1}{2}$

$\Leftrightarrow \cos 2x=-\frac{1}{2}=\cos\frac{2\pi}{3}$

$\Leftrightarrow 2x=\pm\frac{2\pi}{3}+2k\pi \ , k\in\mathbb{Z}$

$\Rightarrow x=\pm\frac{\pi}{3}+k\pi \ , k\in\mathbb{Z}$

ដូចនេះ សមីការមានឫស $\boxed{x=-\frac{\pi}{3}+k\pi \ , \ x=\frac{\pi}{3}+k\pi \ , k\in\mathbb{Z}}$

## ២. សមីការរាង $\sin x=\sin\alpha$

$$\text{សមីការ } \sin x=\sin\alpha \text{ មានចម្លើយ } \begin{bmatrix} x=\alpha+2k\pi \\ x=\pi-\alpha+2k\pi \end{bmatrix}, k\in\mathbb{Z}$$

ឧទាហរណ៍: សមីការ $\sin x = \sin\frac{\pi}{3}$ មានចម្លើយ

$$\begin{cases} x = \frac{\pi}{3} + 2k\pi \\ x = \pi - \frac{\pi}{3} + 2k\pi = \frac{2\pi}{3} + 2k\pi \end{cases}, k \in \mathbb{Z}$$

**ដំណោះស្រាយសមីការ** $\sin x = a$

- បើ $|a| \le 1$ នោះសមីការ $\sin x = a$ មានឫសក្នុងសំណុំចំនួនពិត ។
  យើងត្រូវបម្លែងឲ្យបាន $a = \sin\alpha$
  នោះសមីការទៅជា $\sin x = \sin\alpha$

$$\Leftrightarrow \begin{cases} x = \alpha + 2k\pi \\ x = \pi - \alpha + 2k\pi \end{cases}, k \in \mathbb{Z}$$

- បើ $|a| > 1$ នោះសមីការ $\sin x = a$ គ្មានឫសក្នុងសំណុំចំនួនពិត ។

**Note:** បើ $|a| \le 1$ នោះ សមីការ $\sin x = a$ មានចម្លើយ:

$$\begin{cases} x = \arcsin a + 2k\pi \\ x = \pi - \arcsin a + 2k\pi \end{cases}, k \in \mathbb{Z}$$

☞ $\arcsin a$ ( ឬ $\sin^{-1} a$ ) ជាអនុគមន៍ច្រាសនៃ $\sin a$ ។

លំហាត់គំរូទី១: ដោះស្រាយសមីការ $\sin x = \frac{\sqrt{2}}{2}$

**ដំណោះស្រាយ**

យើងមាន $\sin x = \frac{\sqrt{2}}{2}$

$\Leftrightarrow \sin x = \sin\frac{\pi}{4}$

$$\Leftrightarrow \begin{cases} x=\dfrac{\pi}{4}+2k\pi \\ x=\pi-\dfrac{\pi}{4}+2k\pi=\dfrac{3\pi}{4}+2k\pi \end{cases} ,k\in\mathbb{Z}$$

ដូចនេះ សមីការមានឫស $\boxed{x=\dfrac{\pi}{4}+2k\pi\ ,\ x=\dfrac{3\pi}{4}+2k\pi\ ,\ k\in\mathbb{Z}}$

លំហាត់គំរូទី២: ដោះស្រាយសមីការ $\sin x=\dfrac{\sqrt{3}}{2}$

**ចំណោះស្រាយ**

យើងមាន $\sin x=\dfrac{\sqrt{3}}{2}$

$\Leftrightarrow \sin x=\sin\dfrac{\pi}{3}$

$$\Leftrightarrow \begin{cases} x=\dfrac{\pi}{3}+2k\pi \\ x=\pi-\dfrac{\pi}{3}+2k\pi=\dfrac{2\pi}{3}+2k\pi \end{cases} , k\in\mathbb{Z}$$

ដូចនេះ សមីការមានឫស $\boxed{x=\dfrac{\pi}{3}+2k\pi\ ,\ x=\dfrac{2\pi}{3}+2k\pi\ ,\ k\in\mathbb{Z}}$

លំហាត់គំរូទី៣ : ដោះស្រាយសមីការ $\sin x=-\dfrac{1}{2}$

**ចំណោះស្រាយ**

យើងមាន $\sin x=-\dfrac{1}{2}$

$\Leftrightarrow \sin x=\sin\left(-\dfrac{\pi}{6}\right)$

$$\Leftrightarrow \begin{cases} x = -\dfrac{\pi}{6} + 2k\pi \\ x = \pi + \dfrac{\pi}{6} + 2k\pi = \dfrac{7\pi}{6} + 2k\pi \end{cases} \quad k \in \mathbb{Z}$$

ដូចនេះ សមីការមានឫស $\boxed{x = -\dfrac{\pi}{6} + 2k\pi \ , \ x = \dfrac{7\pi}{6} + 2k\pi, k \in \mathbb{Z}}$

## សម្គាល់ៈ

- $\sin x = 0 \Leftrightarrow x = k\pi$ ឬ $\sin u(x) = 0 \Leftrightarrow u(x) = k\pi \ , k \in \mathbb{Z}$
- $\sin x = 1 \Leftrightarrow x = \dfrac{\pi}{2} + 2k\pi$

  ឬ $\sin u(x) = 1 \Leftrightarrow u(x) = \dfrac{\pi}{2} + 2k\pi \ , k \in \mathbb{Z}$
- $\sin x = -1 \Leftrightarrow x = -\dfrac{\pi}{2} + 2k\pi$

  ឬ $\sin u(x) = -1 \Leftrightarrow u(x) = -\dfrac{\pi}{2} + 2k\pi \ , k \in \mathbb{Z}$
- $\sin x = \pm 1 \Leftrightarrow x = \dfrac{\pi}{2} + k\pi = \dfrac{2k+1}{2}\pi \ , \ k \in \mathbb{Z}$
- $\sin u(x) = \sin v(x) \Leftrightarrow \begin{cases} u(x) = v(x) + 2k\pi \\ u(x) = \pi - v(x) + 2k\pi \end{cases} , \ k \in \mathbb{Z}$

លំហាត់គំរូទី៤: ដោះស្រាយសមីការ $\sin\left(2x + \dfrac{\pi}{4}\right) = 1$

### ចំណោះស្រាយ

យើងមាន $\sin\left(2x + \dfrac{\pi}{4}\right) = 1$

$$\Leftrightarrow 2x+\frac{\pi}{4}=\frac{\pi}{2}+2k\pi \ , k\in\mathbb{Z}$$

$$\Leftrightarrow 2x=\frac{\pi}{2}-\frac{\pi}{4}+2k\pi=\frac{\pi}{4}+2k\pi$$

$$\Rightarrow x=\frac{\pi}{8}+k\pi \ , k\in\mathbb{Z}$$

ដូចនេះ សមីការមានឫស $\boxed{x=\frac{\pi}{8}+k\pi \ , k\in\mathbb{Z}}$

លំហាត់គំរូទី៥: ដោះស្រាយសមីការ $\sin 2x=\sin\left(\frac{\pi}{3}-x\right)$

**ចំណោះស្រាយ**

យើងមាន $\sin 2x=\sin\left(\frac{\pi}{3}-x\right)$

$$\Leftrightarrow \begin{cases} 2x=\frac{\pi}{3}-x+2k\pi \\ 2x=\pi-\left(\frac{\pi}{3}-x\right)+2k\pi \end{cases} , k\in\mathbb{Z}$$

$$\Leftrightarrow \begin{cases} 3x=\frac{\pi}{3}+2k\pi \\ x=\pi-\frac{\pi}{3}+2k\pi \end{cases} , k\in\mathbb{Z}$$

$$\Rightarrow \begin{cases} x=\frac{\pi}{9}+\frac{2k\pi}{3} \\ x=\frac{2\pi}{3}+2k\pi \end{cases} , k\in\mathbb{Z}$$

ដូចនេះ សមីការមានឫស $\boxed{x=\frac{\pi}{9}+\frac{2k\pi}{3}, x=\frac{2\pi}{3}+2k\pi \ , k\in\mathbb{Z}}$

លំហាត់គំរូទី៦: ដោះស្រាយសមីការ $2\sin x\cos x-3\sin 2x=0$

**ចំណោះស្រាយ**

យើងមាន $2\sin x\cos x-3\sin 2x=0$

$\Leftrightarrow \sin 2x-3\sin 2x=0$

$\Leftrightarrow -2\sin 2x=0$

$\Leftrightarrow \sin 2x=0$

$\Leftrightarrow 2x=k\pi \ ,k\in\mathbb{Z}$

$\Rightarrow x=\dfrac{k\pi}{2}\ ,k\in\mathbb{Z}$

ដូចនេះ សមីការមានឫស $\boxed{x=\dfrac{k\pi}{2}\ ,k\in\mathbb{Z}}$

លំហាត់គំរូទី៧: ដោះស្រាយសមីការ $\sin\sqrt{x}=-1$

**ចំណោះស្រាយ**

យើងមាន $\sin\sqrt{x}=-1$

សមីការមានឫសកាលណា $x\geq 0$

យើងបានសមីការសមមូលនឹង $\sqrt{x}=-\dfrac{\pi}{2}+2k\pi$

$(-\dfrac{\pi}{2}+2k\pi\geq 0\Rightarrow k\in\mathbb{N})$

$\Rightarrow x=\left(-\dfrac{\pi}{2}+2k\pi\right)^2\ ,k\in\mathbb{N}$

ដូចនេះ សមីការមានឫស $\boxed{x=\left(-\dfrac{\pi}{2}+2k\pi\right)^2\ ,k\in\mathbb{N}}$

## ៣. សមីការរាង $\tan x = \tan\alpha$

$$\text{សមីការ } \tan x = \tan\alpha \text{ មានចម្លើយ} \begin{cases} x = \alpha + k\pi \\ x \neq \dfrac{\pi}{2} + k\pi \end{cases}, k \in \mathbb{Z}$$

ឧទាហរណ៍: សមីការ $\tan x = \tan\dfrac{\pi}{3}$ មានចម្លើយ $x = \dfrac{\pi}{3} + k\pi \;\; k \in \mathbb{Z}$

**ដំណោះស្រាយសមីការ** $\tan x = a$

សមីការ $\tan x = a$ មានឫសចំពោះ $\forall x \in \mathbb{R}$

យើងត្រូវបម្លែងឲ្យបាន $a = \tan\alpha$ នោះសមីកា $\tan x = a$

អាចសរសេរទៅជា $\tan x = \tan\alpha \Rightarrow x = \alpha + k\pi$ និង

$x \neq \dfrac{\pi}{2} + k\pi, k \in \mathbb{Z}$ ឬ $\tan x = a \Leftrightarrow x = \arctan a + k\pi, \; k \in \mathbb{Z}$

**សម្គាល់ៈ**

- $\tan u(x) = \tan v(x) \Leftrightarrow u(x) = v(x) + k\pi, \;\; k \in \mathbb{Z}$
- ក្នុងការដោះស្រាយសមីការ $\tan u(x) = \tan v(x)$

  យើងត្រូវឲ្យលក្ខខណ្ឌ

$$\begin{cases} \cos u(x) \neq 0 \\ \cos v(x) \neq 0 \end{cases} \Leftrightarrow \begin{cases} u(x) \neq \dfrac{\pi}{2} + k\pi \\ v(x) \neq \dfrac{\pi}{2} + k\pi \end{cases}, k \in \mathbb{Z}$$

ព្រោះ $\tan u(x) = \dfrac{\sin u(x)}{\cos u(x)}$ & $\tan v(x) = \dfrac{\sin v(x)}{\cos v(x)}$

លំហាត់គំរូទី១: ដោះស្រាយសមីការ $\tan x = 1$

**ចំណោះស្រាយ**

យើងមាន សមីការ $\tan x = 1$

សមីការមានឫសកាលណា $x \neq \frac{\pi}{2} + k\pi$

យើងបាន $\tan x = 1 \Leftrightarrow \tan x = \tan \frac{\pi}{4}$

$$\Rightarrow x = \frac{\pi}{4} + k\pi \ , k \in \mathbb{Z}$$

ដូចនេះ សមីការមានឫស $\boxed{x = \frac{\pi}{4} + k\pi \ , k \in \mathbb{Z}}$

លំហាត់គំរូទី២: ដោះស្រាយសមីការ $\tan x = -\frac{\sqrt{3}}{3}$

**ចំណោះស្រាយ**

យើងមាន $\tan x = -\frac{\sqrt{3}}{3}$

$$\Leftrightarrow \tan x = \tan \frac{5\pi}{6}$$

$$\Leftrightarrow x = \frac{5\pi}{6} + k\pi \ , k \in \mathbb{Z}$$

ដូចនេះ សមីការមានឫស $\boxed{x = \frac{5\pi}{6} + k\pi \ , k \in \mathbb{Z}}$

លំហាត់គំរូទី៣: ដោះស្រាយសមីការ $\tan x = \frac{1}{\sqrt{3}}$

**ចំណោះស្រាយ**

យើងមាន $\tan x = \frac{1}{\sqrt{3}}$

$$\Leftrightarrow \tan x = \frac{\sqrt{3}}{3}$$

$$\Leftrightarrow \tan x = \tan\frac{\pi}{6}$$

$$\Rightarrow x = \frac{\pi}{6} + k\pi \ , k \in \mathbb{Z}$$

ដូចនេះ សមីការមានឫស $\boxed{x = \frac{\pi}{6} + k\pi \ , k \in \mathbb{Z}}$

លំហាត់គំរូទី៥: ដោះស្រាយសមីការ $\tan 3x = \sqrt{3}$

**ចំណោះស្រាយ**

យើងមាន $\tan 3x = \sqrt{3}$

សមីការមានឫសកាលណា $3x \neq \frac{\pi}{2} + k\pi \ \Rightarrow x \neq \frac{\pi}{6} + \frac{k\pi}{3}$

យើងបាន $\tan 3x = \sqrt{3}$

$$\Leftrightarrow \tan 3x = \tan\frac{\pi}{3}$$

$$\Leftrightarrow 3x = \frac{\pi}{3} + k\pi \ , k \in \mathbb{Z}$$

$$\Leftrightarrow x = \frac{\pi}{9} + \frac{k\pi}{3} \ , k \in \mathbb{Z}$$

$$\Leftrightarrow x = \frac{\pi}{9} + \frac{k\pi}{3} \ , k \in \mathbb{Z}$$

ដូចនេះ សមីការមានឫស $\boxed{x = \frac{\pi}{9} + \frac{k\pi}{3} \ , k \in \mathbb{Z}}$

លំហាត់គំរូទី៥: ដោះស្រាយសមីការ $\frac{2\tan x}{1-\tan^2 x}=\sqrt{3}$

**ដំណោះស្រាយ**

យើងមាន សមីការ $\frac{2\tan x}{1-\tan^2 x}=\sqrt{3}$

សមីការមានឫសកាលណា $\begin{cases}\cos x\neq 0\\ 1-\tan^2 x\neq 0\end{cases}$

$$\Leftrightarrow \begin{cases}x\neq \frac{\pi}{2}+k\pi\\ x\neq \pm\frac{\pi}{4}+k\pi\end{cases},\ k\in\mathbb{Z}$$

ដោយ $\frac{2\tan x}{1-\tan^2 x}=\tan 2x$

នោះយើងបានសមីការទៅជា $\tan 2x=\sqrt{3}$

$$\Leftrightarrow \tan 2x=\tan\frac{\pi}{3}$$

$$\Leftrightarrow 2x=\frac{\pi}{3}+k\pi\ ,\ k\in\mathbb{Z}$$

$$\Rightarrow x=\frac{\pi}{6}+\frac{k\pi}{2}\ ,\ k\in\mathbb{Z}$$

ដូចនេះ សមីការមានឫស $\boxed{x=\frac{\pi}{6}+\frac{k\pi}{2}\ ,\ k\in\mathbb{Z}}$

លំហាត់គំរូទី៦: ដោះស្រាយសមីការ $2\tan x\cos x+1=2\cos x+\tan x$

**ដំណោះស្រាយ**

យើងមានសមីការ $2\tan x\cos x+1=2\cos x+\tan x$

សមីការមានឫសកាលណា $x\neq\frac{\pi}{2}+k\pi\ ,\ k\in\mathbb{Z}$

យើងបាន $2\tan x\cos x+1=2\cos x+\tan x$

$$\Leftrightarrow 2\tan x\cos x+1-2\cos x-\tan x=0$$
$$\Leftrightarrow 2\cos x(\tan x-1)-(\tan x-1)=0$$
$$\Leftrightarrow (\tan x-1)(2\cos x-1)=0$$
$$\Leftrightarrow \left[\begin{array}{l}\tan x-1=0\\ 2\cos x-1=0\end{array}\right.$$
$$\Leftrightarrow \left[\begin{array}{l}\tan x=1=\tan\dfrac{\pi}{4}\\ \cos x=\dfrac{1}{2}=\cos\dfrac{\pi}{3}\end{array}\right.$$
$$\Rightarrow \left[\begin{array}{l}x=\dfrac{\pi}{4}+k\pi\\ x=\pm\dfrac{\pi}{3}+2k\pi\end{array}\right., k\in\mathbb{Z}$$

ដូចនេះ សមីការមានឫស $\boxed{x=\dfrac{\pi}{4}+k\pi\ ,\ x=\pm\dfrac{\pi}{3}+k\pi\ ,\ k\in\mathbb{Z}}$

## ៤. សមីការរាង $\cot x=\cot\alpha$

$$\text{សមីការ } \cot x=\cot\alpha \text{ មានចម្លើយ } \begin{cases}x=\alpha+k\pi\\ x\neq k\pi\end{cases}, k\in\mathbb{Z}$$

ឧទាហរណ៍: សមីការ $\cot x=\cot\dfrac{\pi}{3}$ មានចម្លើយ $x=\dfrac{\pi}{3}+k\pi\ ,k\in\mathbb{Z}$

**ដំណោះស្រាយសមីការ** $\cot x=a$

សមីការ $\cot x=a$ មានឫស $\forall a\in\mathbb{R}$

យើងត្រូវបម្លែងឲ្យបាន $a=\cot\alpha$

នោះសមីការ $\cot x = a$ អាចសរសេរ $\cot x = \cot \alpha$

$$\Rightarrow x = \alpha + k\pi \ , k \in \mathbb{Z}$$

ឬ $\cot x = a \Leftrightarrow x = arc\cot a + k\pi \ , k \in \mathbb{Z}$

$arc\cot a$ (ឬ $\cot^{-1} a$)ជាអនុគមន៍ច្រាសនៃ $\cot a$ ។

**សម្គាល់ៈ**

- $\cot u(x) = \cot v(x) \Leftrightarrow u(x) = v(x) + k\pi, \ k \in \mathbb{Z}$
- ក្នុងការដោះស្រាយសមីការ $\cot u(x) = \cot v(x)$

  យើងត្រូវឲ្យលក្ខខណ្ឌ

  $$\begin{cases} \sin u(x) \neq 0 \\ \sin v(x) \neq 0 \end{cases} \Leftrightarrow \begin{cases} u(x) \neq k\pi \\ v(x) \neq k\pi \end{cases}, k \in \mathbb{Z}$$

  ព្រោះ $\cot u(x) = \dfrac{\cos u(x)}{\sin u(x)}$ & $\cot v(x) = \dfrac{\cos v(x)}{\sin v(x)}$

លំហាត់គំរូទី១: ដោះស្រាយសមីការ $\cot x = \sqrt{3}$

**ចំណោះស្រាយ**

យើងមាន $\cot x = \sqrt{3}$

សមីការមានឫសកាលណា $x \neq k\pi \ , k \in \mathbb{Z}$

យើងបាន សមីការទៅជា $\cot x = \cot \dfrac{\pi}{6}$

$$\Rightarrow x = \frac{\pi}{6} + k\pi \ , k \in \mathbb{Z}$$

ដូចនេះ សមីការមានឫស $\boxed{x = \dfrac{\pi}{6} + k\pi \ , k \in \mathbb{Z}}$

លំហាត់គំរូទី២: ដោះស្រាយសមីការ $\cot x = -\frac{1}{\sqrt{3}}$

**ចំណោះស្រាយ**

យើងមាន សមីការ $\cot x = -\frac{1}{\sqrt{3}}$

សមីការមានឫស $x \neq k\pi \ , k \in \mathbb{Z}$

យើងបានសមីការទៅជា $\cot x = -\frac{\sqrt{3}}{3}$

$$\Leftrightarrow \cot x = \cot\frac{2\pi}{3}$$

$$\Rightarrow x = \frac{2\pi}{3} + k\pi \ , k \in \mathbb{Z}$$

ដូចនេះ សមីការមានឫស $\boxed{x = \frac{2\pi}{3} + k\pi \ , k \in \mathbb{Z}}$

លំហាត់គំរូទី៣: ដោះស្រាយសមីការ $\cot 3x = \sqrt{3}$

**ចំណោះស្រាយ**

យើងមានសមីការ $\cot 3x = \sqrt{3}$

សមីការមានឫសកាលណា $3x \neq k\pi \Rightarrow x \neq \frac{k\pi}{3} \ , k \in \mathbb{Z}$

ដោយ $\cot\frac{\pi}{6} = \sqrt{3}$

នោះយើងបានសមីការទៅជា $\cot 3x = \cot\frac{\pi}{6}$

$$3x = \frac{\pi}{6} + k\pi, k \in \mathbb{Z}$$

$$\Rightarrow x = \frac{\pi}{18} + \frac{k\pi}{3}, k \in \mathbb{Z}$$

លំហាត់គំរូទី៤: ដោះស្រាយសមីការ $\cot\left(\frac{x}{2}-3\right)=-1$

**ចំណោះស្រាយ**

សមីការមានន័យកាលណា $\frac{x}{2}-3\neq k\pi \Rightarrow x\neq 2k\pi+6\,,k\in\mathbb{Z}$

យើងបាន $\cot\left(\frac{x}{2}-3\right)=-1$

$$\Leftrightarrow \cot\left(\frac{x}{2}-3\right)=\cot\left(-\frac{\pi}{4}\right)$$

$$\Leftrightarrow \frac{x}{2}-3=-\frac{\pi}{4}+k\pi\ ,\ k\in\mathbb{Z}$$

$$\Leftrightarrow \frac{x}{2}=-\frac{\pi}{4}+3+k\pi\ \ ,\ k\in\mathbb{Z}$$

$$\Rightarrow x=2\left(-\frac{\pi}{4}+3+k\pi\right)=6-\frac{\pi}{2}+2k\pi\ \ ,\ k\in\mathbb{Z}$$

ដូចនេះ សមីការមានឫស $\boxed{x=6-\frac{\pi}{2}+2k\pi\ \ ,\ k\in\mathbb{Z}}$

លំហាត់គំរូទី៥: ដោះស្រាយសមីការ $3\cot x-\sqrt{3}=0$

**ចំណោះស្រាយ**

យើងមានសមីការ $3\cot x-\sqrt{3}=0$

សមីការមានឫសកាលណា $x\neq k\pi\ ,k\in\mathbb{Z}$

យើងបាន $3\cot x-\sqrt{3}=0$

$$\Leftrightarrow \cot x=\frac{\sqrt{3}}{3}$$

$$\Leftrightarrow \cot x=\cot\frac{\pi}{3}$$

$$\Rightarrow x=\frac{\pi}{3}+k\pi \ ,k\in\mathbb{Z}$$

ដូចនេះ សមីការមានឫស $\boxed{x=\frac{\pi}{3}+k\pi \ ,k\in\mathbb{Z}}$

លំហាត់គំរូទី៦: ដោះស្រាយសមីការ $\cot\left(2x+\frac{\pi}{3}\right)=2$

**ចំណោះស្រាយ**

សមីការមានឫសកាលណា

$$2x+\frac{\pi}{3}\neq k\pi \Leftrightarrow x\neq -\frac{\pi}{6}+\frac{k\pi}{2} \ ,k\in\mathbb{Z}$$

យើងបាន $\cot\left(2x+\frac{\pi}{3}\right)=2$ ឬ $\cot(2x+60^o)=2$

ដោយ $\cot(26.56)\approx 2$ នោះយើងបាន សមីការទៅជា

$$\cot(2x+60^o)=\cot(26.56^o)$$
$$\Leftrightarrow 2x+60^o=26.56^o+k\cdot 180^o \ ,k\in Z$$
$$\Leftrightarrow 2x=26.56^o-60^o+k\cdot 180^o=-33.44^o+k\cdot 180^o$$
$$\Rightarrow x=-16.72^o+k\cdot 90^o$$

ដូចនេះ សមីការមានឫស $\boxed{x=-16.72^o+k\cdot 90^o \ , k\in\mathbb{Z}}$

**វិធីបម្លាំងទៀត**

$$\cot\left(2x+\frac{\pi}{3}\right)=2$$
$$\Leftrightarrow 2x+\frac{\pi}{3}=arc\cot 2+k\pi \ ,k\in\mathbb{Z}$$
$$\Leftrightarrow 2x=-\frac{\pi}{3}+arc\cot 2+k\pi$$

$$\Rightarrow x=-\frac{\pi}{6}+\frac{1}{2}arc\cot 2+\frac{k\pi}{2}\quad k\in\mathbb{Z}$$

ដូចនេះ សមីការមានឫស $\boxed{x=-\frac{\pi}{6}+\frac{1}{2}arc\cot 2+\frac{k\pi}{2},k\in\mathbb{Z}}$

## របៀបកំណត់ចំនួនចុងធ្នូ(ចម្លើយ)

- ☞ បើសមីការមានចម្លើយ $x=\alpha+\frac{2k\pi}{n}$ នោះមានចុងធ្នូចំនួន $n$ ។
- ☞ បើសមីការមានចម្លើយ $x=\alpha+n\cdot k\pi$ (*) នោះ ៖
  - បើ $n$ ជាចំនួនគត់គូ នោះ (*) មានចុងធ្នូចំនួន 1 ។
  - បើ $n$ ជាចំនួនគត់សេស នោះ (*) មានចុងធ្នូចំនួន 2 ។
- ☞ បើសមីការមានចម្លើយ $\left[\begin{array}{l}x=\alpha+\frac{2k\pi}{n}\\ x=\beta+\frac{2k\pi}{m}\end{array}\right.$ នោះ
  - មានចុងធ្នូចំនួន $(m+n)$ បើ $\alpha\neq\beta$
  - មានចុងធ្នូចំនួន $(m+n-1)$ បើ $\alpha=\beta$ ។

លំហាត់គំរូទី៧: ដោះស្រាយសមីការ $2\sin 2x-2\cos x=\sqrt{3}\sin x-\sqrt{3}$ រួចកំណត់ចំនួនចុងធ្នូ និងដៅចំណុចចុងធ្នូលើរង្វង់ត្រីកោណមាត្រ។

### ដំណោះស្រាយ

យើងមានសមីការ $2\sin 2x-2\cos x=2\sqrt{3}\sin x-\sqrt{3}$

$$\Leftrightarrow 4\sin x\cos x-2\cos x=\sqrt{3}\left(2\sin x-1\right)$$
$$\Leftrightarrow 2\cos x\left(2\sin x-1\right)-\sqrt{3}\left(2\sin x-1\right)=0$$
$$\Leftrightarrow \left(2\sin x-1\right)\left(2\cos x-\sqrt{3}\right)=0$$

$$\Leftrightarrow \left[\begin{array}{l} 2\sin x-1=0 \\ 2\cos x-\sqrt{3}=0 \end{array}\right.$$

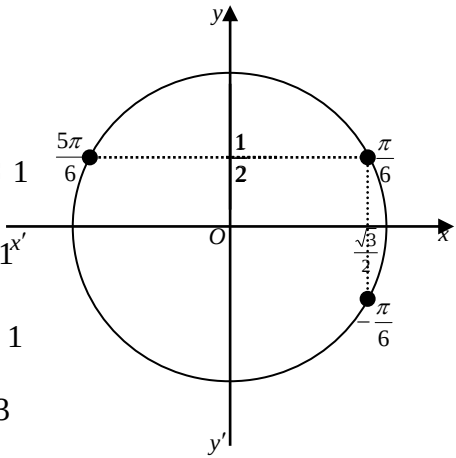
$$\Leftrightarrow \left[\begin{array}{l} \sin x=\dfrac{1}{2}=\sin\dfrac{\pi}{6} \\ \cos x=\dfrac{\sqrt{3}}{2}=\cos\dfrac{\pi}{6} \end{array}\right.$$

$$\Rightarrow \left[\begin{array}{l} \left[\begin{array}{l} x=\dfrac{\pi}{6}+2k\pi \\ x=\pi-\dfrac{\pi}{6}+2k\pi=\dfrac{5\pi}{6}+2k\pi \end{array}\right. \\ x=\pm\dfrac{\pi}{6}+2k\pi \end{array}\right. , k\in\mathbb{Z}$$

ដូចនេះ សមីការមានចម្លើយ

$$x=-\frac{\pi}{6}+2k\pi, x=\frac{\pi}{6}+2k\pi, x=\frac{5\pi}{6}+2k\pi \ , k\in Z$$

កំណត់ចំនួនចុងធ្នូ




- $x=-\frac{\pi}{6}+2k\pi$ មានចុងធ្នូចំនួន 1
- $x=\frac{\pi}{6}+2k\pi$ មានចុងធ្នូចំនួន 1
- $x=\frac{5\pi}{6}+2k\pi$ មានចុងធ្នូចំនួន 1

សរុបចុងធ្នូទាំងអស់មានចំនួន 3

## ៥. សមីការទម្រង់ $a\cos x + b\sin x = c$

លក្ខខណ្ឌដើម្បីឲ្យសមីការមានឫសជាចំនួនពិត គឺ $a^2 + b^2 \geq c^2$ ។

**របៀបដោះស្រាយ**

**របៀបទី១:**

សមីការ $a\cos x + b\sin x = c$ (*)

តាង $r = \sqrt{a^2 + b^2}$

ចែកអង្គទាំងពីរនៃ (*) នឹង $r = \sqrt{a^2 + b^2}$ យើងបាន

$$\frac{a}{r}\cos x + \frac{b}{r}\sin x = \frac{c}{r} \quad (i)$$

ដោយ $\left(\frac{a}{r}\right)^2 + \left(\frac{b}{r}\right)^2 = \frac{a^2}{a^2+b^2} + \frac{b^2}{a^2+b^2} = 1$

នោះតាង $\left\{\begin{array}{l} \cos\alpha = \frac{a}{r} \\ \sin\alpha = \frac{b}{r} \end{array}\right. \Rightarrow \alpha = \arctan\left(\frac{b}{a}\right)$

នោះយើងបាន $(i)$ ទៅជា $\cos\alpha\cos x + \sin\alpha\sin x = \frac{c}{r}$

$$\Leftrightarrow \boxed{\cos(x - \alpha) = \frac{c}{r}}$$

តាម $(i)$ បើតាង $\frac{a}{r} = \sin\alpha$ និង $\frac{b}{r} = \cos\alpha$

នោះ $(i)$ ទៅជា $\sin\alpha\cos x + \cos\alpha\sin x = \frac{c}{r}$

$$\Leftrightarrow \boxed{\sin(x + \alpha) = \frac{c}{r}}$$

**របៀបទី២:**

សមីការ $a\cos x + b\sin x = c$ (*)

$\Leftrightarrow \cos x+\frac{b}{a}\sin x=\frac{c}{a}$

តាង $\tan\alpha=\frac{b}{a}=\frac{\sin\alpha}{\cos\alpha}$ នោះយើងបាន

$\cos x+\frac{\sin\alpha}{\cos\alpha}\sin\alpha=\frac{c}{a}$

$\cos x\cos\alpha+\sin x\sin\alpha=\frac{c}{a}\cos\alpha$

$\boxed{\cos(x-\alpha)=\frac{c}{a}\cos\alpha}$ ដែល $\tan\alpha=\frac{b}{a}$

## វិធីបទទី៣:

សមីការ $a\cos x+b\sin x=c$ (*)

$\Leftrightarrow \cos x+\frac{b}{a}\sin x=\frac{c}{a}$

តាង $\cot\alpha=\frac{b}{a}=\frac{\cos\alpha}{\sin\alpha}$

យើងបាន $\cos x+\frac{\cos\alpha}{\sin\alpha}\sin x=\frac{c}{a}$

$\Leftrightarrow \sin\alpha\cos x+\sin x\cos\alpha=\frac{c}{a}\sin\alpha$

$\boxed{\sin(x+\alpha)=\frac{c}{a}\sin\alpha}$ ដែល $\cot\alpha=\frac{b}{a}$ ឬ $\tan\alpha=\frac{a}{b}$

## វិធីបទទី៤:

សមីការ $a\cos x+b\sin x=c$ (*)

តាង $t=\tan\frac{x}{2}$ ( បើ $x\neq\pi+2k\pi, k\in\mathbb{Z}$ ) នោះ $\begin{cases}\cos x=\frac{1-t^2}{1+t^2}\\ \sin x=\frac{2t}{1+t^2}\end{cases}$

យើងបានសមីការទៅជា $a\frac{1-t^2}{1+t^2}+b\frac{2t}{1+t^2}=c$

$$\Leftrightarrow a(1-t^2)+2bt=c(1+t^2)$$

$$\boxed{(c+a)t^2-2bt+c-a=0}$$

**Note:** ដើម្បីដោះស្រាយតាមវិធីនេះយើងត្រូវផ្សេងផ្ទាត់ថាតើ $x=\pi+2k\pi$ ជាឫសរបស់ សមីការនេះ ដែរឬទេ ?

**វិធីទី៥:**

ដោយ $\cos^2 x+\sin^2 x=1 \Rightarrow \sin x=\pm\sqrt{1-\cos^2 x}$

នោះសមីការសមមូលនឹង $a\cos x\pm b\sqrt{1-\cos^2 x}=c$

$$a\cos x-c=\mp b\sqrt{1-\cos^2 x}$$

$$a^2\cos^2 x-2ac\cos x+c^2=b^2\left(1-\cos^2 x\right)$$

$$\boxed{\left(a^2+b^2\right)\cos^2 x-2ac\cos x+c^2-b^2=0}$$

ជាសមីការដឺក្រទី២នៃ $\cos x$

**វិធីទី៦:**

$$a\cos x+b\sin x=c$$

$$\Leftrightarrow \left(a\cos x+b\sin x\right)^2=c^2$$

$$\Leftrightarrow a^2\cos^2 x+2ab\sin x\cos x+b^2\sin^2 x=c^2$$

ចែកអង្គទាំងពីរនឹង $\cos^2 x\neq 0$ យើងបាន

$$a^2+2ab\tan x+b^2\tan^2 x=\frac{c^2}{\cos^2 x}$$

$$a^2+2ab\tan x+b^2\tan^2 x=c^2\left(1+\tan^2 x\right)$$

$$\boxed{\left(b^2-c^2\right)\tan^2 x+2ab\tan x+a^2-c^2=0}$$

លំហាត់គំរូទី១: ដោះស្រាយសមីការ $\sin x+\sqrt{3}\cos x=0$

## ចំណោះស្រាយ

ដោយ $1^2+\sqrt{3}^2>0^2$ នោះ សមីការមានឫសជាចំនួនពិត

### វិធីទី១

យើងមានសមីការ $\sin x+\sqrt{3}\cos x=0$ (*)

$$r=\sqrt{1^2+\left(\sqrt{3}\right)^2}=\sqrt{4}=2$$

$$\left.\begin{aligned}\cos\alpha&=\frac{a}{r}=\frac{1}{2}\\ \sin\alpha&=\frac{b}{r}=\frac{\sqrt{3}}{2}\end{aligned}\right| \Rightarrow \alpha=\frac{\pi}{6}$$

យើងបាន សមីការ (*) ទៅជា $\cos\left(x-\frac{\pi}{6}\right)=0$

$$\Leftrightarrow x-\frac{\pi}{6}=\frac{\pi}{2}+k\pi\ ,k\in\mathbb{Z}$$

$$\Rightarrow x=\frac{\pi}{2}+\frac{\pi}{6}+k\pi=\frac{2\pi}{3}+k\pi\ ,k\in\mathbb{Z}$$

ដូចនេះ សមីការមានឫស $\boxed{x=\frac{2\pi}{3}+k\pi\ ,k\in\mathbb{Z}}$

### វិធីទី២

យើងមានសមីការ $\sin x+\sqrt{3}\cos x=0$ (*)

$\Leftrightarrow \frac{1}{2}\sin x+\frac{\sqrt{3}}{2}\cos x=0$ (ចែកអង្គទាំងពីរនឹង 2)

$\Leftrightarrow \cos\frac{\pi}{3}\sin x+\sin\frac{\pi}{3}\cos x=0$

$\Leftrightarrow \sin\left(x+\frac{\pi}{3}\right)=0$

$$\Leftrightarrow x+\frac{\pi}{3}=k\pi \ , k\in\mathbb{Z}$$

$$\Rightarrow x=-\frac{\pi}{3}+k\pi \ , k\in\mathbb{Z}$$

ដូចនេះ សមីការមានឫស $\boxed{x=-\frac{\pi}{3}+k\pi \ , k\in\mathbb{Z}}$

**វិធីបីទី៣**

យើងមានសមីការ $\sin x+\sqrt{3}\cos x=0$ (*)

តាង $\tan\alpha=\sqrt{3} \Rightarrow \alpha=\frac{\pi}{3}$

យើងបានសមីការ (*) ទៅជា $\sin x+\tan\frac{\pi}{3}\cos x=0$

$$\Leftrightarrow \sin x\cos\frac{\pi}{3}+\sin\frac{\pi}{3}\cos x=0$$

$$\Leftrightarrow \sin\left(x+\frac{\pi}{3}\right)=0$$

$$\Leftrightarrow x+\frac{\pi}{3}=k\pi \ ,k\in\mathbb{Z}$$

$$\Rightarrow x=-\frac{\pi}{3}+k\pi \ ,k\in\mathbb{Z}$$

**វិធីបីទី៤**

តាង $t=\tan\frac{x}{2} \Rightarrow \begin{cases}\cos x=\frac{1-t^2}{1+t^2}\\ \sin x=\frac{2t}{1+t^2}\end{cases}$

យើងបានសមីកា (*) ទៅជា $\frac{2t}{1+t^2}+\sqrt{3}\frac{1-t^2}{1+t^2}=0$

$$\Leftrightarrow 2t+\sqrt{3}\left(1-t^2\right)=0$$

$$\Leftrightarrow -\sqrt{3}t^2+2t+\sqrt{3}=0$$

$$\Delta'=1+3=4=2^2$$

$$x_1=\frac{-1-2}{-\sqrt{3}}=\sqrt{3}$$

$$x_2=\frac{-1+2}{-\sqrt{3}}=-\frac{\sqrt{3}}{3}$$

$$t=\sqrt{3}\Leftrightarrow \tan\frac{x}{2}=\sqrt{3}=\tan\frac{\pi}{3}$$

$$\Leftrightarrow \frac{x}{2}=\frac{\pi}{3}+k\pi\ ,k\in\mathbb{Z}$$

$$\Rightarrow x=\frac{2\pi}{3}+2k\pi\ ,k\in\mathbb{Z}$$

$$t=-\frac{\sqrt{3}}{3}\Leftrightarrow \tan\frac{x}{2}=-\frac{\sqrt{3}}{3}=\tan\left(-\frac{\pi}{6}\right)$$

$$\Leftrightarrow \frac{x}{2}=-\frac{\pi}{6}+k\pi\ ,k\in\mathbb{Z}$$

$$\Rightarrow x=-\frac{\pi}{3}+2k\pi\ ,k\in\mathbb{Z}$$

ដូចនេះ សមីការមានចម្លើយ $\boxed{x=\frac{2\pi}{3}+2k\pi\ ,\ x=-\frac{\pi}{3}+2k\pi\ ,\in\mathbb{Z}}$

**វរៀបទី៨**

$$\sin x+\sqrt{3}\cos x=0$$

$$\sin x=-\sqrt{3}\cos x$$

$$\sin^2 x=3\cos^2 x$$

$3\cos^2 x-\sin^2 x=0$

$4\cos^2 x-1=0$

$4\cos^2 x-1=0$

$(2\cos x-1)(2\cos x+1)=0$

$$\left[\begin{array}{l}2\cos x-1=0\\2\cos x+1=0\end{array}\right.$$

$$\Leftrightarrow \left[\begin{array}{l}\cos x=1/2\\\cos x=-1/2\end{array}\right.$$

$$\Leftrightarrow \left[\begin{array}{l}x=\pm\dfrac{\pi}{3}+2k\pi\\x=\pm\dfrac{2\pi}{3}+2k\pi\end{array}\right. ,k\in\mathbb{Z}$$

តែសមីការ (*)មានចម្លើយកាលណា $\begin{cases}\cos x>0\\\sin x<0\end{cases}$ ឬ $\begin{cases}\cos x<0\\\sin x>0\end{cases}$

$$\Rightarrow x\in\left(\frac{\pi}{2},\pi\right)\cup\left(\frac{3\pi}{2},2\pi\right)$$

ដូចនេះ សមីការមានចម្លើយ $\boxed{x=-\frac{\pi}{3}+2k\pi, x=\frac{2\pi}{3}+2k\pi, k\in\mathbb{Z}}$

**វិធីទី២**

យើងមានសមីការ $\sin x+\sqrt{3}\cos x=0$ (*)

$$\Leftrightarrow \sin x=-\sqrt{3}\cos x$$

$$\Leftrightarrow \frac{\sin x}{\cos x}=-\sqrt{3}$$

$$\Leftrightarrow \tan x=\tan\frac{2\pi}{3}$$

$$\Rightarrow x=\frac{2\pi}{3}+k\pi \ ,k\in\mathbb{Z}$$

ដូចនេះ សមីការមានឫស $\boxed{x=\frac{2\pi}{3}+k\pi \ ,k\in\mathbb{Z}}$

លំហាត់គំរូទី២: ដោះស្រាយសមីការ $\sin x+7\cos x=5$

**ដំណោះស្រាយ**

ដោយ $1^2+7^2>5^2$ នោះសមីការមានឫសជាចំនួនពិត

**វិធីទី១**

តាង $\tan\alpha=7=\frac{\sin\alpha}{\cos\alpha} \Rightarrow \alpha=\arctan 7$

យើងបាន $\sin x+\frac{\sin\alpha}{\cos\alpha}\cos\alpha=5$

$\sin x\cos\alpha+\sin\alpha\cos x=5\cos\alpha$

$\sin(x+\alpha)=5\cos\alpha \quad (*)$

ដោយ $\tan\alpha=7 \Rightarrow \alpha\approx 81.87^o$

យើងបាន $(*)\Leftrightarrow \sin\left(x+81.87^o\right)=5\cos(81.87^o)=0.7071$

$\sin(x+81.87^o)=\sin 45^o$

$$\begin{cases} x+81.87^o=45^o+k\cdot 360^o \\ x+81.87^o=180^o-45^o+k\cdot 360^o \end{cases}, k\in\mathbb{Z}$$

$$\begin{cases} x=-36.87^o+k\cdot 360^o \\ x=53.13^o+k\cdot 360^o \end{cases}, k\in Z$$

ដូចនេះ សមីការមានឫស

$$\boxed{x=-36.87^o+k\cdot 360^o \ , \ x=53.13^o+k\cdot 360^o}$$

## វិធីទី២

យើងមាន សមីការ $\sin x+7\cos x=5$

$$r=\sqrt{1^2+7^2}=\sqrt{50}=5\sqrt{2}$$

$$\left.\begin{array}{l}\cos\alpha=\dfrac{1}{5\sqrt{2}} \Leftrightarrow 5\sqrt{2}\cos\alpha=1\\ \sin\alpha=\dfrac{7}{5\sqrt{2}} \Leftrightarrow 5\sqrt{2}\sin\alpha=1\end{array}\right| \Leftrightarrow \tan\alpha=7 \Rightarrow \alpha=\arctan 7$$

យើងបានសមីការ ទៅជា $5\sqrt{2}(\cos\alpha\sin x+\sin\alpha\cos x)=5$

$$\sin x\cos\alpha+\sin\alpha\cos x=\frac{\sqrt{2}}{2}$$

$$\sin(x+\alpha)=\sin\frac{\pi}{4}$$

$$\left[\begin{array}{l}x+\alpha=\dfrac{\pi}{4}+2k\pi\\ x+\alpha=\pi-\dfrac{\pi}{4}+2k\pi\end{array}\right. , k\in\mathbb{Z}$$

$$\left[\begin{array}{l}x=\dfrac{\pi}{4}-\alpha+2k\pi\\ x=\dfrac{3\pi}{4}-\alpha+2k\pi\end{array}\right. , k\in\mathbb{Z}$$

ដូចនេះ សមីកាមានឫស

$$\boxed{x=\frac{\pi}{4}-\alpha+2k\pi \ , \ x=\frac{3\pi}{4}-\alpha+2k\pi \ , k\in\mathbb{Z}}$$

ដែល $\tan\alpha=7 \Leftrightarrow \alpha=\arctan 7$

លំហាត់គំរូទី៣: ដោះស្រាយសមីការ $2\sin x-3\cos x=3$

## ចំណោះស្រាយ

ដោយ $2^2+(-3)^2>3^2$ នោះសមីការមានឫសជាចំនួនពិត

**វិធីទី១**

យើងមានសមីការ $2\sin x-3\cos x=3$ (*)

$$r=\sqrt{2^2+(-3)^2}=\sqrt{13}$$

$$\cos\alpha=\frac{2}{\sqrt{13}} \ \& \ \sin\alpha=\frac{3}{\sqrt{13}}$$

នោះ $\tan\alpha=\frac{3}{2}\Leftrightarrow\alpha=\arctan\left(\frac{3}{2}\right)$

នោះតាម (*) ចែកអង្គទាំងពីរ នឹង $r=\sqrt{13}$ យើងបាន

$$\frac{2}{\sqrt{13}}\sin x-\frac{3}{\sqrt{13}}\cos x=\frac{3}{\sqrt{13}}$$

$$\cos\alpha\sin x-\sin\alpha\cos x=\sin\alpha$$

$$\sin(x-\alpha)=\sin\alpha$$

$$\begin{bmatrix}x-\alpha=\alpha+2k\pi\\x-\alpha=\pi-\alpha+2k\pi\end{bmatrix},k\in\mathbb{Z}$$

$$\begin{bmatrix}x=2\alpha+2k\pi\\x=\pi+2k\pi\end{bmatrix},k\in\mathbb{Z}$$

ដូចនេះ សមីការមានឫស $\boxed{x=2\alpha+2k\pi,x=\pi+2k\pi,k\in\mathbb{Z}}$

ដែល $\tan\alpha=\frac{3}{2}\Leftrightarrow\alpha=\arctan\left(\frac{3}{2}\right)$

**វិធីទី២**

យើងមានសមីការ $2\sin x-3\cos x=3$ (*)

$$\Leftrightarrow\frac{2}{3}\sin x-\cos x=1$$

តាង $\tan\alpha=\frac{2}{3}=\frac{\sin\alpha}{\cos\alpha} \Leftrightarrow \alpha=\arctan\left(\frac{2}{3}\right)$

យើងបានសមីការទៅជា $\frac{\sin\alpha}{\cos\alpha}\sin x-\cos x=1$

$\sin\alpha\sin x-\cos\alpha\cos x=\cos\alpha$

$-(\cos x\cos\alpha-\sin x\sin\alpha)=\cos\alpha$

$\cos(x+\alpha)=-\cos\alpha=\cos(\pi+\alpha)$

$x+\alpha=\pm(\pi+\alpha)+2k\pi\ ,\ k\in\mathbb{Z}$

$x=\pi+2k\pi\ ,\ x=-\pi-2\alpha+2k\pi\ ,\ k\in\mathbb{Z}$

ដូចនេះ សមីការមានឫស $\boxed{x=\pi+2k\pi, x=-\pi-2\alpha+2k\pi, k\in\mathbb{Z}}$

ដែល $\tan\alpha=\frac{2}{3} \Leftrightarrow \alpha=\arctan\left(\frac{2}{3}\right)$

## វរៀបទី៣

យើងមានសមីការ $2\sin x-3\cos x=3$ (*)

➢ បើ $x\neq\pi+2k\pi$ តាង $t=\tan\frac{x}{2}$

នោះ $\cos x=\frac{1-t^2}{1+t^2}$ និង $\sin x=\frac{2t}{1+t^2}$

យើងបាន សមីការទៅជា $2\left(\frac{2t}{1+t^2}\right)-3\left(\frac{1-t^2}{1+t^2}\right)=3$

$4t-3(1-t^2)=3(1+t^2)$

$4t=6$

$t=\frac{6}{4}=\frac{3}{2}$

ដោយ $t=\tan\frac{x}{2}$ នោះយើងបាន $\tan\frac{x}{2}=\frac{3}{2}$

$\frac{x}{2}=\arctan\left(\frac{3}{2}\right)+k\pi\ ,\ k\in\mathbb{Z}$

$$x = 2\arctan\left(\frac{3}{2}\right) + 2k\pi \ , \ k \in \mathbb{Z}$$

➢ បើ $x = \pi + 2k\pi$ នោះយើងបាន $2\sin(\pi + 2k\pi) - 3\cos(\pi + 2k\pi) = 3$

$$2\times 0 - 3\times(-1) = 3$$
$$3 = 3 \text{ (ផ្ទៀងផ្ទាត់)}$$

ដូចនេះ សមីការមានឫស

$$\boxed{x = 2\arctan\left(\frac{3}{2}\right) + 2k\pi \ , \ x = \pi + 2k\pi \ , \ k \in \mathbb{Z}}$$

លំហាត់គំរូទី៣: ដោះស្រាយសមីការ $\cos 2x - \sin 2x = -1$

**ចំណោះស្រាយ**

ដោយ $1^2 + 1^2 > (-1)^2$ នោះសមីការមានឫសជាចំនួនពិត

**វិធីទី១**

យើងមាន សមីការ $\cos 2x - \sin 2x = -1$

$$r = \sqrt{1^2 + 1^2} = \sqrt{2}$$

$$\left.\begin{aligned}\cos\alpha &= \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}\\ \sin\alpha &= \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}\end{aligned}\right| \Rightarrow \alpha = \frac{\pi}{4}$$

យើងបាន $\cos 2x - \sin 2x = -1$

$$\Leftrightarrow \frac{\sqrt{2}}{2}\cos 2x - \frac{\sqrt{2}}{2}\sin 2x = -\frac{\sqrt{2}}{2}$$

$$\Leftrightarrow \cos\frac{\pi}{4}\cos 2x - \sin\frac{\pi}{4}\sin 2x = \cos\frac{3\pi}{4}$$

$$\Leftrightarrow \cos\left(2x+\frac{\pi}{4}\right)=\cos\frac{3\pi}{4}$$

$$\Leftrightarrow 2x+\frac{\pi}{4}=\pm\frac{3\pi}{4}+2k\pi \quad , k\in\mathbb{Z}$$

$$\Leftrightarrow \begin{cases} 2x+\frac{\pi}{4}=\frac{3\pi}{4}+2k\pi \\ 2x+\frac{\pi}{4}=-\frac{3\pi}{4}+2k\pi \end{cases}$$

$$\Leftrightarrow \begin{cases} 2x=\frac{\pi}{2}+2k\pi \\ 2x=-\pi+2k\pi \end{cases}$$

$$\Rightarrow \begin{cases} x=\frac{\pi}{4}+k\pi \\ x=-\frac{\pi}{2}+k\pi \end{cases}$$

ដូចនេះ សមីការមានឫស $\boxed{x=\frac{\pi}{4}+k\pi \ , \ x=-\frac{\pi}{2}+k\pi}$

## វិធីទី២

យើងមាន សមីការ $\cos 2x-\sin 2x=-1$

$$\Leftrightarrow \cos 2x+1-\sin 2x=0$$

$$\Leftrightarrow 2\cos^2 x-2\sin x\cos x=0$$

$$\Leftrightarrow 2\cos x(\cos x-\sin x)=0$$

$$\Leftrightarrow \begin{cases} 2\cos x=0 \\ \cos x-\sin x=0 \end{cases}$$

$$\Leftrightarrow \begin{cases} \cos x=0 \\ \cos x=\sin x=\cos\left(\frac{\pi}{2}-x\right) \end{cases}$$

$$\Leftrightarrow \begin{bmatrix} x=\dfrac{\pi}{2}+k\pi \\ x=\pm\left(\dfrac{\pi}{2}-x\right)+2k\pi \end{bmatrix}, k\in\mathbb{Z}$$

$$\begin{bmatrix} x=\dfrac{\pi}{2}+k\pi \\ x=\dfrac{\pi}{4}+k\pi \end{bmatrix}$$

ដូចនេះ សមីការមានឫស $\boxed{x=\dfrac{\pi}{2}+k\pi \ , \ x=\dfrac{\pi}{4}+k\pi \ , k\in\mathbb{Z}}$

## វិធីទី៣

តាង $t=\tan x$ នោះ $\cos x=\dfrac{1-t^2}{1+t^2}$ & $\sin x=\dfrac{2t}{1+t^2}$

យើងបានសមីការទៅជា $\dfrac{1-t^2}{1+t^2}-\dfrac{2t}{1+t^2}=-1$

$$\Leftrightarrow 1-t^2-2t=-1-t^2$$
$$\Leftrightarrow -2t=-2$$
$$\Rightarrow t=1$$

$t=1 \Leftrightarrow \tan x=1=\tan\dfrac{\pi}{4}$

$$\Rightarrow x=\frac{\pi}{4}+k\pi \ , k\in\mathbb{Z}$$

បើ $x=\dfrac{\pi}{2}+k\pi$ នោះយើងបាន

$$\cos 2\left(\frac{\pi}{2}+k\pi\right)-\sin 2\left(\frac{\pi}{2}+k\pi\right)=-1$$
$$\cos\pi-\sin\pi=-1$$
$$-1-0=-1 \quad \text{ផ្ទៀងផ្ទាត់}$$

ដូចនេះ សមីការមានឫស $x=\frac{\pi}{4}+k\pi\ ,\ x=\frac{\pi}{2}+k\pi\ ,k\in\mathbb{Z}$

លំហាត់គំរូទី៤: ដោះស្រាយសមីការ $\cos x+\sqrt{3}\sin x=1$

**ដំណោះស្រាយ**

ដោយ $1^2+\sqrt{3}^2>1^2$ នោះសមីការមានឫសជាចំនួនពិត

**វិធីទី១**

យើងមានសមីការ $\cos x+\sqrt{3}\sin x=1$

$$r=\sqrt{1^2+\sqrt{3}^2}=\sqrt{4}=2$$

$$\left.\begin{array}{l}\cos\alpha=\frac{1}{2}\\ \sin\alpha=\frac{\sqrt{3}}{2}\end{array}\right|\Rightarrow\alpha=\frac{\pi}{3}$$

គេបានសមីការទៅជា $\frac{1}{2}\cos x+\frac{\sqrt{3}}{2}\sin x=\frac{1}{2}$

$$\cos\frac{\pi}{3}\cos x+\sin\frac{\pi}{3}\sin x=\cos\frac{\pi}{3}$$

$$\cos\left(x-\frac{\pi}{3}\right)=\cos\frac{\pi}{3}$$

$$\left[\begin{array}{l}x-\frac{\pi}{3}=\frac{\pi}{3}+2k\pi\\ x-\frac{\pi}{3}=-\frac{\pi}{3}+2k\pi\end{array}\right.\ ,k\in\mathbb{Z}$$

$$\left[\begin{array}{l}x=\frac{2\pi}{3}+2k\pi\\ x=2k\pi\end{array}\right.\ ,k\in\mathbb{Z}$$

ដូចនេះ សមីការមានឫស $\boxed{x=\frac{2\pi}{3}+2k\pi \ , \ x=2k\pi \ , k\in\mathbb{Z}}$

## វិធីទី២

តាង $\tan\alpha=\sqrt{3} \Leftrightarrow \alpha=\frac{\pi}{3}$

នោះសមីការ $\cos x+\sqrt{3}\sin x=1$

$$\cos x+\tan\alpha\sin x=1$$

$$\cos x+\frac{\sin\alpha}{\cos\alpha}\sin x=1$$

$$\cos x\cos\alpha+\sin x\sin\alpha=\cos\alpha$$

$$\cos(x-\alpha)=\cos\alpha$$

$$\begin{cases} x-\alpha=\alpha+2k\pi \\ x-\alpha=-\alpha+2k\pi \end{cases} , k\in\mathbb{Z}$$

$$\begin{cases} x=2\alpha+2k\pi=\frac{2\pi}{3}+2k\pi \\ x=2k\pi \end{cases} , k\in\mathbb{Z}$$

ដូចនេះ សមីការមានឫស $\boxed{x=\frac{2\pi}{3}+2k\pi \ , \ x=2k\pi \ , \ k\in\mathbb{Z}}$

## វិធីទី៣

យើងមាន សមីការ $\cos x+\sqrt{3}\sin x=1$

$$\Leftrightarrow \sqrt{3}\sin x+\cos x-1=0$$

$$\Leftrightarrow 2\sqrt{3}\sin\frac{x}{2}\cos\frac{x}{2}-2\sin^2\frac{x}{2}=0$$

$$\Leftrightarrow 2\sin\frac{x}{2}\left(\sqrt{3}\cos\frac{x}{2}-\sin\frac{x}{2}\right)=0$$

$$\Leftrightarrow \begin{cases} 2\sin\dfrac{x}{2}=0 \\ \sqrt{3}\cos\dfrac{x}{2}-\sin\dfrac{x}{2}=0 \end{cases}$$

$$\Leftrightarrow \begin{cases} \sin\dfrac{x}{2}=0 \\ \tan\dfrac{x}{2}=\sqrt{3} \end{cases}$$

$$\Leftrightarrow \begin{cases} \dfrac{x}{2}=k\pi \\ \dfrac{x}{2}=\dfrac{\pi}{3}+k\pi \end{cases} , k\in\mathbb{Z}$$

$$\Leftrightarrow \begin{cases} x=2k\pi \\ x=\dfrac{2\pi}{3}+2k\pi \end{cases} , k\in\mathbb{Z}$$

ដូចនេះ សមីការមានឫស $\boxed{x=2k\pi \ , \ x=\dfrac{2\pi}{3}+2k\pi \ , k\in\mathbb{Z}}$

**វរៀបទី៨**

យើងមានសមីការ $\cos x+\sqrt{3}\sin x=1$

តាង $t=\tan\dfrac{x}{2}$ នោះ $\cos x=\dfrac{1-t^2}{1+t^2}$ & $\sin x=\dfrac{2t}{1+t^2}$

យើងបានសមីការទៅជា $\dfrac{1-t^2}{1+t^2}+\sqrt{3}\dfrac{2t}{1+t^2}=1$

$$\Leftrightarrow 1-t^2+2\sqrt{3}t=1+t^2$$

$$\Leftrightarrow 2t^2-2\sqrt{3}t=0$$

$$\Leftrightarrow 2t\left(t-\sqrt{3}\right)=0$$

$$\Leftrightarrow \begin{bmatrix} 2t = 0 \\ t - \sqrt{3} = 0 \end{bmatrix}$$

$$\Leftrightarrow \begin{bmatrix} t = 0 \\ t = \sqrt{3} \end{bmatrix}$$

$$\Leftrightarrow \begin{bmatrix} \tan\frac{x}{2} = 0 \\ \tan\frac{x}{2} = \sqrt{3} \end{bmatrix}$$

$$\Leftrightarrow \begin{bmatrix} \frac{x}{2} = k\pi \\ \frac{x}{2} = \frac{\pi}{3} + k\pi \end{bmatrix}, k \in \mathbb{Z}$$

$$\Leftrightarrow \begin{bmatrix} x = 2k\pi \\ x = \frac{2\pi}{3} + 2k\pi \end{bmatrix}, k \in \mathbb{Z}$$

បើ $x = \pi + 2k\pi$ នោះ $\cos(\pi + 2k\pi) + \sqrt{3}\sin(\pi + 2k\pi) = 1$

$\Leftrightarrow \cos\pi + \sqrt{3}\sin\pi = 1$

$\Leftrightarrow -1 + 0 = 1$ មិនផ្ទៀងផ្ទាត់

ដូចនេះ សមីការមានឫស $\boxed{x = 2k\pi \text{ , } x = \frac{2\pi}{3} + 2k\pi \text{ , } k \in \mathbb{Z}}$

លំហាត់គំរូទី៥: ដោះស្រាយសមីការ $\cos x - \sqrt{3}\sin x = 3$ (*)

**ចំណោះស្រាយ**

យើងមានសមីការ $\cos x - \sqrt{3}\sin x = 3$ (*)

សមីការ $a\cos x + b\sin x = c$ មានឫសកាលណា $a^2 + b^2 > c^2$

ដោយ $a^2 + b^2 = 1^2 + (-\sqrt{3})^2 = 4$ ហើយ $c^2 = 3^2 = 9$

នោះ $a^2+b^2<c^2$

ដូចនេះ | សមីការ (*) គ្មានឫស |

**វិធីទី២**

យើងមានសមីការ $\cos x-\sqrt{3}\sin x=3$ (*)

$r=\sqrt{1^2+(-\sqrt{3})^2}=2$

ចែកអង្គទាំងពីរនៃ (*) នឹង $r=2$ យើងបាន

$$\frac{1}{2}\cos x-\frac{\sqrt{3}}{2}\sin x=\frac{3}{2}$$

$$\cos\frac{\pi}{3}\cos x-\sin\frac{\pi}{3}\sin x=\frac{3}{2}$$

$$\cos\left(x+\frac{\pi}{3}\right)=\frac{3}{2}$$

ដោយ $\forall x\in\mathbb{R}$ , $-1\le\cos\left(x+\frac{\pi}{3}\right)\le 1$

នោះ សមីការ $\cos\left(x+\frac{\pi}{3}\right)=\frac{3}{2}>1$ គ្មានឫសជាចំនួនពិត

ដូចនេះ | សមីការ (*) គ្មានឫស |

## ៦. សមីការដឺក្រេទី២នៃអនុគមន៍ត្រីកោណមាត្រ

**សមីការរាង ៖**

**ក.** $a\cos^2x+b\cos x+c=0$

**ខ.** $a\sin^2x+b\sin x+c=0$

**គ.** $a\tan^2x+b\tan x+c=0$

**ឃ.** $a\cot^2x+b\cot x+c=0$

## របៀបដោះស្រាយ

**ក.** $a\cos^2x+b\cos x+c=0$ (1)

តាង $t=\cos x$ ដែល $-1\le t\le 1$ ឬ $|t|\le 1$

យើងបានសមីការ (1) ទៅជា $at^2+bt+c=0$

ដោះស្រាយរកតម្លៃ $t$

- បើ $-1\le t\le 1$ នោះសមីការ (1) មានឫសជាចំនួនពិត
- បើ $t>1$ នោះសមីការ (1) គ្មានឫសជាចំនួនពិត

**ខ.** $a\sin^2x+b\sin x+c=0$

តាង $t=\sin x$ ដែល $-1\le t\le 1$ ឬ $|t|\le 1$

យើងបានសមីការ (2) ទៅជា $at^2+bt+c=0$

ដោះស្រាយរកតម្លៃ $t$

- បើ $-1\le t\le 1$ នោះសមីការ (2) មានឫសជាចំនួនពិត
- បើ $t>1$ នោះសមីការ (2) គ្មានឫសជាចំនួនពិត

**គ.** $a\tan^2x+b\tan x+c=0$ (3)

តាង $t=\tan x$ , $t\in\mathbb{R}$ ឬ $t\in(-\infty,+\infty)$

យើងបាន សមីការ (3) ទៅជា $at^2+bt+c=0$

ដោះស្រាយរកតម្លៃ $t$ យើងបានសមីការ (3) មានឫស

$x=\arctan t+k\pi$ , $k\in\mathbb{Z}$ ចំពោះគ្រប់តម្លៃចំនួនពិត $t$

**ឃ.** $a\cot^2x+b\cot x+c=0$ (4)

តាង $t=\cot x$ , $t\in\mathbb{R}$ ឬ $t\in(-\infty,+\infty)$

យើងបាន សមីការ (4) ទៅជា $at^2+bt+c=0$

ដោះស្រាយរកតម្លៃ $t$ យើងបានសមីការ (4) មានឫស

$x = arc\cot t + k\pi \,, k \in \mathbb{Z}$ ចំពោះគ្រប់តម្លៃចំនួនពិត $t$

លំហាត់គំរូទី១: ដោះស្រាយសមីការ $2\cos^2 x - 3\sqrt{2}\cos x + 2 = 0$

**ចំណោះស្រាយ**

យើងមានសមីការ $2\cos^2 x - 3\sqrt{2}\cos x + 2 = 0$

តាង $t = \cos x \quad , -1 \le t \le 1$

នោះសមីការទៅជា $2t^2 - 3\sqrt{2}t + 2 = 0$

$$\Delta = (-3\sqrt{2})^2 - 4 \times 2 \times 2 = 18 - 16 = 2$$

$$t_1 = \frac{3\sqrt{2} + \sqrt{2}}{2 \times 2} = \sqrt{2} > 1 \text{ (មិនយក)}$$

$$t_2 = \frac{3\sqrt{2} - \sqrt{2}}{2 \times 2} = \frac{\sqrt{2}}{2}$$

តែ $t = \cos x$ យើងបាន $\cos x = \frac{\sqrt{2}}{2} = \cos\frac{\pi}{4}$

$$\Rightarrow x = \pm\frac{\pi}{4} + 2k\pi \ , k \in \mathbb{Z}$$

ដូចនេះ សមីការមានឫស $\boxed{x = \pm\frac{\pi}{4} + 2k\pi \ , k \in \mathbb{Z}}$

លំហាត់គំរូទី២: ដោះស្រាយសមីការ $8\sin^2 x - 6\sin x = 5$

**ចំណោះស្រាយ**

យើងមាន $8\sin^2 x - 6\sin x = 5$

តាង $t = \sin x \quad , -1 \le t \le 1$

យើងបានសមីការទៅជា $8t^2-6t=5$

$$\Leftrightarrow 8t^2-6t-5=0$$

$$\Delta'=9+40=49=7^2$$

$$t_1=\frac{3+7}{8}=\frac{10}{8}=\frac{5}{4}>1 \quad (\text{មិនយក})$$

$$t_2=\frac{3-7}{8}=-\frac{1}{2}$$

តែ $t=\cos x$ យើងបាន $\sin x=-\frac{1}{2}=\sin\left(-\frac{\pi}{6}\right)$

$$\Rightarrow \begin{cases} x=-\frac{\pi}{6}+2k\pi \\ x=\pi+\frac{\pi}{6}+2k\pi=\frac{7\pi}{6}+2k\pi \end{cases}, k\in\mathbb{Z}$$

ដូចនេះ សមីការមានឫស $\boxed{x=-\frac{\pi}{6}+2k\pi\ ,x=\frac{7\pi}{6}+2k\pi\ ,k\in\mathbb{Z}}$

លំហាត់គំរូទី៣: ដោះស្រាយសមីការ $5\tan^2 y+5\tan y=2(1+\tan^2 y)$

## ដំណោះស្រាយ

សមីការមានន័យកាលណា $y\neq k\pi$

តាង $t=\tan y\ ,t\in\mathbb{R}$

នោះសមីការ $5\tan^2 y+5\tan y=2(1+\tan^2 y)$ ទៅជា

$$5t^2+5t=2(1+t^2)$$

$$\Leftrightarrow 3t^2+5t-2=0$$

$$\Delta=25+24=49=7^2$$

$$t_1=\frac{-5-7}{2\times 3}=\frac{-12}{6}=-2 \quad , \quad t_2=\frac{-5+7}{2\times 3}=\frac{1}{3}$$

- $t=-2 \Leftrightarrow \tan y=-2$

$$\Rightarrow y=\arctan(-2)+k\pi,\ k\in\mathbb{Z}$$

- $t=\dfrac{1}{3} \Leftrightarrow \tan y=\dfrac{1}{3}$

$$\Rightarrow y=\arctan\left(\frac{1}{3}\right)+k\pi,\ k\in\mathbb{Z}$$

ដូចនេះ សមីការមានឫស

$$\boxed{y=\arctan(-2)+k\pi,\ y=\arctan\left(\frac{1}{3}\right)+k\pi,\ k\in\mathbb{Z}}$$

លំហាត់គំរូទី៥: ដោះស្រាយសមីការ $\cot^2 x-(\sqrt{3}+1)\cot x+\sqrt{3}=0$

**ចំណោះស្រាយ**

សមីការមានន័យកាលណា $x\neq\dfrac{\pi}{2}+k\pi,\ k\in\mathbb{Z}$

តាង $t=\cot x$ នោះសមីការទៅជា $t^2-(\sqrt{3}+1)t+\sqrt{3}=0$

ដោយ $a+b+c=1-(\sqrt{3}+1)+\sqrt{3}=0$

$\Rightarrow t_1=1,\ t_2=\sqrt{3}$

- $t=1\Leftrightarrow \cot x=1=\cot\dfrac{\pi}{4}$

$$\Rightarrow x=\frac{\pi}{4}+k\pi,\ k\in\mathbb{Z}$$

- $t=\sqrt{3}\Leftrightarrow \cot x=\sqrt{3}=\cot\dfrac{\pi}{6}$

$$\Rightarrow x=\frac{\pi}{6}+k\pi,\ k\in\mathbb{Z}$$

ដូចនេះ សមីការមានឫស $\boxed{x=\dfrac{\pi}{4}+k\pi,\ x=\dfrac{\pi}{6}+k\pi,\ k\in\mathbb{Z}}$

លំហាត់គំរូទី៥: ដោះស្រាយសមីការ $\frac{1}{\sin^2 x}=\cot x+3$

**ដំណោះស្រាយ**

សមីការមានន័យកាលណា $\sin x\neq 0\Leftrightarrow x\neq k\pi\ ,k\in\mathbb{Z}$

ដោយ $1+\cot^2 x=\frac{1}{\sin^2 x}$

នោះយើងបានសមីការទៅជា $1+\cot^2 x=\cot x+3$

$$\Leftrightarrow \cot^2 x-\cot x-2=0$$

ដោយ $b=a+c$ គឺ $-1=1-2\ \Rightarrow \cot x=-1\ \vee\ \cot x=2$

- $\cot x=-1\Leftrightarrow \cot x=\cot\left(-\frac{\pi}{4}\right)\Rightarrow x=-\frac{\pi}{4}+k\pi,\ k\in\mathbb{Z}$
- $\cot x=2\ \Rightarrow\ x=arc\cot 2+k\pi\ ,k\in\mathbb{Z}$

ដូចនេះ សមីការមានឫស $\boxed{x=-\frac{\pi}{4}+k\pi, x=arc\cot 2+k\pi, k\in\mathbb{Z}}$

## ៧ . សមីការរាង $a\sin^2 x+b\sin x\cos x+c\cos^2 x=d$ ដែល $a,b,c\neq 0$

**របៀបដោះស្រាយ**

យើងមានសមីការ $a\sin^2 x+b\sin x\cos x+c\cos^2 x=d\quad (*)$

**របៀបទី១**

ចែកអង្គទាំងពីរនៃ (*) នឹង $\cos^2 x\neq 0\Leftrightarrow x\neq\frac{\pi}{2}+k\pi\ ,k\in\mathbb{Z}$

យើងបាន $a\frac{\sin^2 x}{\cos^2 x}+b\frac{\sin x\cos x}{\cos^2 x}+c\frac{\cos^2 x}{\cos^2 x}=\frac{d}{\cos^2 x}$

$$a\tan^2 x+b\tan x+c=d\left(1+\tan^2 x\right)$$

$$\boxed{(a-d)\tan^2 x+b\tan x+c-d=0}$$

**វិធីទី២**

ដោយ $\sin^2 x = \dfrac{1-\cos 2x}{2}$ , $\cos^2 x = \dfrac{1+\cos 2x}{2}$

នោះ $(*) \Leftrightarrow a\dfrac{1-\cos 2x}{2} + \dfrac{b}{2}2\sin x\cos x + c\dfrac{1+\cos 2x}{2} = d$

$\Leftrightarrow a1(-\cos 2x) + b\sin 2x + c(1+\cos 2x) = 2d$

$$\boxed{(c-a)\cos 2x + b\sin 2x = 2d - a - c}$$

ដោះស្រាយតាមរបៀបសមីការ $a\cos x + b\sin x = c$

**លំហាត់គំរូទី១: ដោះស្រាយសមីការ**

$7\cos^2 x + 6\sqrt{3}\sin x\cos x - \sin^2 x + 2 = 0$

**ដំណោះស្រាយ**

យើងមានសមីការ $7\cos^2 x + 6\sqrt{3}\sin x\cos x - \sin^2 x + 2 = 0$

**វិធីទី១**

ចែកអង្គទាំងពីរនៃសមីការនឹង $\cos^2 x \neq 0$ , $x \neq \dfrac{\pi}{2} + k\pi$ , $k \in \mathbb{Z}$

យើងបាន $7 + 6\sqrt{3}\tan x - \tan^2 x + \dfrac{2}{\cos^2 x} = 0$

$7 + 6\sqrt{3}\tan x - \tan^2 x + 2\left(1 + \tan^2 x\right) = 0$

$\tan^2 x + 6\sqrt{3}\tan x + 9 = 0$

$\Delta' = (3\sqrt{3})^2 - 9 = 27 - 9 = 18 = (3\sqrt{2})^2$

$\tan x_1 = -3\sqrt{3} - 3\sqrt{2}$ , $\tan x_2 = -3\sqrt{3} + 3\sqrt{2}$

- $\tan x = -3\sqrt{3} - 3\sqrt{2} \Rightarrow x = \arctan(-3\sqrt{3} - 3\sqrt{2}) + k\pi$ , $k \in \mathbb{Z}$
- $\tan x = -3\sqrt{3} + 3\sqrt{2} \Rightarrow x = \arctan(-3\sqrt{3} + 3\sqrt{2}) + k\pi$ , $k \in \mathbb{Z}$

បើ $x=\frac{\pi}{2}+k\pi$ នោះ សមីការមិនផ្ទៀងផ្ទាត់

ដូចនេះ សមីការមានឫស

$$\boxed{\begin{array}{l} x=\arctan(-3\sqrt{3}-3\sqrt{2})+k\pi \\ x=\arctan(-3\sqrt{3}+3\sqrt{2})+k\pi\ ,\ k\in\mathbb{Z} \end{array}}$$

## វិធីទី២

ដោយ $\cos^2 x=\frac{1}{2}(1+\cos 2x)$ , $\sin^2 x=\frac{1}{2}(1-\cos^2 x)$

នោះយើងបានសមីការទៅជា

$$\frac{7}{2}(1+\cos 2x)+6\sqrt{3}\sin x\cos x-\frac{1}{2}(1-\cos 2x)+2=0$$

$$\Leftrightarrow 7+7\cos 2x+6\sqrt{3}\sin 2x-1+\cos 2x+4=0$$

$$\Leftrightarrow 8\cos 2x+6\sqrt{3}\sin 2x=-10 \qquad (*)$$

ដោយ $8^2+(6\sqrt{3})^2>(-10)^2$

នោះសមីការ $(*)$ មានឫសជាចំនួនពិត

$$r=\sqrt{8^2+(6\sqrt{3})^2}=\sqrt{172}=2\sqrt{43}$$

$$\left.\begin{array}{l} \cos\alpha=\frac{8}{2\sqrt{43}}=\frac{4}{\sqrt{43}} \\ \sin\alpha=\frac{6\sqrt{3}}{2\sqrt{43}}=\frac{3\sqrt{3}}{\sqrt{43}} \end{array}\right| \Leftrightarrow \tan\alpha=\frac{3\sqrt{3}}{4}$$

$$\Rightarrow \alpha=\arctan\left(\frac{3\sqrt{3}}{4}\right)$$

នោះ $(*)\Leftrightarrow \frac{8}{2\sqrt{43}}\cos 2x+\frac{6\sqrt{3}}{2\sqrt{43}}\sin 2x=-\frac{10}{2\sqrt{43}}$

$$\Leftrightarrow \cos\alpha\cos 2x+\sin\alpha\sin 2x=-\frac{5}{\sqrt{43}}$$

$$\Leftrightarrow \cos(2x-\alpha)=-\frac{5}{\sqrt{43}}$$

$$\Leftrightarrow 2x-\alpha=\pm\arccos\left(-\frac{5}{\sqrt{43}}\right)+2k\pi\ ,\ k\in\mathbb{Z}$$

$$\Leftrightarrow 2x=\alpha\pm\arccos\left(-\frac{5}{\sqrt{43}}\right)+2k\pi$$

$$\Rightarrow x=\frac{\alpha}{2}\pm\frac{1}{2}\arccos\left(-\frac{5}{\sqrt{43}}\right)+k\pi$$

$$x=\frac{1}{2}\arctan\left(\frac{3\sqrt{3}}{4}\right)\pm\frac{1}{2}\arccos\left(-\frac{5}{\sqrt{43}}\right)+k\pi\ ,\ k\in\mathbb{Z}$$

ដូចនេះ សមីការមានឫស

$$\boxed{x=\frac{1}{2}\arctan\left(\frac{3\sqrt{3}}{4}\right)\pm\frac{1}{2}\arccos\left(-\frac{5}{\sqrt{43}}\right)+k\pi\ ,\ k\in\mathbb{Z}}$$

លំហាត់គំរូទី២: ដោះស្រាយសមីការ $\cos^2 x+\cot^2 x=\frac{\sin^3 x+1}{\sin^2 x}$

**ដំណោះស្រាយ**

សមីការមានឫសកាលណា $\sin x\neq 0 \Leftrightarrow x\neq k\pi\ ,\ k\in\mathbb{Z}$

យើងបានសមីការ $\cos^2 x+\cot^2 x=\frac{\sin^3 x+1}{\sin^2 x}$

$$\Leftrightarrow \cos^2 x+\cot^2 x=\sin x+\frac{1}{\sin^2 x}$$

$$\Leftrightarrow \cos^2 x+\cot^2 x=\sin x+1+\cot^2 x$$

$$\Leftrightarrow \cos^2 x-\sin x-1=0$$

$$\Leftrightarrow 1-\sin^2 x-\sin x-1=0$$
$$\Leftrightarrow -\sin^2 x-\sin x=0$$
$$\Leftrightarrow -\sin x(\sin x+1)=0$$

តែ $\sin x\neq 0$ នោះ $\sin x+1=0$

$$\Leftrightarrow \sin x=-1$$
$$\Rightarrow x=-\frac{\pi}{2}+2k\pi\ ,\ k\in\mathbb{Z}$$

ដូចនេះ សមីការមានឫស $\boxed{x=-\frac{\pi}{2}+2k\pi\ ,\ k\in\mathbb{Z}}$

លំហាត់គំរូទី៣: ដោះស្រាយសមីការ $\tan^3 x+\tan^2 x-3\tan x=3$

**ដំណោះស្រាយ**

យើងមានសមីការ $\tan^3 x+\tan^2 x-3\tan x=3$

សមីការមានន័យកាលណា $x\neq\frac{\pi}{2}+k\pi\ ,\ k\in\mathbb{Z}$

តាង $t=\tan x$

នោះសមីការ ទៅជា $t^3+t^2-3t=3$

$$\Leftrightarrow t^3+t^2-3t-3=0$$
$$\Leftrightarrow t^2(t+1)-3(t+1)=0$$
$$\Leftrightarrow (t+1)(t^2-3)=0$$
$$\Leftrightarrow (t+1)(t+\sqrt{3})(t-\sqrt{3})=0$$
$$\Rightarrow\ t=-1\ ,\ t=-\sqrt{3}\ ,\ t=\sqrt{3}$$

តែ $t=\tan x$ នោះយើងបាន

- $t=-1\Leftrightarrow \tan x=-1=\tan\left(-\frac{\pi}{4}\right)\ \Rightarrow x=-\frac{\pi}{4}+k\pi\ ,k\in\mathbb{Z}$

- $t=-\sqrt{3}\Leftrightarrow \tan x=-\sqrt{3}=\tan\left(-\frac{\pi}{3}\right)\Rightarrow x=-\frac{\pi}{3}+k\pi\,,\ k\in\mathbb{Z}$
- $t=\sqrt{3}\Leftrightarrow \tan x=\sqrt{3}=\tan\frac{\pi}{3}\Rightarrow x=\frac{\pi}{3}+k\pi\,,k\in\mathbb{Z}$

ដូចនេះ សមីការមានឫស $\boxed{x=-\frac{\pi}{4}+k\pi,\ x=\pm\frac{\pi}{3}+k\pi\,,\ k\in\mathbb{Z}}$

លំហាត់គរូទី៤: ដោះស្រាយសមីការ $\sin^2 x+3\cos^2 x-2\sin 2x=0$

## ចំណោះស្រាយ

### វិធីទី១

យើងមានសមីការ $\sin^2 x+3\cos^2 x-2\sin 2x=0$

$\Leftrightarrow \sin^2 x-4\sin x\cos x+3\cos^2 x=0$

ចែកអង្គទាំងពីរនឹង $\cos^2 x\neq 0$ យើងបាន

$\tan^2 x-4\tan x+3=0$

$\Rightarrow \tan x_1=1\ ,\ \tan x_2=3$

- $\tan x=1=\tan\frac{\pi}{4}\Rightarrow x=\frac{\pi}{4}+k\pi\ ,\in\mathbb{Z}$
- $\tan x=3\Rightarrow x=\arctan 3+k\pi\ ,\ k\in\mathbb{Z}$

បើ $x=\frac{\pi}{2}+k\pi$ នោះ សមីការ មិនផ្ទៀងផ្ទាត់

ដូចនេះ សមីការមានឫស $\boxed{x=\frac{\pi}{4}+k\pi,x=\arctan 3+k\pi,k\in\mathbb{Z}}$

### វិធីទី២

យើងមានសមីការ $\sin^2 x+3\cos^2 x-2\sin 2x=0$

$$\Leftrightarrow \sin^2 x - 4\sin x\cos x + 3\cos^2 x = 0$$

$$\Leftrightarrow \sin^2 x - 3\sin x\cos x - \sin x\cos x + 3\cos^2 x = 0$$

$$\Leftrightarrow \sin x(\sin x - 3\cos x) - \cos x(\sin x - 3\cos x) = 0$$

$$\Leftrightarrow (\sin x - 3\cos x)(\sin x - \cos x) = 0$$

$$\Leftrightarrow \sin x - 3\cos x = 0 \vee \sin x - \cos x = 0$$

- $\sin x - 3\cos x = 0$

$$\Leftrightarrow \sin x = 3\cos x$$

$$\Leftrightarrow \tan x = 3$$

$$\Rightarrow x = \arctan 3 + k\pi, k \in \mathbb{Z}$$

( $x = \frac{\pi}{2}$ មិនមែនជាឫសនៃសមីការ )

- $\sin x - \cos x = 0$

$$\Leftrightarrow \sin x = \cos x$$

$$\Leftrightarrow \tan x = 1 = \tan\frac{\pi}{4}$$

$$\Rightarrow x = \frac{\pi}{4} + k\pi, k \in \mathbb{Z}$$

ដូចនេះ សមីការមានឫស $\boxed{x = \frac{\pi}{4} + k\pi, x = \arctan 3 + k\pi, k \in \mathbb{Z}}$

## ៨. សមីការរាង $a(\sin x \pm \cos x) + b\sin x\cos x + c = 0$

**របៀបដោះស្រាយ**

**របៀបទី១**

តាង $t = \sin x \pm \cos x$

$$t^2 = (\sin x \pm \cos x)^2$$

$$= \sin^2 x \pm 2\sin x\cos x + \cos^2 x$$
$$= 1 \pm 2\sin x\cos x$$

$$\Rightarrow \sin x\cos x = \frac{t^2-1}{\pm 2}$$

នោះសមីការទៅជា $at + b\frac{t^2-1}{\pm 2} + c = 0$

$$\boxed{\pm\frac{b}{2}(t^2-1) + at + c = 0}$$

ដែល $-\sqrt{2} \le t \le \sqrt{2}$

Note:

$$\sin x \pm \cos x = \sqrt{2}\left(\frac{\sqrt{2}}{2}\sin x \pm \frac{\sqrt{2}}{2}\cos x\right)$$
$$= \sqrt{2}\left(\sin x\sin\frac{\pi}{4} \pm \cos x\cos\frac{\pi}{4}\right)$$
$$= \sqrt{2}\sin\left(x \pm \frac{\pi}{4}\right)$$

ឬ $\sin x \pm \cos x = \pm(\cos x + \sin x)$

$$= \pm\sqrt{2}\left(\frac{\sqrt{2}}{2}\cos x + \frac{\sqrt{2}}{2}\sin x\right)$$
$$= \pm\sqrt{2}\cos\left(x - \frac{\pi}{4}\right)$$

ដូចនេះ លក្ខខណ្ឌនៃ $t$ គឺ $-\sqrt{2} \le t \le \sqrt{2}$

## វិធីទី២

យើងមានសមីការ $a(\sin x \pm \cos x) + b\sin x\cos x + c = 0$

ដោយ $\sin x \pm \cos x = \pm(\cos x + \sin x) = \pm\sqrt{2}\cos\left(x - \frac{\pi}{4}\right)$

តាង $t = x - \frac{\pi}{4} \Leftrightarrow x = t + \frac{\pi}{4}$

នោះ $\sin x \pm \cos x = \pm\sqrt{2}\cos t$

$$(\sin x \pm \cos x)^2 = (\pm\sqrt{2}\cos t)^2$$
$$\sin^2 x \pm 2\sin x\cos x + \cos^2 x = 2\cos^2 t$$
$$1 \pm 2\sin x\cos x = 2\cos^2 t$$
$$\Rightarrow \sin x\cos x = \frac{2\cos^2 t - 1}{\pm 2}$$

យើងបានសមីការទៅជា $\pm a\sqrt{2}\cos t + b\frac{2\cos^2 t - 1}{\pm 2} + c = 0$

$$2a\sqrt{2}\cos t + 2b\cos^2 t - b \pm 2c = 0$$

$$\boxed{b\cos^2 t + a\sqrt{2}\cos t - \frac{b}{2} \pm c = 0}$$

លំហាត់គំរូទី១: ដោះស្រាយសមីការ

$$(\sin x + \cos x)^2 - 4\sqrt{2}\sin x\cos x - 5\sqrt{2} = 0$$

**ចំណោះស្រាយ**

**វិធីទី១**

យើងមានសមីការ $8(\sin x + \cos x) - 4\sqrt{2}\sin x\cos x - 5\sqrt{2} = 0$ (*)

តាង $t = \sin x + \cos x$ , $-\sqrt{2} \le t \le \sqrt{2}$

$$t^2 = \sin^2 x + 2\sin x\cos x + \cos^2 x = 1 + 2\sin x\cos x$$
$$\Rightarrow \sin x\cos x = \frac{t^2 - 1}{2}$$

នោះសមីការ (*) ទៅជា $8t-4\sqrt{2}\dfrac{t^2-1}{2}-5\sqrt{2}=0$

$$8t-2\sqrt{2}t^2+2\sqrt{2}-5\sqrt{2}=0$$

$$2t^2-4\sqrt{2}t+3=0$$

$$\Delta'=(-2\sqrt{2})^2-2\times 3=2$$

$$t_1=\frac{2\sqrt{2}-\sqrt{2}}{2}=\frac{\sqrt{2}}{2}$$

$$t_2=\frac{2\sqrt{2}+\sqrt{2}}{2}=\frac{3\sqrt{2}}{2}>\sqrt{2}\quad \text{(មិនយក)}$$

តែ $t=\sin x+\cos x=\sqrt{2}\cos\left(x-\dfrac{\pi}{4}\right)$

យើងបាន $\sqrt{2}\cos\left(x-\dfrac{\pi}{4}\right)=\dfrac{\sqrt{2}}{2}$

$$\cos\left(x-\frac{\pi}{4}\right)=\frac{1}{2}=\cos\frac{\pi}{3}$$

$$x-\frac{\pi}{4}=\pm\frac{\pi}{3}+2k\pi\ ,\ k\in\mathbb{Z}$$

$$x-\frac{\pi}{4}=-\frac{\pi}{3}+2k\pi\vee x-\frac{\pi}{4}=\frac{\pi}{3}+2k\pi$$

$$\Rightarrow x=-\frac{\pi}{12}+2k\pi\ ,\ x=\frac{7\pi}{12}+2k\pi\ ,\ k\in\mathbb{Z}s$$

ដូចនេះ សមីការមានឫស $\boxed{x=-\dfrac{\pi}{12}+2k\pi, x=\dfrac{7\pi}{12}+2k\pi,\ k\in\mathbb{Z}}$

## វិធីទី២

យើងមានសមីការ $8(\sin x+\cos x)-4\sqrt{2}\sin x\cos x-5\sqrt{2}=0$ (*)

ដោយ $\sin x+\cos x=\sqrt{2}\cos\left(x-\dfrac{\pi}{4}\right)$

តាង $t = x - \frac{\pi}{4} \Leftrightarrow x = t + \frac{\pi}{4}$

នោះ $\begin{cases} \sin x + \cos x = \sqrt{2}\cos t \\ \sin x \cos x = \frac{2\cos^2 t - 1}{2} = \cos^2 t - \frac{1}{2} \end{cases}$

យើងបានសមីការ (*) ទៅជា $8\sqrt{2}t - 4\sqrt{2}\left(\cos^2 t - \frac{1}{2}\right) - 5\sqrt{2} = 0$

$$8\sqrt{2}\cos t - 4\sqrt{2}\cos^2 t + 2\sqrt{2} - 5\sqrt{2} = 0$$

$$-4\sqrt{2}\cos^2 t + 8\sqrt{2}\cos t - 3\sqrt{2} = 0$$

$$4\cos^2 t - 8\cos t + 3 = 0$$

$$\Delta' = 16 - 12 = 4 = 2^2$$

$$\cos t_1 = \frac{4-2}{4} = \frac{1}{2}$$

$$\cos t_2 = \frac{4+2}{4} = \frac{3}{2} > 1 \quad \text{(មិនយក)}$$

យើងបាន $\cos t = \frac{1}{2} \Leftrightarrow \cos t = \cos\frac{\pi}{3}$

$$\Rightarrow t = \pm\frac{\pi}{3} + 2k\pi \ \ k \in \mathbb{Z}$$

តែ $x = \frac{\pi}{4} + t$ គេបាន:

- $t = -\frac{\pi}{3} + 2k\pi \Rightarrow x = \frac{\pi}{4} - \frac{\pi}{3} + 2k\pi = -\frac{\pi}{12} + 2k\pi \ , \ k \in \mathbb{Z}$
- $t = \frac{\pi}{3} + 2k\pi \Rightarrow x = \frac{\pi}{4} + \frac{\pi}{3} + 2k\pi = \frac{7\pi}{12} + 2k\pi \ , \ k \in \mathbb{Z}$

ដូចនេះ សមីការមានឫស $\boxed{x = -\frac{\pi}{12} + 2k\pi, x = \frac{7\pi}{12} + 2k\pi, k \in \mathbb{Z}}$

លំហាត់គំរូទី២:

ដោះស្រាយសមីការ $3(\sin x+\cos x)-2\sin x\cos x-3=0$

## ចំណោះស្រាយ

**វិធីទី១**

យើងមានសមីការ $3(\sin x+\cos x)-2\sin x\cos x-3=0$

តាង $t=\sin x+\cos x=\sqrt{2}\cos\left(x-\frac{\pi}{4}\right)$ ដែល $-\sqrt{2}\le t\le\sqrt{2}$

$t^2=\sin^2 x+2\sin x\cos x+\cos^2 x=1+2\sin x\cos x$

$\Rightarrow 2\sin x\cos x=t^2-1$

នោះយើងបានសមីការទៅជា $3t-(t^2-1)-3=0$

$$\Leftrightarrow -t^2+3t-2=0$$

ដោយ $a+b+c=-1+3-2=0$

នោះ $t_1=1$ , $t_2=\frac{c}{a}=2$

តែ $-\sqrt{2}\le t\le\sqrt{2}$ នោះ $t=2\notin\left[-\sqrt{2},\sqrt{2}\right]$ (មិនយក)

$$\Rightarrow t=1\Leftrightarrow\sqrt{2}\cos\left(x-\frac{\pi}{4}\right)=1$$

$$\Leftrightarrow \cos\left(x-\frac{\pi}{4}\right)=\frac{\sqrt{2}}{2}=\cos\frac{\pi}{4}$$

$$\Leftrightarrow\begin{cases}x-\frac{\pi}{4}=\frac{\pi}{4}+2k\pi\\ x-\frac{\pi}{4}=-\frac{\pi}{4}+2k\pi\end{cases},\ k\in\mathbb{Z}$$

$$\Rightarrow\begin{cases}x=\frac{\pi}{2}+2k\pi\\ x=2k\pi\end{cases},\ k\in\mathbb{Z}$$

ដូចនេះ សមីការមានឫស $x=\frac{\pi}{2}+2k\pi\ ,\ x=2k\pi\ ,k\in\mathbb{Z}$

## វិធីទី២

យើងមានសមីការ $3(\sin x+\cos x)-2\sin x\cos x-3=0$

ដោយ $\sin x+\cos x=\sqrt{2}\cos\left(x-\frac{\pi}{4}\right)$

តាង $t=x-\frac{\pi}{4}\ \Leftrightarrow x=t+\frac{\pi}{4}$

នោះ $\sin x+\cos x=\sqrt{2}\cos t$

$$\sin^2 x+\cos^2 x+2\sin x\cos x=2\cos^2 t$$
$$1+2\sin x\cos x=2\cos^2 t$$
$$\Rightarrow 2\sin x\cos x=2\cos^2 t-1$$

យើងបានសមីការទៅជា $3\sqrt{2}\cos t-(2\cos^2 t-1)-3=0$

$$3\sqrt{2}\cos t-2\cos^2 t-2=0$$
$$2\cos^2 t-3\sqrt{2}\cos t+2=0$$
$$\Delta=(-3\sqrt{2})^2-4\times 2\times 2=18-16=2$$
$$\cos t_1=\frac{3\sqrt{2}-\sqrt{2}}{2\times 2}=\frac{\sqrt{2}}{2}$$
$$\cos t_2=\frac{3\sqrt{2}+\sqrt{2}}{2\times 2}=\sqrt{2}\notin[-1\ ,\ 1]\ \text{(មិនយក)}$$

នោះយើងបាន $\cos t=\frac{\sqrt{2}}{2}\ \Leftrightarrow\ \cos t=\cos\frac{\pi}{4}$

$$\Rightarrow t=\pm\frac{\pi}{4}+2k\pi\ ,\ k\in\mathbb{Z}$$

យើងបាន $x=\frac{\pi}{4}+t=\frac{\pi}{4}\pm\frac{\pi}{4}+2k\pi$

$$x=\frac{\pi}{4}-\frac{\pi}{4}+2k\pi \ \vee \ x=\frac{\pi}{4}+\frac{\pi}{4}+2k\pi$$

$$x=2k\pi \ \vee \ x=\frac{\pi}{2}+2k\pi \ , \ k\in\mathbb{Z}$$

ដូចនេះ សមីការមានឫស $\boxed{x=2k\pi \ , \ x=\frac{\pi}{2}+2k\pi \ , \ k\in\mathbb{Z}}$

## ៩. សមីការ $\sin^m x$ ឈមនឹង $\cos^n x$ ដែលនិទស្សន្ត m និង n ជាចំនួនគត់គូ

**របៀបដោះស្រាយ**

**របៀបទី១**

តាង $t=\cos 2x \ , \ -1\le t\le 1$

$$\sin^2 x=\frac{1-\cos 2x}{2}=\frac{1-t}{2}$$

$$\cos^2 x=\frac{1+\cos 2x}{2}=\frac{1+t}{2}$$

យកតម្លៃ $\sin^2 x$ និង $\cos^2 x$ ទៅជំនួសក្នុង $\sin^m x$ និង $\cos^n x$

**របៀបទី២**

$$\text{តាង } t=\sin^2 x \Rightarrow \cos^2 x=1-t \quad (0\le t\le 1)$$

$$\text{ឬ តាង } t=\cos^2 x \Rightarrow \sin^2 x=1-t \quad (0\le t\le 1)$$

លំហាត់គំរូទី១: ដោះស្រាយសមីការ $4\cos^4 x+7\sin^2 x=4$

**ដំណោះស្រាយ**

**របៀបទី១**

យើងមានសមីការ $4\cos^4 x+7\sin^2 x=4 \quad (*)$

តាង $t=\cos 2x \quad , -1\le t\le 1$

- $\cos^2 x=\dfrac{1+\cos 2x}{2}=\dfrac{1+t}{2}$
- $\sin^2 x=\dfrac{1-\cos 2x}{2}=\dfrac{1-t}{2}$

នោះយើងបានសមីការ $(*)\Leftrightarrow 4\left(\cos^2 x\right)^2+7\sin^2 x=4$

$$\Leftrightarrow 4\left(\frac{1+t}{2}\right)^2+7\left(\frac{1-t}{2}\right)=4$$

$$\Leftrightarrow 2(1+2t+t^2)+7(1-t)=8$$

$$\Leftrightarrow 2t^2-3t+1=0$$

ដោយ $a+b+c=2-3+1=0$

នោះ $t_1=1\ ,\ t_2=\dfrac{c}{a}=\dfrac{1}{2}$

តែ $t=\cos 2x$ យើងបាន

- $t=1\Leftrightarrow \cos 2x=1$

$$\Leftrightarrow 2x=2k\pi \ ,k\in\mathbb{Z}$$

$$\Rightarrow x=k\pi \ , k\in\mathbb{Z}$$

- $t=\dfrac{1}{2}\Leftrightarrow \cos 2x=\dfrac{1}{2}=\cos\dfrac{\pi}{3}$

$$\Leftrightarrow 2x=\pm\frac{\pi}{3}+2k\pi,\ k\in\mathbb{Z} \quad \Rightarrow x=\pm\frac{\pi}{6}+k\pi \ \ k\in\mathbb{Z}$$

ដូចនេះ សមីការមានឫស $\boxed{x=k\pi \ ,\ x=\pm\dfrac{\pi}{6}+k\pi \ ,k\in\mathbb{Z}}$

**វិធីទី២**

យើងមានសមីការ $4\cos^4 x+7\sin^2 x=4 \quad (*)$

តាង $t=\cos^2 x \quad , 0\le t\le 1$

យើងបាន $(*) \Leftrightarrow 4\left(\cos^2 x\right)^2 + 7\left(1-\cos^2 x\right) = 4$

$\Leftrightarrow 4t^2 + 7(1-t) = 4$

$\Leftrightarrow 4t^2 - 7t + 3 = 0$

ដោយ $a+b+c = 4-7+3 = 0$

នោះ $t_1 = 1$ , $t_2 = \frac{c}{a} = \frac{3}{4}$

- $t = 1 \Leftrightarrow \cos^2 x = 1$

$\Leftrightarrow \cos x = \pm 1$

$\Rightarrow x = k\pi$ , $k \in \mathbb{Z}$

- $t = \frac{3}{4} \Leftrightarrow \cos^2 x = \frac{3}{4}$

$\Leftrightarrow \cos x = \pm\frac{\sqrt{3}}{2}$

$\circ$ $\cos x = \frac{\sqrt{3}}{2} = \cos\frac{\pi}{6}$

$\Rightarrow x = \pm\frac{\pi}{6} + 2k\pi$ , $k \in \mathbb{Z}$

$\circ$ $\cos x = -\frac{\sqrt{3}}{2} = \cos\frac{5\pi}{6}$

$\Rightarrow x = \pm\frac{5\pi}{6} + 2k\pi$ , $k \in \mathbb{Z}$

ដូចនេះ សមីការមានឫស

$$\boxed{x = k\pi \text{ , } x = \pm\frac{\pi}{6} + 2k\pi \text{ , } x = \pm\frac{5\pi}{6} + 2k\pi \text{ , } k \in \mathbb{Z}}$$

**Note:** បើ $A = \{2k\pi, k \in \mathbb{Z}\}$ និង $B = \{k\pi, k \in \mathbb{Z}\}$ នោះ $A \subseteq B$

**វរៀបចំ**

យើងមានសមីការ $4\cos^4 x+7\sin^2 x=4$ (*)

បើ $\cos x=0 \Leftrightarrow x=\frac{\pi}{2}+k\pi \Rightarrow \sin x=\pm 1$

នោះ (*) : $4\times 0^4+7\times(\pm 1)^2=4 \Leftrightarrow 7=4$ មិនផ្ទៀងផ្ទាត់

$\Rightarrow x=\frac{\pi}{2}$ មិនមែនជាឫសនៃ សមីការ (*)

នោះ ចែកអង្គទាំងពីរនៃសមីកា (*) នឹង $\cos^2 x$ យើងបាន

$$\frac{4\cos^4 x}{\cos^4 x}+\frac{7\sin^2 x}{\cos^4 x}=\frac{4}{\cos^4 x}$$

$$4+\frac{7\tan^2 x}{\cos^2 x}=\frac{4}{\left(\cos^2 x\right)^2}$$

$$4+7\tan^2 x(1+\tan^2 x)=4(1+\tan^2 x)^2$$

$$4+7\tan^2 x+7\tan^4 x=4+8\tan^2 x+4\tan^4 x$$

$$3\tan^4 x-\tan^2 x=0$$

$$\tan^2 x\left(3\tan^2 x-1\right)=0$$

$$\tan^2 x=0 \ \vee \ 3\tan^2 x-1=0$$

- $\tan^2 x=0 \Leftrightarrow \tan x=0$

$$\Rightarrow x=k\pi,\ k\in\mathbb{Z}$$

- $3\tan^2 x-1=0 \Leftrightarrow \tan^2 x=\frac{1}{3}$

$$\Leftrightarrow \tan x=\pm\frac{\sqrt{3}}{3}=\tan\left(\pm\frac{\pi}{6}\right)$$

$$\Rightarrow x=\pm\frac{\pi}{6}+k\pi,\ k\in\mathbb{Z}$$

ដូចនេះ សមីការមានឫស

$$x=k\pi\ ,\ x=\frac{\pi}{6}+k\pi\ ,\ x=-\frac{\pi}{6}+k\pi\ ,k\in\mathbb{Z}$$

លំហាត់គំរូទី២: ដោះស្រាយសមីការ $\cos^4 x+\sin^6 x=\cos 2x$ (#)

## ចំណោះស្រាយ

### វិធីទី១

យើងមានសមីការ $\cos^4 x+\sin^6 x=\cos 2x$ (#)

តាង $t=\cos 2x \quad , -1\le t\le 1$

$$\cos^2 x=\frac{1+\cos 2x}{2}=\frac{1+t}{2}$$

$$\sin^2 x=\frac{1-\cos 2x}{2}=\frac{1-t}{2}$$

យើងបាន $(\#)\Leftrightarrow \left(\cos^2 x\right)^2+\left(\sin^2 x\right)^3=\cos 2x$

$$\Leftrightarrow \left(\frac{1+t}{2}\right)^2+\left(\frac{1-t}{2}\right)^3=t$$

$$\Leftrightarrow 2(1+t)^2+(1-t)^3=8t$$

$$\Leftrightarrow 2+4t+2t^2+1-3t+3t^2-t^3=8t$$

$$\Leftrightarrow t^3-5t^2+7t-3=0$$

$$\Leftrightarrow (t-3)(t-1)^2=0$$

$$\Leftrightarrow t=3\ \vee\ t=1$$

តែ $t=3\notin[-1,1]$ (មិនយក)

នោះយើងបាន $t=1\Leftrightarrow \cos 2x=1$

$$\Leftrightarrow\ 2x=2k\pi\ ,\ k\in\mathbb{Z}$$

$$\Rightarrow x = k\pi \ , k \in \mathbb{Z}$$

ដូចនេះ សមីការមានឫស $\boxed{x = k\pi \ , k \in \mathbb{Z}}$

## វិធីទី២

យើងមានសមីការ $\cos^4 x + \sin^6 x = \cos 2x \ \ (\#)$

តាង $t = \cos^2 x \ \ , \ 0 \le t \le 1$

យើងបាន $(\#) \Leftrightarrow \left(\cos^2 x\right)^2 + \left(1 - \cos^2 x\right)^3 = 2\cos^2 x - 1$

$$\Leftrightarrow t^2 + (1-t)^3 = 2t - 1$$
$$\Leftrightarrow t^2 + 1 - 3t + 3t^2 - t^3 = 2t - 1$$
$$\Leftrightarrow t^3 - 4t^2 + 5t - 2 = 0$$
$$\Leftrightarrow (t-1)\left(t^2 - t + 2\right) = 0$$

- $t - 1 = 0 \Rightarrow t = 1$
- $t^2 - t + 2 = 0$

$\Delta = 1 - 8 = -7 < 0$

$\Rightarrow$ សមីការ $t^2 - t + 2 = 0$ គ្មានឫស $\forall t \in \mathbb{R}$

យើងបាន $t = 1 \Leftrightarrow \cos^2 x = 1$

$$\Leftrightarrow \cos x = \pm 1$$
$$\Rightarrow x = k\pi \ , \ k \in \mathbb{Z}$$

ដូចនេះ សមីការមានឫស $\boxed{x = k\pi \ , \ k \in \mathbb{Z}}$

## ១O. សមីការ $\sin^m x$ ឈមនឹង $\cos^n x$ ដែល $m,n$ ជាចំនួនគត់សេសមិនស្មើគ្នា

☞ បើ $m<n$ ចែកអង្គទាំងពីរនៃសមីការនឹង $\cos^m x$

$$(\cos x \neq 0\ ,\ x \neq \frac{\pi}{2}+k\pi, k\in\mathbb{Z})$$

☞ បើ $m>n$ ចែកអង្គទាំងពីរនៃសមីការនឹង $\sin^m x$

$$(\sin x \neq 0\ ,\ x \neq k\pi\ \ , k\in\mathbb{Z})$$

រួចប្រើរូបមន្ត:

$$1+\tan^2 x=\frac{1}{\cos^2 x} \Leftrightarrow \cos^2 x=\frac{1}{1+\tan^2 x}$$

$$1+\cot^2 x=\frac{1}{\sin^2 x} \Leftrightarrow \sin^2 x=\frac{1}{1+\cot^2 x}$$

យើងបានសមីការដែលមានអញ្ញាត $\tan x$ ឬ $\cot x$

### ចំណាំ:

ក្នុងការដោះស្រាយសមីការក្នុងករណីនេះ យើងត្រូវសិក្សាពេល $\cos x=0$ ឬ $\sin x=0$ ថាតើតម្លៃ $x$ ជាឫសនៃសមីការដែរឬទេ

☞ បើចែកអង្គទាំងពីរនឹង $\cos x$ ត្រូវសិក្សាពេល $\cos x=0$

$$x=\frac{\pi}{2}+k\pi, k\in\mathbb{Z}$$

☞ បើចែកអង្គទាំងពីរនឹង $\sin x$ ត្រូវសិក្សាពេល $\sin x=0$

$$x=k\pi\ , k\in\mathbb{Z}$$

លំហាត់គំរូទី១: ដោះស្រាយសមីការ $2\cos^5 x+\sin^3 x=2\cos^3 x$ (*)

## ចំណោះស្រាយ

យើងមានសមីការ $2\cos^5 x+\sin^3 x=2\cos^3 x$ (*)

យើងឃើញថា និទស្សន្តនៃ $\cos x$ ធំជាងនិទស្សន្តនៃ $\sin x$

នោះ ចែកអង្គទាំងពីរនៃ (*) នឹង $\cos^3 x$ ( បើ $\cos x\neq 0$ )

យើងបាន $(*)\Leftrightarrow 2\cos^2 x+\tan^3 x=2$

$$\Leftrightarrow \frac{2}{1+\tan^2 x}+\tan^3 x=2$$
$$\Leftrightarrow 2+\tan^3 x\left(1+\tan^2 x\right)=2\left(1+\tan^2 x\right)$$
$$\Leftrightarrow \tan^5 x+\tan^3 x-2\tan^2 x=0$$
$$\Leftrightarrow \tan^2 x\left(\tan^3 x+\tan x-2\right)=0$$
$$\Leftrightarrow \tan^2 x\left(\tan x-1\right)\left(\tan^2 x+\tan x+2\right)=0$$
$$\Leftrightarrow \tan^2 x=0\vee \tan x-1=0\vee \tan^2 x+\tan x+2=0$$

- $\tan^2 x=0\Leftrightarrow \tan x=0\Rightarrow x=k\pi$ , $k\in\mathbb{Z}$
- $\tan x-1=0\Leftrightarrow \tan x=1=\tan\frac{\pi}{4}\Rightarrow x=\frac{\pi}{4}+k\pi$ , $k\in\mathbb{Z}$
- $\tan^2 x+\tan x+2=0$ $(\circ)$

  $\Delta=1-8=-7<0$

  នោះ សមីការ$(\circ)$ គ្មានឫស $\forall x\in\mathbb{R}$

  បើ $\cos x=0\Rightarrow \sin x=\pm 1$ នោះយើងបាន

$$(*):2\times(0)^5+(\pm 1)^3=2\times 0^3$$
$$\pm 1=0 \text{ (មិនផ្ទៀងផ្ទាត់)}$$

ដូចនេះ សមីការមានឫស $\boxed{x=k\pi\ ,\ x=\frac{\pi}{4}+k\pi\ ,\ k\in\mathbb{Z}}$

លំហាត់គំរូទី២: ដោះស្រាយសមីការ $2\sin^3 x-3\sin x+2\cos x=0$ (#)

## ចំណោះស្រាយ

យើងមានសមីការ $2\sin^3 x-3\sin x+2\cos x=0$ (#)

ដោយនិទស្សន្តនៃ $\sin x$ ធំជាង និទស្សន្តនៃ $\cos x$

នោះចែកអង្គទាំងពីរនៃសមីការនឹង $\sin x$ (បើ $\sin x\neq 0$)

យើងបាន $2\sin^2 x-3+\cot x=0$

$$\frac{2}{1+\cot^2 x}-3+2\cot x=0$$
$$2-3\left(1+\cot^2 x\right)+2\cot x\left(1+\cot^2 x\right)=0$$
$$2\cot^3 x-3\cot^2 x+2\cot x-1=0$$
$$\left(\cot x-1\right)\left(2\cot^2 x-\cot x+1\right)=0$$
$$\cot x-1=0\ \vee\ 2\cot^2 x-\cot x+1=0$$

- $\cot x-1=0\ \Leftrightarrow\ \cot x=1=\cot\dfrac{\pi}{4}$

$$\Rightarrow\ x=\frac{\pi}{4}+k\pi\ ,\ k\in\mathbb{Z}$$

- $2\cot^2 x-\cot x+1=0$ $(\oplus)$

$\Delta=1-8=-7<0$

នោះ សមីការ $(\oplus)$ គ្មានឫស $\forall x\in\mathbb{R}$

បើ $\sin x=0\Leftrightarrow x=k\pi\ ,\ k\in Z\ \Rightarrow\cos x=\pm 1$

នោះ(#): $2\times 0^3-3\times 0+2(\pm 1)=0\ \Leftrightarrow\ \pm 2=0$ (មិនផ្ទៀងផ្ទាត់)

ដូចនេះ សមីការមានឫស $\boxed{x=\dfrac{\pi}{4}+k\pi\ ,\ k\in\mathbb{Z}}$

## ១១. ដោះស្រាយសមីការ $\cos^n x + \sin^n x = 1$ $(n \in \mathbb{N})$

១. ករណី $n = 2$

បើ $n = 2$ នោះសមីការទៅជា $\cos^2 x + \sin^2 x = 1$ ផ្ទៀងផ្ទាត់ $\forall x \in \mathbb{R}$

នោះសមីការមានឫសរាប់មិនអស់

២. ករណី $n \neq 2$

☞ បើ $n < 2$ (មានន័យថា $n = 1$)

យើងបានសមីការទៅជា $\cos x + \sin x = 1$

$$\Leftrightarrow \frac{\sqrt{2}}{2}\cos x + \frac{\sqrt{2}}{2}\sin x = \frac{\sqrt{2}}{2}$$

$$\Leftrightarrow \cos\frac{\pi}{4}\cos x + \sin\frac{\pi}{4}\sin x = \cos\frac{\pi}{4}$$

$$\Leftrightarrow \cos\left(x - \frac{\pi}{4}\right) = \cos\frac{\pi}{4}$$

$$\Leftrightarrow \begin{cases} x - \frac{\pi}{4} = \frac{\pi}{4} + 2k\pi \\ x - \frac{\pi}{4} = -\frac{\pi}{4} + 2k\pi \end{cases}, k \in \mathbb{Z}$$

$$\Rightarrow \begin{cases} x = \frac{\pi}{2} + 2k\pi \\ x = 2k\pi \end{cases}, k \in \mathbb{Z}$$

នោះសមីការមានឫស $x = \frac{\pi}{2} + 2k\pi$ , $x = 2k\pi$ , $k \in \mathbb{Z}$ ចំពោះ $n = 1$

☞ បើ $n > 2$ នោះ $n - 2 > 0$

ដោយ $\cos^2 x + \sin^2 x = 1$

នោះយើងបាន $\cos^n x + \sin^n x = \cos^2 x + \sin^2 x$

$$\cos^n x-\cos^2 x=\sin^2 x-\sin^n x$$

$$\cos^2 x\left(\cos^{n-2} x-1\right)=\sin^2 x\left(1-\sin^{n-2} x\right) \quad (*)$$

ដោយ $\left.\begin{matrix}\cos^{n-2} x-1\le 0\\ \cos^2 x\ge 0\end{matrix}\right| \Rightarrow \cos^2 x\left(\cos^{n-2} x-1\right)\le 0$

ហើយ $\left.\begin{matrix}\sin^2 x\ge 0\\ 1-\sin^{n-2} x\ge 0\end{matrix}\right| \Rightarrow \sin^2 x\left(1-\sin^{n-2} x\right)\ge 0$

នោះ សមីការ (*) ធ្វៀងផ្ទាត់លុះត្រាតែ៖

$$\begin{cases}\cos^2 x\left(\cos^{n-2} x-1\right)=0 & (1)\\ \sin^2 x\left(1-\sin^{n-2} x\right)=0 & (2)\end{cases}$$

☞ បើ $n$ ជាចំនួនគត់គូ នោះ $n-2$ ក៏ជាចំនួនគត់គូ នោះ ៖

$$(1):\ \cos^2 x\left(\cos^{n-2} x-1\right)=0$$

$$\cos x=0\vee\cos^{n-2} x-1=0$$

$$\cos x=0\vee\cos x=\pm 1$$

$$\Rightarrow x=\frac{\pi}{2}+k\pi\vee x=k\pi\ ,\ k\in\mathbb{Z}$$

$$(2):\sin^2 x\left(1-\sin^{n-2} x\right)=0$$

$$\sin^2 x=0\vee 1-\sin^{n-2}=0$$

$$\sin x=0\vee\sin x=\pm 1$$

$$\Rightarrow x=k\pi\vee\ x=\frac{\pi}{2}+k\pi\ ,\ k\in\mathbb{Z}$$

ដូចនេះ បើ $n$ ជាចំនួនគត់គូ នោះសមីការមានចម្លើយ

$$\boxed{x=k\pi\ ,\ x=\frac{\pi}{2}+k\pi\ ,\ k\in\mathbb{Z}}$$

☞ បើ $n$ ជាចំនួនគត់សេស នោះ $n-2$ ក៏ជាចំនួនគត់សេស នោះ ៖

$$(1):\ \cos^2 x\left(\cos^{n-2} x-1\right)=0$$

$$\cos x=0 \vee \cos^{n-2} x-1=0$$

$$\cos x=0 \vee \cos x=1$$

$$\Rightarrow x=\frac{\pi}{2}+k\pi \vee x=2k\pi\ ,\ k\in\mathbb{Z} \quad (I)$$

$$(2):\sin^2 x\left(1-\sin^{n-2} x\right)=0$$

$$\sin^2 x=0 \vee 1-\sin^{n-2}=0$$

$$\sin x=0 \vee \sin x=1$$

$$\Rightarrow x=k\pi \vee x=\frac{\pi}{2}+2k\pi\ ,\ k\in\mathbb{Z} \quad (II)$$

$$(I)\cap(II) \Rightarrow x=2k\pi\ ,\ x=\frac{\pi}{2}+2k\pi\ ,\ k\in\mathbb{Z}$$

ដូចនេះ បើ $n$ ជាចំនួនគត់សេស នោះសមីការមានចម្លើយ

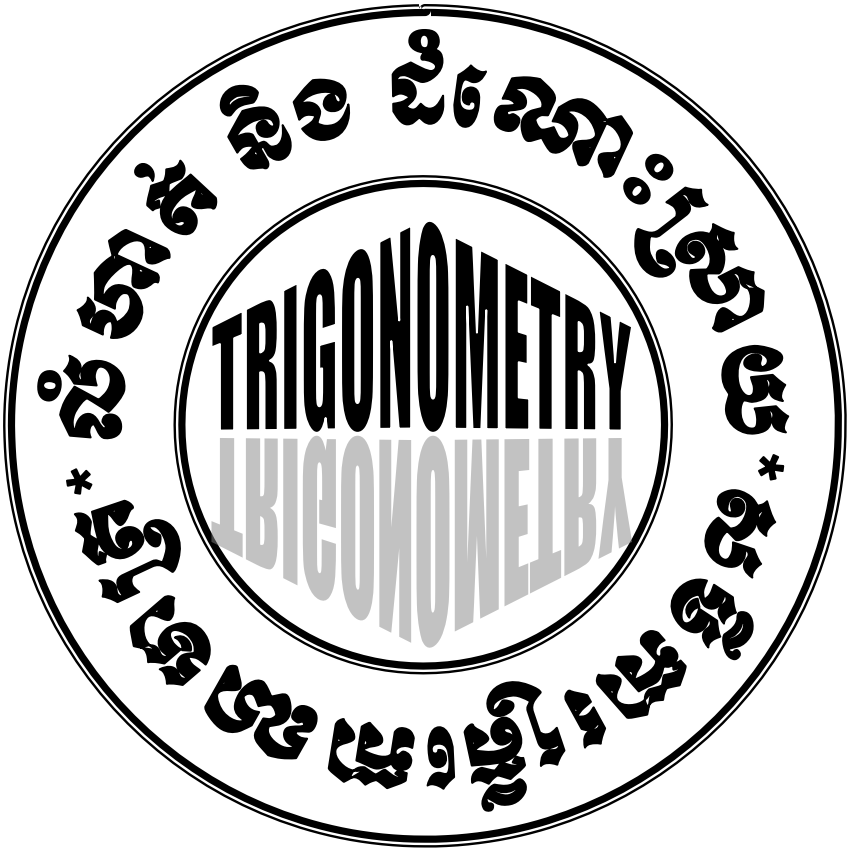
$$\boxed{x=2k\pi, x=\frac{\pi}{2}+2k\pi, k\in\mathbb{Z}}$$

សរុបមក សមីការ $\cos^n x+\sin^n x=1\ ,(n\in\mathbb{N})$ មានចម្លើយ៖

★ $x=k\pi\ ,\ x=\frac{\pi}{2}+k\pi\ ,\ k\in\mathbb{Z}$ បើ $n$ ជាចំនួនគត់គូ ( $n\neq 2$ )

★ $x=2k\pi, x=\frac{\pi}{2}+2k\pi, k\in\mathbb{Z}$ បើ $n$ ជាចំនួនគត់សេស

★ ហើយសមីការ $\cos^n x+\sin^n x=1$ មានចម្លើយច្រើនរាប់មិនអស់ ចំពោះ $n=2$


TRIGONOMETRY

# លំហាត់ និង ដំណោះស្រាយ

លំហាត់ទី១ : ដោះស្រាយសមីការខាងក្រោម ៖

១. $\sin 2x = 1$

២. $\tan x = -\frac{1}{\sqrt{3}}$

៣. $\cos 3x = \frac{1}{2}$

៤. $\cot 2x = 1$

## ចំណោះស្រាយ

១. $\sin 2x = 1$

$\Leftrightarrow 2x = \frac{\pi}{2} + 2k\pi \ , k \in \mathbb{Z}$

$\Rightarrow x = \frac{\pi}{4} + k\pi \ , k \in \mathbb{Z}$

ដូចនេះ សមីការមានឫស $\boxed{x = \frac{\pi}{4} + k\pi \ , k \in \mathbb{Z}}$

២. $\tan x = -\frac{1}{\sqrt{3}}$

(សមីការមានឫសកាលណា $x \neq \frac{\pi}{2} + k\pi \ , k \in \mathbb{Z}$)

$\Leftrightarrow \tan x = -\frac{\sqrt{3}}{3}$

$\Leftrightarrow \tan x = \tan\left(-\frac{\pi}{6}\right)$

$\Rightarrow x = -\frac{\pi}{6} + k\pi \ , k \in \mathbb{Z}$

ដូចនេះ សមីការមានចម្លើយ $\boxed{x = -\frac{\pi}{6} + k\pi \ , k \in \mathbb{Z}}$

៣. $\cos 3x = \frac{1}{2}$

$\Leftrightarrow \cos 3x = \cos\frac{\pi}{3}\ , k \in \mathbb{Z}$

$\Leftrightarrow 3x = \pm\frac{\pi}{3} + 2k\pi$

$\Rightarrow x = \pm\frac{\pi}{9} + \frac{2k\pi}{3}$

ដូចនេះ សមីការមានចម្លើយ $\boxed{x = \pm\frac{\pi}{9} + \frac{2k\pi}{3}\ ,\ k \in \mathbb{Z}}$

៤. $\cot 2x = 1$

សមីការមានឫសកាលណា $2x \neq k\pi \Rightarrow x \neq \frac{k\pi}{2}\ ,\ k \in \mathbb{Z}$

ដោយ $\cot\frac{\pi}{4} = 1$ នោះ សមីការទៅជា $\cot 2x = \cot\frac{\pi}{4}$

$\Leftrightarrow 2x = \frac{\pi}{4} + k\pi\ ,\ k \in \mathbb{Z}$

$\Rightarrow x = \frac{\pi}{8} + \frac{k\pi}{2}\ , k \in \mathbb{Z}$

ដូចនេះ សមីការមានចម្លើយ $\boxed{x = \frac{\pi}{8} + \frac{k\pi}{2}\ , k \in \mathbb{Z}}$

លំហាត់ទី២ : ដោះស្រាយសមីការត្រីកោណមាត្រខាងក្រោម៖

ក. $\sin(x+2) = \frac{1}{2}$ ខ. $\cos(\pi - x) = 1$

គ. $\tan(2x-1) = \sqrt{3}$ ឃ. $\cot(3x-1) = -\sqrt{3}$

## ចំណោះស្រាយ

ក. $\sin(x+2)=\frac{1}{2}$

$$\Leftrightarrow \sin(x+2)=\sin\frac{\pi}{6}$$

$$\Leftrightarrow \begin{cases} x+2=\frac{\pi}{6}+2k\pi \\ x+2=\pi-\frac{\pi}{6}+2k\pi \end{cases}, k\in\mathbb{Z}$$

$$\Rightarrow \begin{cases} x=(\frac{\pi}{6}-2)+2k\pi \\ x=(\frac{5\pi}{6}-2)+2k\pi \end{cases}, \ k\in\mathbb{Z}$$

ដូចនេះ សមីការមានចម្លើយ

$$\boxed{x=(\frac{\pi}{6}-2)+2k\pi\ ,\ x=(\frac{5\pi}{6}-2)+2k\pi\ \ ,k\in\mathbb{Z}}$$

ខ. $\cos(\pi-x)=1$

$$\pi-x=2k\pi\ \ ,k\in\mathbb{Z}$$

$$\Rightarrow x=\pi-2k\pi\ ,k\in\mathbb{Z}$$

ដូចនេះ សមីការមានឫស $\boxed{x=\pi-2k\pi,k\in\mathbb{Z}}$ ឬ $\boxed{x=\pi+2k\pi,k\in\mathbb{Z}}$

គ. $\tan(2x-1)=\sqrt{3}$

សមីការមានឫសកាលណា $2x-1\neq\frac{\pi}{2}+k\pi\ \Rightarrow x\neq\frac{\pi}{4}+\frac{k\pi}{2}+\frac{1}{2}$

ដោយ $\tan\frac{\pi}{3}=\sqrt{3}$ នោះសមីការអាចសរសេរ $\tan(2x-1)=\tan\frac{\pi}{3}$

$$\Leftrightarrow 2x-1=\frac{\pi}{3}+k\pi\,,\,k\in\mathbb{Z}$$

$$\Rightarrow x=\frac{\pi}{6}+\frac{k\pi}{2}+\frac{1}{2}\,,\,k\in\mathbb{Z}$$

ដូចនេះ សមីការមានឫស $\boxed{x=\frac{\pi}{6}+\frac{k\pi}{2}+\frac{1}{2}\,,\,k\in\mathbb{Z}}$

ឃ. $\cot(3x-1)=-\sqrt{3}$

សមីការមានឫសកាលណា $3x-1\neq k\pi \Rightarrow x\neq\frac{k\pi}{3}+\frac{1}{3}$

ដោយ $\cot\left(-\frac{\pi}{6}\right)=-\sqrt{3}$

នោះសមីការអាចសរសេរ $\cot(3x-1)=\cot\left(-\frac{\pi}{6}\right)$

$$\Leftrightarrow 3x-1=-\frac{\pi}{6}+k\pi\ \,,\,k\in\mathbb{Z}$$

$$\Rightarrow x=-\frac{\pi}{18}+\frac{k\pi}{3}+\frac{1}{3}\ \,,k\in\mathbb{Z}$$

ដូចនេះ សមីការមានឫស $\boxed{x=-\frac{\pi}{18}+\frac{k\pi}{3}+\frac{1}{3}\ \,,k\in\mathbb{Z}}$

**លំហាត់ទី៣ : ដោះស្រាយសមីការត្រីកោណមាត្រខាងក្រោម :**

ក. $\tan\left(x+\frac{\pi}{4}\right)=\tan\left(2x+\frac{\pi}{3}\right)$

ខ. $\tan\left(x+\frac{\pi}{4}\right)=-\cot\left(2x-\frac{\pi}{3}\right)$

គ. $\sin x=\cos\left(x+\pi\right)$

## ចំណោះស្រាយ

ក. $\tan\left(x+\frac{\pi}{4}\right)=\tan\left(2x+\frac{\pi}{3}\right)$

$$\text{សមីការមានឫសកាលណា}\begin{cases}x+\frac{\pi}{4}\neq\frac{\pi}{2}+k\pi\\2x+\frac{\pi}{3}\neq\frac{\pi}{2}+k\pi\end{cases},k\in\mathbb{Z}$$

$$\Rightarrow\begin{cases}x\neq-\frac{\pi}{4}+k\pi\\x\neq\frac{\pi}{12}+\frac{k\pi}{2}\end{cases},k\in\mathbb{Z}$$

គេបានសមីការសមមូលនឹង $x+\frac{\pi}{4}=2x+\frac{\pi}{3}+k\pi\ ,\ k\in\mathbb{Z}$

$$\Leftrightarrow -x=\frac{\pi}{12}+k\pi\ \ k\in\mathbb{Z}$$

$$\Rightarrow x=-\frac{\pi}{12}-k\pi\ \ ,k\in\mathbb{Z}$$

ដូចនេះ សមីការមានឫស $\boxed{x=-\frac{\pi}{12}-k\pi\ \ ,k\in\mathbb{Z}}$

ឬ $\boxed{x=-\frac{\pi}{12}+k\pi\ \ ,k\in\mathbb{Z}}$

ខ. $\tan\left(x+\frac{\pi}{4}\right)=-\cot\left(2x-\frac{\pi}{3}\right)$

$$\text{សមីការមានឫសកាលណា}\begin{cases}x+\frac{\pi}{4}\neq\frac{\pi}{2}+k\pi\\2x-\frac{\pi}{3}\neq k\pi\end{cases}$$

$$\Rightarrow \begin{cases} x \neq \frac{\pi}{4} + k\pi \\ x \neq \frac{\pi}{6} + \frac{k\pi}{2} \end{cases}, k \in \mathbb{Z}$$

ដោយ $\tan\left(\frac{\pi}{2}+\alpha\right) = -\cot\alpha$

នោះ $-\cot\left(2x-\frac{\pi}{3}\right) = \tan\left(\frac{\pi}{2}+2x-\frac{\pi}{3}\right) = \tan\left(2x+\frac{\pi}{6}\right)$

គេបាន សមីការទៅជា $\tan\left(x+\frac{\pi}{4}\right) = \tan\left(2x+\frac{\pi}{6}\right)$

$$\Leftrightarrow x+\frac{\pi}{4} = 2x+\frac{\pi}{6}+k\pi,\ k \in \mathbb{Z}$$

$$\Leftrightarrow -x = -\frac{\pi}{12}+k\pi, k \in \mathbb{Z}$$

$$\Rightarrow x = \frac{\pi}{12}-k\pi, k \in \mathbb{Z}$$

ដូចនេះ សមីការមានចម្លើយ $x = \frac{\pi}{12}-k\pi, k \in \mathbb{Z}$

ឬ សមីការមានចម្លើយ $x = \frac{\pi}{12}+k\pi, k \in \mathbb{Z}$

គ. $\sin x = \cos(x+\pi)$

ដោយ $\sin x = \cos\left(\frac{\pi}{2}-x\right)$

នោះសមីការទៅជា $\cos(x+\pi) = \cos\left(\frac{\pi}{2}-x\right)$

យើងបាន $x+\pi = \pm\left(\frac{\pi}{2}-x\right)+2k\pi\ , k \in \mathbb{Z}$

$$\Leftrightarrow \left[\begin{array}{l} x+\pi=\dfrac{\pi}{2}-x+2k\pi \\ x+\pi=-\dfrac{\pi}{2}+x+2k\pi \end{array}\right. , k\in\mathbb{Z}$$

$$\Rightarrow x=-\frac{\pi}{4}+k\pi \ , k\in\mathbb{Z}$$

ដូចនេះ សមីការមានចម្លើយ $\boxed{x=-\frac{\pi}{4}+k\pi \ , k\in\mathbb{Z}}$

លំហាត់ទី៤ : ដោះស្រាយសមីការត្រីកោណមាត្រខាងក្រោម ៖

ក. $\tan x+\cot x=-\dfrac{2}{\cos x}$

ខ. $\dfrac{\sin x}{\sin 2x}-\dfrac{\cos x}{\cos 2x}=0$

គ. $(2\cos x-1)(2\sin x+\cos x)=\sin 2x-\sin x$

## ដំណោះស្រាយ

ក. $\tan x+\cot x=-\dfrac{2}{\cos x}$

សមីការមានឫសកាលណា $\begin{cases}\sin x\neq 0\\ \cos x\neq 0\end{cases}$

$$\Leftrightarrow \begin{cases} x\neq \dfrac{\pi}{2}+k\pi \\ x\neq k\pi \end{cases} \Rightarrow x\neq \frac{k\pi}{2}, k\in\mathbb{Z}$$

យើងបានសមមូលនឹង $\dfrac{\sin x}{\cos x}+\dfrac{\cos x}{\sin x}=-\dfrac{2}{\cos x}$

$$\Leftrightarrow \frac{\sin^2 x+\cos^2 x}{\sin x\cos x}=-\frac{2}{\cos x}$$

$$\Leftrightarrow \frac{1}{\sin x}=-2$$

$$\Leftrightarrow \sin x=-\frac{1}{2}=\sin\left(-\frac{\pi}{6}\right)$$

$$\Leftrightarrow \begin{cases} x=-\dfrac{\pi}{6}+2k\pi \\ x=\dfrac{7\pi}{6}+2k\pi \end{cases},\ k\in\mathbb{Z}$$

ដូចនេះ សមីការមានចម្លើយ $\boxed{x=-\frac{\pi}{6}+2k\pi\ ,\ x=\frac{7\pi}{6}+2k\pi\ ,\ k\in\mathbb{Z}}$

ខ. $\dfrac{\sin x}{\sin 2x}-\dfrac{\cos x}{\cos 2x}=0$

សមីការមានចម្លើយកាលណា $\begin{cases}\sin 2x\neq 0\\ \cos 2x\neq 0\end{cases}$

$$\Leftrightarrow \begin{cases} 2x\neq k\pi \\ 2x\neq \dfrac{\pi}{2}+k\pi \end{cases} \Rightarrow x\neq\frac{k\pi}{4},\ k\in\mathbb{Z}$$

យើងបាន $\dfrac{\sin x}{\sin 2x}-\dfrac{\cos x}{\cos 2x}=0$

$$\Leftrightarrow \frac{\sin x\cos 2x-\cos x\sin 2x}{\sin 2x\cos 2x}=0$$

$$\Leftrightarrow \sin(x-2x)=0$$

$$\Leftrightarrow \sin x=0$$

តែ $\sin 2x\neq 0\Leftrightarrow 2\sin x\cos x\neq 0\Rightarrow \sin x\neq 0$

ដូចនេះ សមីការគ្មានចម្លើយ

គ. $(2\cos x-1)(2\sin x+\cos x)=\sin 2x-\sin x$

$\Leftrightarrow (2\cos x-1)(2\sin x+\cos x)=\sin x(2\cos x-1)$

$\Leftrightarrow (2\cos x-1)(2\sin x+\cos x-\sin x)=0$

$\Leftrightarrow (2\cos x-1)(\sin x-\cos x)=0$

$$\Leftrightarrow \left[\begin{array}{l} 2\cos x-1=0 \\ \sin x-\cos x=0 \end{array}\right.$$

$$\Leftrightarrow \left[\begin{array}{l} \cos x=\dfrac{1}{2}=\cos\dfrac{\pi}{3} \\ \tan x=1=\tan\dfrac{\pi}{4} \end{array}\right.$$

$$\Rightarrow \left[\begin{array}{l} x=\pm\dfrac{\pi}{3}+2k\pi \\ x=\dfrac{\pi}{4}+k\pi \end{array}\right. , k\in\mathbb{Z}$$

ដូចនេះ សមីការមានចម្លើយ $\boxed{x=\pm\frac{\pi}{3}+2k\pi, x=\frac{\pi}{4}+k\pi}$

លំហាត់ទី៥ : ដោះស្រាយសមីការត្រីកោណមាត្រខាងក្រោម៖

ក. $\sqrt{3}\tan(2x-36^o)-1=0$ , $x\in\left[90^o,180^o\right]$

ខ. $\sin(3x+1)+\sin(3x-1)=0$ , $x\in[-\pi,\pi]$

គ. $\cos 3x-4\cos 2x+3\cos x-4=0$ , $x\in[0,14]$

## ចំណោះស្រាយ

ក. $\sqrt{3}\tan(2x-36^o)-1=0$ , $x\in\left[90^o,180^o\right]$

សមីការមានឫសកាលណា $2x-36^o \neq 90^o + k\cdot 180^o$

$$\Rightarrow x \neq 63^o + k\cdot 90^o$$

យើងបាន សមីការ $\sqrt{3}\tan(2x-36^o)-1=0$

$$\Leftrightarrow \tan(2x-36^o)=\frac{\sqrt{3}}{3}=\tan 30^o$$

$$\Leftrightarrow 2x-36^o=30^o+k\cdot 180^o$$

$$\Rightarrow x=33^o+k\cdot 90^o$$

បើ $k=0\Rightarrow x=33^o \notin \left[90^o, 180^o\right]$(មិនយក)

បើ $k=1\Rightarrow x=33^o+90^o=123^o \in \left[90^o, 180^o\right]$

បើ $k=2\Rightarrow x=33^o+180^o \notin \left[90^o, 180^o\right]$(មិនយក)

ដូចនេះ សមីការមានចម្លើយ $x=123^o$

ខ. $\sin(3x+1)+\sin(3x-1)=0$ , $x\in\left[-\pi, \pi\right]$

$$\Leftrightarrow \sin(3x+1)=-\sin(3x-1)$$

$$\Leftrightarrow \sin(3x+1)=\sin(1-3x)$$

$$\Leftrightarrow \begin{bmatrix} 3x+1=1-3x+2k\pi \\ 3x+1=\pi-1+3x+2k\pi \end{bmatrix}, k\in\mathbb{Z}$$

$$\Leftrightarrow \begin{bmatrix} 6x=2k\pi \\ 0x=-2+\pi+2k\pi \end{bmatrix}, k\in\mathbb{Z}$$

$$\Rightarrow x=\frac{k\pi}{3}, k\in\mathbb{Z}$$

ដោយ $x\in\left[-\pi, \pi\right]$ឬ $-\pi\le x\le\pi$

$$-\pi\le\frac{k\pi}{3}\le\pi$$

$$\Rightarrow -3 \le k \le 3 \ , k \in \mathbb{Z}$$

$$\Rightarrow k = 0 , \pm 1 , \pm 2 , \pm 3$$

យើងបាន $x = 0 , \pm \frac{\pi}{3} , \pm \frac{2\pi}{3} , \pm \pi$

ដូចនេះ សមីការមានចម្លើយ $x \in \left\{ 0 , \pm \frac{\pi}{3} , \pm \frac{2\pi}{3} , \pm \pi \right\}$

គ. $\cos 3x - 4\cos 2x + 3\cos x - 4 = 0 \ , x \in [0 , 14]$

$$\Leftrightarrow (4\cos^3 x - 3\cos x) - 4(2\cos^2 x - 1) + 3\cos x - 4 = 0$$

$$\Leftrightarrow 4\cos^3 x - 8\cos^2 x = 0$$

$$\Leftrightarrow 4\cos^2 x(\cos x - 2) = 0$$

$$\Leftrightarrow \cos x = 0 \ \ (\text{ព្រោះ } \cos x - 2 < 0 , \forall x \in \mathbb{R})$$

$$\Rightarrow x = \frac{\pi}{2} + k\pi , k \in \mathbb{Z}$$

ដោយ $0 \le x \le 14 \Leftrightarrow 0 \le \frac{\pi}{2} + k\pi \le 14$

$$\Leftrightarrow -\frac{1}{2} \le k \le \frac{14}{\pi} - \frac{1}{2} \ , k \in \mathbb{Z}$$

$$\Leftrightarrow k \in \{0 , 1 , 2 , 3\}$$

យើងបាន $x \in \left\{ \frac{\pi}{2} , \frac{3\pi}{2} , \frac{5\pi}{2} , \frac{7\pi}{2} \right\}$

ដូចនេះ សមីការមានចម្លើយ $\boxed{x \in \left\{ \frac{\pi}{2} , \frac{3\pi}{2} , \frac{5\pi}{2} , \frac{7\pi}{2} \right\}}$

លំហាត់ទី៦ : ដោះស្រាយសមីការត្រីកោណមាត្រខាងក្រោម ៖

ក. $\sin(2x+\frac{5\pi}{2})-3\cos(x-\frac{7\pi}{2})=1+2\sin x \ , x\in\left[\frac{\pi}{2},3\pi\right]$

ខ. $\frac{\sin 3x-\sin x}{\sqrt{1-\cos 2x}}=\sin 2x+\cos 2x \ , x\in(0,2\pi)$

## ដំណោះស្រាយ

ក. $\sin(2x+\frac{5\pi}{2})-3\cos(x-\frac{7\pi}{2})=1+2\sin x \ , x\in\left[\frac{\pi}{2},3\pi\right]$

ដោយ $\sin(2x+\frac{5\pi}{2})=\sin(2x+\frac{\pi}{2}+2\pi)=\cos 2x$

$\cos(x-\frac{7\pi}{2})=\cos(x+\frac{\pi}{2}-4\pi)=-\sin x$

យើងបានសមីការទៅជា $\cos 2x+3\sin x=1+2\sin x$

$\Leftrightarrow(1-2\sin^2 x)+3\sin x-(1+2\sin x)=0$

$\Leftrightarrow -2\sin^2 x+\sin x=0$

$\Leftrightarrow -\sin x(2\sin x-1)=0$

$\Leftrightarrow\begin{cases}-\sin x=0\\2\sin x-1=0\end{cases}$

$\Leftrightarrow\begin{cases}\sin x=0\\\sin x=\frac{1}{2}=\sin\frac{\pi}{6}\end{cases}$

$$\Rightarrow \begin{cases} x = k\pi \\ \begin{cases} x = \dfrac{\pi}{6} + 2k\pi \\ x = \pi - \dfrac{\pi}{6} + 2k\pi = \dfrac{5\pi}{6} + 2k\pi \end{cases} \end{cases} \quad , k \in \mathbb{Z}$$

ដោយ $x \in \left[\frac{\pi}{2}, 3\pi\right]$ ឬ $\frac{\pi}{2} \le x \le 3\pi$

ដូចនេះសមីការមានឫស $\boxed{x \in \left\{\pi, 2\pi, 3\pi, \frac{13\pi}{6}, \frac{5\pi}{6}, \frac{17\pi}{6}\right\}}$

ខ. $\dfrac{\sin 3x - \sin x}{\sqrt{1-\cos 2x}} = \sin 2x + \cos 2x \ , x \in (0, 2\pi)$

សមីការមានឫសកាលណា $1-\cos 2x \neq 0$

$$\Leftrightarrow 2\sin^2 x \neq 0 \Rightarrow x \neq k\pi, k \in \mathbb{Z}$$

យើងបានសមីការអាចសរសេរ $\dfrac{2\cos 2x \sin x}{\sqrt{2\sin^2 x}} = \sin 2x + \cos 2x$

$$\Leftrightarrow \frac{2\cos 2x \sin x}{\sqrt{2}|\sin x|} = \sqrt{2}\cos\left(2x - \frac{\pi}{4}\right)$$

$$\Leftrightarrow \frac{\cos 2x \sin x}{|\sin x|} = \cos\left(2x - \frac{\pi}{4}\right) \quad (*)$$

☞ បើ $x \in (0, \pi)$ នោះ $\sin x > 0$

យើងបាន (*) ទៅជា $\dfrac{\cos 2x \sin x}{\sin x} = \cos\left(2x - \frac{\pi}{4}\right)$

$$\Leftrightarrow \cos 2x = \cos\left(2x - \frac{\pi}{4}\right)$$

$$\Leftrightarrow \begin{cases} 2x = 2x - \dfrac{\pi}{4} + 2k\pi \\ 2x = -2x + \dfrac{\pi}{4} + 2k\pi \end{cases}, k \in \mathbb{Z}$$

$$\Leftrightarrow \begin{cases} 0x = -\dfrac{\pi}{4} + 2k\pi \\ 4x = \dfrac{\pi}{4} + 2k\pi \end{cases}, k \in \mathbb{Z}$$

$$\Leftrightarrow x = \frac{\pi}{16} + \frac{k\pi}{2}$$

ដោយ $x \in (0, \pi)$ យើងបាន $x \in \left\{ \dfrac{\pi}{16}, \dfrac{9\pi}{16} \right\}$

☞ បើ $x \in (\pi, 2\pi)$ នោះ $\sin x < 0$

យើងបានសមីការ (*) ទៅជា $\dfrac{\cos 2x \sin x}{-\sin x} = \cos\left(2x - \dfrac{\pi}{4}\right)$

$$\Leftrightarrow -\cos 2x = \cos\left(2x - \frac{\pi}{4}\right)$$

$$\Leftrightarrow \cos(2x - \pi) = \cos\left(2x - \frac{\pi}{4}\right)$$

$$\Leftrightarrow \begin{cases} 2x - \pi = 2x - \dfrac{\pi}{4} + 2k\pi \\ 2x - \pi = -2x + \dfrac{\pi}{4} + 2k\pi \end{cases}, k \in \mathbb{Z}$$

$$\Leftrightarrow \begin{cases} 0x = \dfrac{3\pi}{4} + 2k\pi \\ 4x = \dfrac{5\pi}{4} + 2k\pi \end{cases}, k \in \mathbb{Z}$$

$$\Leftrightarrow x = \frac{5\pi}{16} + \frac{k\pi}{2}, k \in \mathbb{Z}$$

ដោយ $x\in(\pi,2\pi)$ យើងបាន $x\in\left\{\frac{21\pi}{16},\frac{29\pi}{16}\right\}$

ដូចនេះ សមីការមានចម្លើយ $\boxed{x\in\left\{\frac{\pi}{16},\frac{9\pi}{16},\frac{21\pi}{16},\frac{29\pi}{16}\right\}}$

លំហាត់ទី៧ : ដោះស្រាយសមីការត្រីកោណមាត្រខាងក្រោម ៖

ក. $\sin x+\cos x+1+\sin 2x+\cos 2x=0$

ខ. $\cot x+\sin x(1+\tan x\tan\frac{x}{2})=4$

គ. $\frac{1}{\sin x}+\frac{1}{\sin(x-\frac{3\pi}{2})}=4\sin\left(\frac{7\pi}{4}-x\right)$

## ចំណោះស្រាយ

ក. $\sin x+\cos x+1+\sin 2x+\cos 2x=0$

$\Leftrightarrow \sin x+\cos x+1+2\sin x\cos x+2\cos^2 x-1=0$

$\Leftrightarrow (\sin x+\cos x)+2\cos x(\sin x+\cos x)=0$

$\Leftrightarrow (\sin x+\cos x)(1+2\cos x)=0$

$\Leftrightarrow \left[\begin{array}{l}\sin x+\cos x=0\\1+2\cos x=0\end{array}\right.$

$\Leftrightarrow \left[\begin{array}{l}\tan x=-1=\tan\left(-\frac{\pi}{4}\right)\\\cos x=-\frac{1}{2}=\cos\left(\frac{2\pi}{3}\right)\end{array}\right.$

$$\Leftrightarrow \begin{bmatrix} x=-\frac{\pi}{4}+k\pi \\ x=\pm\frac{2\pi}{3}+2k\pi \end{bmatrix}, k\in\mathbb{Z}$$

ដូចនេះ សមីការមានចម្លើយ $\boxed{x=-\frac{\pi}{4}+k\pi\ ,\ x=\pm\frac{2\pi}{3}+2k\pi\ ,\ k\in\mathbb{Z}}$

ខ. $\cot x+\sin x(1+\tan x\tan\frac{x}{2})=4$

សមីការមានឫសកាលណា $\begin{cases}\cos x\neq 0\\ \cos\frac{x}{2}\neq 0\\ \sin x\neq 0\end{cases}$

$$\Leftrightarrow \begin{cases} x\neq\frac{\pi}{2}+k\pi=\frac{(2k+1)\pi}{2} \\ x\neq\pi+2k\pi=(2k+1)\pi \\ x\neq k\pi \end{cases}$$

$$\Leftrightarrow x\neq\frac{k\pi}{2}\ ,\ k\in\mathbb{Z}$$

ដោយ $1+\tan x\tan\frac{x}{2}=1+\frac{\sin x\cdot\sin\frac{x}{2}}{\cos x\cdot\cos\frac{x}{2}}$

$$=\frac{\cos x\cos\frac{x}{2}+\sin x\sin\frac{x}{2}}{\cos x\cos\frac{x}{2}}$$

$$= \frac{\cos\left(x-\frac{x}{2}\right)}{\cos x \cos\frac{x}{2}} = \frac{1}{\cos x}$$

យើងបាន សមីការអាចសរសេរ $\cot x + \sin x \cdot \frac{1}{\cos x} = 4$

$$\Leftrightarrow \frac{\cos^2 x + \sin^2 x}{\sin x \cos x} = 4$$

$$\Leftrightarrow \frac{2}{\sin 2x} = 4$$

$$\Leftrightarrow \sin 2x = \frac{1}{2} = \sin\frac{\pi}{6}$$

$$\Leftrightarrow \begin{cases} 2x = \frac{\pi}{6} + 2k\pi \\ 2x = \pi - \frac{\pi}{6} + 2k\pi \end{cases}, k \in \mathbb{Z}$$

$$\Rightarrow \begin{cases} x = \frac{\pi}{12} + k\pi \\ x = \frac{5\pi}{12} + k\pi \end{cases}, k \in \mathbb{Z}$$

ដូចនេះ សមីការមានចម្លើយ $\boxed{x = \frac{\pi}{12} + k\pi \text{ , } x = \frac{5\pi}{12} + k\pi \text{ , } k \in \mathbb{Z}}$

គ. $\frac{1}{\sin x} + \frac{1}{\sin(x-\frac{3\pi}{2})} = 4\sin\left(\frac{7\pi}{4} - x\right)$

ដោយ $\sin\left(x-\frac{3\pi}{2}\right) = \cos x$

សមីការមានឫសកាលណា $\begin{cases}\sin x \neq 0\\ \cos x \neq 0\end{cases}$

$$\Leftrightarrow \sin 2x \neq 0$$

$$\Leftrightarrow x \neq \frac{k\pi}{2}, k \in \mathbb{Z}$$

យើងបានសមីការទៅជា $\frac{1}{\sin x}+\frac{1}{\cos x}=4\sin\left(\frac{7\pi}{4}-x\right)$

$$\Leftrightarrow \frac{1}{\sin x}+\frac{1}{\cos x}=4\left(\sin\frac{7\pi}{4}\cos x-\sin x\cos\frac{7\pi}{4}\right)$$

$$\Leftrightarrow \frac{\cos x+\sin x}{\sin x\cos x}=4\left(-\frac{\sqrt{2}}{2}\cos x-\frac{\sqrt{2}}{2}\sin x\right)$$

$$\Leftrightarrow \frac{2(\sin x+\cos x)}{\sin 2x}=-2\sqrt{2}(\cos x+\sin x)$$

$$\Leftrightarrow 2(\sin x+\cos x)\left(\frac{1}{\sin 2x}+\sqrt{2}\right)=0$$

$$\Leftrightarrow \left[\begin{array}{l}\sin x+\cos x=0\\ \dfrac{1}{\sin 2x}+\sqrt{2}=0\end{array}\right.$$

$$\Leftrightarrow \left[\begin{array}{l}\tan x=-1=\tan\left(-\dfrac{\pi}{4}\right)\\ \sin 2x=-\dfrac{\sqrt{2}}{2}=\sin\left(-\dfrac{\pi}{4}\right)\end{array}\right.$$

$$\Leftrightarrow \begin{cases} x=-\dfrac{\pi}{4}+k\pi \\ 2x=-\dfrac{\pi}{4}+2k\pi \quad k\in\mathbb{Z} \\ 2x=\pi+\dfrac{\pi}{4}+2k\pi \end{cases}$$

$$\Leftrightarrow \begin{cases} x=-\dfrac{\pi}{4}+k\pi \\ x=-\dfrac{\pi}{8}+k\pi \ , k\in\mathbb{Z} \\ x=\dfrac{5\pi}{8}+k\pi \end{cases}$$

ដូចនេះ សមីការមានចម្លើយ :

$$x=-\frac{\pi}{4}+k\pi\ ,\ x=-\frac{\pi}{8}+k\pi\ ,\ x=\frac{5\pi}{8}+k\pi\ ,\ k\in\mathbb{Z}$$

**លំហាត់ទី៨:ដោះស្រាយសមីការ** $\cos\left[\dfrac{\pi}{4}\left(3x-\sqrt{9x^2-16x-80}\right)\right]=1$

## ចំណោះស្រាយ

សមីការមានឫសកាលណា $9x^2-16x-80\geq 0$

នោះ $x\in(-\infty\ ,-\dfrac{20}{9}]\cup[4\ ,+\infty)$

យើងបាន $\cos\left[\dfrac{\pi}{4}\left(3x-\sqrt{9x^2-16x-80}\right)\right]=1$

$$\Leftrightarrow \frac{\pi}{4}\left(3x-\sqrt{9x^2-16x-80}\right)=2k\pi\ ,\ k\in\mathbb{Z}$$

$\Leftrightarrow 3x-\sqrt{9x^2-16x-80}=8k$

$\Leftrightarrow 3x-8k=\sqrt{9x^2-16x-80}$ (#)

នោះ សមីការមានឫសកាលណា $3x-8k\geq 0\Rightarrow x\geq \frac{8k}{3}$

លើកសមីការ (#) ជាការេគេបាន ៖

$\Leftrightarrow (3x-8k)^2=\left(\sqrt{9x^2-16x-80}\right)^2$

$\Leftrightarrow 9x^2-48kx+64k^2=9x^2-16x-80$

$\Leftrightarrow (16-48k)x=-64k^2-80$

$\Rightarrow x=\frac{-64k^2-80}{16-48k}=\frac{4k^2+5}{3k-1}$

ដូចនេះ សមីការមានឫស $\boxed{x=\frac{4k^2+5}{3k-1},\ k\in\mathbb{Z}}$

លំហាត់ទី៩: ដោះស្រាយសមីការត្រីកោណមាត្រខាងក្រោម ៖

ក. $2\sin^2 x+3\sin x-5=0$

ខ. $2\cos^2 2x+3\cos 2x+1=0$

គ. $\tan^4 x-2\tan^2 x+1=0$

ឃ. $\cot^2 x+2\cot x-3=0$

## ដំណោះស្រាយ

ក. $2\sin^2 x+3\sin x-5=0$

តាង $t=\sin x$ ដែល $-1\leq t\leq 1$

យើងបានសមីការទៅជា $2t^2+3t-5=0$

$$\Delta = 9+40 = 49 = 7^2$$

$$t_1 = \frac{-3+7}{2\cdot 2} = 1$$

$$t_2 = \frac{-3-7}{2\cdot 22} = -\frac{5}{2} < -1 \text{ (មិនយក)}$$

តែ $t=\sin x$ យើងបាន $\sin x = 1 \Rightarrow x = \frac{\pi}{2}+2k\pi \ , k\in\mathbb{Z}$

ដូចនេះ សមីការមានចម្លើយ $\boxed{x=\frac{\pi}{2}+2k\pi \ , k\in\mathbb{Z}}$

ខ. $2\cos^2 2x+3\cos 2x+1=0$

តាង $t=\cos 2x$ ដែល $-1\le t\le 1$

យើងបានសមីការទៅជា $2t^2+3t+1=0$

ដោយ $a-b+c=2-3+1=0$

នោះ $t_1=-1, t_2=-\frac{c}{a}=-\frac{1}{2}$

តែ $t=\cos 2x$ យើងបាន:

- $t=1 \Leftrightarrow \cos 2x=-1$

$$\Leftrightarrow 2x=\pi+2k\pi \ , k\in\mathbb{Z}$$

$$\Rightarrow x=\frac{\pi}{2}+k\pi \ , k\in\mathbb{Z}$$

$$t=-\frac{1}{2} \Leftrightarrow \cos 2x=-\frac{1}{2}=\cos\left(\frac{2\pi}{3}\right)$$

$$\Leftrightarrow 2x=\pm\frac{2\pi}{3}+2k\pi$$

$$\Rightarrow x=\pm\frac{\pi}{3}+k\pi\ ,k\in\mathbb{Z}$$

ដូចនេះ សមីការមានចម្លើយ $\boxed{x=\frac{\pi}{2}+k\pi\ ,x=\pm\frac{\pi}{3}+k\pi\ ,k\in\mathbb{Z}}$

គ. $\tan^4 x-2\tan^2 x+1=0$

សមីការមានចម្លើយកាលណា $\cos x\neq 0\Rightarrow x\neq\frac{\pi}{2}+k\pi\ ,k\in\mathbb{Z}$

តាង $t=\tan x\ ,t\in\mathbb{R}$

យើងបានសមីការទៅជា $t^4-2t^2+1=0$

$$\Leftrightarrow (t^2-1)^2=0$$
$$\Leftrightarrow t^2-1=0$$
$$\Rightarrow t=\pm 1$$

- $t=1\Leftrightarrow\tan x=1=\tan\frac{\pi}{4}$

$$\Rightarrow x=\frac{\pi}{4}+k\pi\ ,k\in\mathbb{Z}$$

- $t=-1\Leftrightarrow\tan x=-1=\tan\left(-\frac{\pi}{4}\right)$

$$\Rightarrow x=-\frac{\pi}{4}+k\pi\ ,k\in\mathbb{Z}$$

ដូចនេះ សមីការមានចម្លើយ $\boxed{x=\pm\frac{\pi}{4}+k\pi\ ,k\in\mathbb{Z}}$

ឃ. $\cot^2 x+2\cot x-3=0$

សមីការមានន័យកាលណា $\sin x\neq 0\Rightarrow x\neq k\pi\ ,k\in\mathbb{Z}$

តាង $t=\cot x\ ,t\in\mathbb{R}$

យើងបានសមីការទៅជា $t^2+2t-3=0$

ដោយ $a+b+c=1+2-3=0$

នោះ $t_1=1, t_2=\frac{c}{a}=-\frac{3}{1}=-3$

- $t=1\Leftrightarrow \cot x=1=\cot\frac{\pi}{4}$

$$\Rightarrow x=\frac{\pi}{4}+k\pi \ , k\in\mathbb{Z}$$

- $t=-3\Leftrightarrow \cot x=-3$

$$\Rightarrow x=\operatorname{arc}\cot(-3)+k\pi, k\in\mathbb{Z}$$

ដូចនេះ សមីការមានចម្លើយ

$$x=\frac{\pi}{4}+k\pi \ , \ x=\operatorname{arc}\cot(-3)=k\pi, k\in\mathbb{Z}$$

លំហាត់ទី១០ : ដោះស្រាយសមីការត្រីកោណមាត្រខាងក្រោម៖

ក. $\sin^4 x+\cos^4 x=\sin 2x-\frac{1}{2}$

ខ. $\sin^4 x+\cos^4 x=-\frac{1}{2}\sin 2x$

គ. $\frac{\sin^4 x+\cos^4 x}{\cos^2 x-\sin^2 x}=-\frac{1}{2}\tan 2x$

ឃ. $\sin^4 x+\cos^4 x=\frac{11}{8}-\sin 2x$

## ចំណោះស្រាយ

ក. $\sin^4 x+\cos^4 x=\sin 2x-\frac{1}{2}$

$\Leftrightarrow (\cos^2 x+\sin^2 x)^2-2\sin^2 x\cos^2 x=\sin 2x-\frac{1}{2}$

$\Leftrightarrow 1^2-\frac{1}{2}\sin^2 2x-\sin 2x+\frac{1}{2}=0$

$\Leftrightarrow \sin^2 2x+2\sin 2x-3=0$

តាង $t=\sin 2x$ ដែល $-1\le t\le 1$

នោះសមីការទៅជា $t^2+2t-3=0$

ដោយ $a+b+c=1+2-3=0$

នោះ $t_1=1$ , $t_2=\frac{c}{a}=-3<-1$ (មិនយក)

- $\sin 2x=1$

$\Leftrightarrow 2x=\frac{\pi}{2}+2k\pi$ , $k\in\mathbb{Z}$

$\Rightarrow x=\frac{\pi}{4}+k\pi$ , $k\in\mathbb{Z}$

ដូចនេះ សមីការមានចម្លើយ $\boxed{x=\frac{\pi}{4}+k\pi \ , k\in\mathbb{Z}}$

ខ. $\sin^4 x+\cos^4 x=-\frac{1}{2}\sin 2x$

$\Leftrightarrow (\sin^2 x+\cos^2 x)^2-2\sin^2 x\cos^2 x=-\frac{1}{2}\sin 2x$

$\Leftrightarrow 1^2-\frac{1}{2}\sin^2 2x+\frac{1}{2}\sin 2x=0$

$\Leftrightarrow \sin^2 2x-\sin 2x-2=0$

តាង $\sin 2x = t$ ដែល $-1 \le t \le 1$

យើងបានសមីការទៅជា $t^2 - t - 2 = 0$

ដោយ $b = a + c$ គឺ $-1 = 1 - 2$

នោះ $t_1 = -1$ , $t_2 = -\frac{c}{a} = 2 \notin [-1,1]$ (មិនយក)

តែ $t = \sin 2x$ យើងបាន $\sin 2x = -1$

$$\Leftrightarrow 2x = -\frac{\pi}{2} + 2k\pi \ , \ k \in \mathbb{Z}$$

$$\Rightarrow x = -\frac{\pi}{4} + k\pi \ , k \in \mathbb{Z}$$

ដូចនេះ សមីការមានឫស $\boxed{x = -\frac{\pi}{4} + k\pi \ , k \in \mathbb{Z}}$

គ. $\frac{\sin^4 x + \cos^4 x}{\cos^2 x - \sin^2 x} = -\frac{1}{2}\tan 2x$

$\Leftrightarrow \frac{\sin^4 x + \cos^4 x}{\cos 2x} = -\frac{\sin 2x}{2\cos 2x}$

សមីការមានន័យកាលណា $\cos 2x \neq 0$

$$\Leftrightarrow 2x \neq \frac{\pi}{2} + k\pi \ , k \in \mathbb{Z}$$

$$\Rightarrow x \neq \frac{\pi}{4} + \frac{k\pi}{2} \ , k \in \mathbb{Z}$$

ដោយ $\sin^4 x + \cos^4 x = (\sin^2 x + \cos^2 x)^2 - 2\sin^2 x \cos^2 x$

$$= 1 - \frac{1}{2}\sin^2 2x$$

នោះសមីការទៅ $\frac{1 - \frac{1}{2}\sin^2 2x}{\cos 2x} = -\frac{\sin 2x}{2\cos 2x}$

$$\Leftrightarrow 1-\frac{1}{2}\sin^2 2x=-\frac{1}{2}\sin 2x$$

$$\Leftrightarrow \sin^2 2x-\sin 2x-2=0$$

តាង $\sin 2x=t$ ដែល $-1\le t\le 1$

យើងបានសមីការទៅជា $t^2-t-2=0$

ដោយ $b=a+c$ គឺ $-1=1-2$

នោះ $t_1=-1$ , $t_2=-\frac{c}{a}=2\notin[-1,1]$ (មិនយក)

តែ $t=\sin 2x$ យើងបាន $\sin 2x=-1$

$$\Leftrightarrow 2x=-\frac{\pi}{2}+2k\pi \ , \ k\in\mathbb{Z}$$

$$\Rightarrow x=-\frac{\pi}{4}+k\pi \ , k\in\mathbb{Z}$$

ដូចនេះ សមីការមានឫស $\boxed{x=-\frac{\pi}{4}+k\pi \ , k\in\mathbb{Z}}$

ឃ. $\sin^4 x+\cos^4 x=\frac{11}{8}-\sin 2x$

$$\Leftrightarrow (\sin^2 x+\cos^2 x)^2-2\sin^2 x\cos^2 x=\frac{11}{8}-\sin 2x$$

$$\Leftrightarrow 1-\frac{1}{2}\sin^2 2x=\frac{11}{8}-\sin 2x$$

$$\Leftrightarrow 4\sin^2 2x-8\sin 2x+3=0$$

តាង $t=\sin 2x$ ដែល $-1\le t\le 1$

យើងបានសមីការទៅជា $4t^2-8t+3=0$

$$\Delta'=16-12=4=2^2$$

$$t_1=\frac{4-2}{4}=\frac{1}{2}$$

$$t_2=\frac{4+2}{4}=\frac{3}{2}>1 \quad (\text{មិនយក})$$

តែ $t=\sin 2x$ យើងបាន $\sin 2x=\frac{1}{2}=\sin\frac{\pi}{6}$

$$\Leftrightarrow\begin{cases}2x=\frac{\pi}{6}+2k\pi\\ 2x=\pi-\frac{\pi}{6}+2k\pi\end{cases},\ k\in\mathbb{Z}$$

$$\Rightarrow\begin{cases}x=\frac{\pi}{12}+k\pi\\ x=\frac{5\pi}{12}+k\pi\end{cases},\ k\in\mathbb{Z}$$

ដូចនេះ សមីការមានចម្លើយ $\boxed{x=\frac{\pi}{12}+k\pi\ ,\ x=\frac{5\pi}{12}+k\pi\ ,\ k\in\mathbb{Z}}$

លំហាត់ទី១១ : ដោះស្រាយសមីការត្រីកោណមាត្រខាងក្រោម៖

ក. $\cos^2\left(x+\frac{\pi}{3}\right)+2\sin\left(-x+\frac{\pi}{6}\right)=3$

ខ. $\sin^4 2x+\cos^4 2x=\sin 2x\cos 2x$

គ. $\sin^8 x+\cos^8 x=\frac{17}{16}\cos^2 2x$

ឃ. $\sin^6 x+\cos^6 x-\sin 2x=0$

ង. $\cos^4\frac{x}{5}+\sin^2\frac{x}{5}=1$

## ចំណោះស្រាយ

ក. $\cos^2\left(x+\frac{\pi}{3}\right)+2\sin\left(-x+\frac{\pi}{6}\right)=3$

ដោយ $\sin\left(-x+\frac{\pi}{6}\right)=\cos\left[\frac{\pi}{2}-\left(-x+\frac{\pi}{6}\right)\right]=\cos\left(x+\frac{\pi}{3}\right)$

តាង $t=\cos\left(x+\frac{\pi}{3}\right)$ ដែល $-1\le t\le 1$

យើងបានសមីការទៅជា $t^2+2t=3\Leftrightarrow t^2+2t-3=0$

ដោយ $a+b+c=1+2-3=0$

នោះ $t_1=1$ , $t_2=\frac{c}{a}=-\frac{3}{1}=-3<-1$ (មិនយក)

តែ $t=\cos\left(x+\frac{\pi}{3}\right)$ យើងបាន $\cos\left(x+\frac{\pi}{3}\right)=1$

$$x+\frac{\pi}{3}=2k\pi \ , \ k\in\mathbb{Z}$$

$$\Rightarrow x=-\frac{\pi}{3}+2k\pi \ , \ k\in\mathbb{Z}$$

ដូចនេះ សមីការមានចម្លើយ $\boxed{x=-\frac{\pi}{3}+2k\pi \ , \ k\in\mathbb{Z}}$

ខ. $\sin^4 2x+\cos^4 2x=\sin 2x\cos 2x$

$$\Leftrightarrow (\sin^2 2x+\cos^2 2x)^2-2\sin^2 2x\cos^2 2x=\frac{1}{2}\sin 4x$$

$$\Leftrightarrow 1-\frac{1}{2}\sin^2 4x=\frac{1}{2}\sin 4x$$

$$\Leftrightarrow \sin^2 4x+\sin 4x-2=0$$

តាង $t=\sin 4x$ ដែល $-1\le t\le 1$

យើងបានសមីការទៅជា $t^2+t-2=0$

ដោយ $a+b+c=1+1-2=0$

នោះ $t_1=1$ , $t_2=\dfrac{c}{a}=-2<-1$ (មិនយក)

តែ $t=\sin 4x$ យើងបាន $\sin 4x=1$

$$\Leftrightarrow 4x=\frac{\pi}{2}+2k\pi \ , \ k\in\mathbb{Z}$$

$$\Rightarrow x=\frac{\pi}{8}+\frac{k\pi}{2} \ , \ k\in\mathbb{Z}$$

ដូចនេះ សមីការមានឫស $\boxed{x=\dfrac{\pi}{8}+\dfrac{k\pi}{2} \ , \ k\in\mathbb{Z}}$

គ. $\sin^8 x+\cos^8 x=\dfrac{17}{16}\cos^2 2x$

យើងមាន $\sin^8 x+\cos^8 x=(\sin^4 x+\cos^4 x)^2-2\sin^4 x\cos^4 x$

$$=\left[(\sin^2 x+\cos^2 x)^2-2\sin^2 x\cos^2 x\right]^2-2\left(\frac{1}{2}\sin 2x\right)^4$$

$$=\left(1-\frac{1}{2}\sin^2 2x\right)^2-\frac{1}{8}\sin^4 2x$$

យើងបាន $\left(1-\dfrac{1}{2}\sin^2 2x\right)^2-\dfrac{1}{8}\sin^4 2x=\dfrac{17}{16}\cos^2 2x$

$$\Leftrightarrow\left(1-\frac{1}{2}\sin^2 2x\right)^2-\frac{1}{8}\sin^4 2x=\frac{17}{16}\left(1-\sin^2 2x\right)$$

តាង $t=\sin^2 2x$ ដែល $0\le t\le 1$

យើងបាន $\left(1-\dfrac{t}{2}\right)^2-\dfrac{1}{8}t^2=\dfrac{17}{16}(1-t)$

$$\Leftrightarrow 1-t+\frac{t^2}{4}-\frac{t^2}{8}=\frac{17}{16}-\frac{17}{16}t$$

$$\Leftrightarrow 2t^2+t-1=0$$

ដោយ $a-b+c=2-1+(-1)=0$

នោះ $t_1=-1<0$ (មិនយក)

$$t_2=-\frac{c}{a}=\frac{1}{2}$$

តែ $t=\sin^2 2x$ យើងបាន $\sin^2 2x=\frac{1}{2}$

$$\Leftrightarrow \frac{1-\cos 4x}{2}=\frac{1}{2}$$

$$\Leftrightarrow \cos 4x=0$$

$$\Leftrightarrow 4x=\frac{\pi}{2}+k\pi \ , \ k\in\mathbb{Z}$$

$$\Rightarrow x=\frac{\pi}{8}+\frac{k\pi}{4} \ , k\in\mathbb{Z}$$

ដូចនេះ សមីការមានចម្លើយ $\boxed{x=\frac{\pi}{8}+\frac{k\pi}{4} \ , k\in\mathbb{Z}}$

ឃ. $\sin^6 x+\cos^6 x-\sin 2x=0$

$$\Leftrightarrow (\sin^2 x+\cos^2 x)^3-3\sin^2 x\cos^2 x(\sin^2 x+\cos^2 x)-\sin 2x=0$$

$$\Leftrightarrow 1-\frac{3}{4}\sin^2 2x-\sin 2x=0$$

$$\Leftrightarrow 3\sin^2 2x+4\sin 2x+4=0$$

តាង $t=\sin 2x$ ដែល $-1\le t\le 1$

យើងបានសមីការទៅជា $3t^2+4t-4=0$

$$\Delta' = 4+12 = 16$$

$$t_1 = \frac{-2+4}{3} = \frac{2}{3}$$

$$t_2 = \frac{-2-4}{3} = -2 < -1 (\text{មិនយក})$$

តែ $t = \sin 2x$ យើងបាន $\sin 2x = \frac{2}{3}$

$$\Leftrightarrow \begin{cases} 2x = \arcsin\left(\frac{2}{3}\right) + 2k\pi \\ 2x = \pi - \arcsin\left(\frac{2}{3}\right) + 2k\pi \end{cases}, k \in \mathbb{Z}$$

$$\Rightarrow \begin{cases} x = \frac{1}{2}\arcsin\left(\frac{2}{3}\right) + k\pi \\ x = \frac{\pi}{2} - \frac{1}{2}\arcsin\left(\frac{2}{3}\right) + k\pi \end{cases}, \ k \in \mathbb{Z}$$

ដូចនេះ សមីការមានចម្លើយ

$$\boxed{x = \frac{1}{2}\arcsin\left(\frac{2}{3}\right) + k\pi \ , \ x = \frac{\pi}{2} - \frac{1}{2}\arcsin\left(\frac{2}{3}\right) + k\pi , k \in \mathbb{Z}}$$

ង. $\cos^4 \frac{x}{5} + \sin^2 \frac{x}{5} = 1$

$$\Leftrightarrow \cos^4 \frac{x}{5} + 1 - \cos^2 \frac{x}{5} = 1$$

$$\Leftrightarrow \cos^4 \frac{x}{5} - \cos^2 \frac{x}{5} = 0$$

$$\Leftrightarrow \cos^2 \frac{x}{5}\left(\cos^2 \frac{x}{5} - 1\right) = 0$$

$$\Leftrightarrow \begin{cases} \cos^2\dfrac{x}{5}=0 \\ \cos^2\dfrac{x}{5}-1=0 \end{cases}$$

$$\Leftrightarrow \begin{cases} \cos\dfrac{x}{5}=0 \\ \cos\dfrac{x}{5}=\pm 1 \end{cases}$$

$$\Leftrightarrow \begin{cases} \dfrac{x}{5}=\dfrac{\pi}{2}+k\pi \\ \dfrac{x}{5}=k\pi \end{cases}, \ k\in\mathbb{Z}$$

$$\Rightarrow \begin{cases} x=\dfrac{5\pi}{2}+5k\pi \\ x=5k\pi \end{cases}, \ k\in\mathbb{Z}$$

ដូចនេះ សមីការមានចម្លើយ $\boxed{x=\dfrac{5\pi}{2}+5k\pi\ ,\ x=5k\pi\ ,\ k\in\mathbb{Z}}$

**លំហាត់ទី១២៖ ដោះស្រាយសមីការត្រីកោណមាត្រខាងក្រោម៖**

ក. $\dfrac{\cos x(2\sin x+3\sqrt{2})-2\cos^2 x-1}{1+\sin 2x}=1$

ខ. $\cos\left(2x+\dfrac{\pi}{4}\right)+\cos\left(2x-\dfrac{\pi}{4}\right)+4\sin x=2+\sqrt{2}\left(1-\sin x\right)$

គ. $3\cot^2 x+2\sqrt{2}\sin^2 x=(2+3\sqrt{2})\cos x$

ឃ. $\dfrac{4\sin^2 2x+6\sin^2 x-9-3\cos 2x}{\cos x}=0$

ង. $\sin 2x(\cot x+\tan 2x)=4\cos^2 x$

## ចំណោះស្រាយ

ក. $\dfrac{\cos x(2\sin x+3\sqrt{2})-2\cos^2 x-1}{1+\sin 2x}=1$

សមីការមានឫសកាលណា $1+\sin 2x\neq 0$

$$\Leftrightarrow \sin 2x\neq -1$$

$$\Leftrightarrow 2x\neq -\frac{\pi}{2}+2k\pi \ , \ k\in\mathbb{Z}$$

$$\Rightarrow x\neq -\frac{\pi}{4}+k\pi \ , k\in\mathbb{Z}$$

យើងបានសមីការទៅជា

$$\cos x(2\sin x+3\sqrt{2})-2\cos^2 x-1=1+\sin 2x$$

$$\Leftrightarrow \sin 2x+3\sqrt{2}\cos x-2\cos^2 x-1=1+\sin 2x$$

$$\Leftrightarrow 2\cos^2 x-3\sqrt{2}\cos x+2=0$$

តាង $t=\cos x$ ដែល $-1\le t\le 1$

យើងបាន $2t^2-3\sqrt{2}t+2=0$

$$\Delta=18-16=2$$

$$t_1=\frac{3\sqrt{2}-\sqrt{2}}{2\cdot 2}=\frac{\sqrt{2}}{2}$$

$$t_2=\frac{3\sqrt{2}+\sqrt{2}}{2\cdot 2}=\sqrt{2}>1 \text{ (មិនយក)}$$

តែ $t=\cos x$ យើងបាន $\cos x=\dfrac{\sqrt{2}}{2}=\cos\dfrac{\pi}{4}$

$$\Rightarrow x=\pm\frac{\pi}{4}+2k\pi \ , \ k\in\mathbb{Z}$$

តែ $x\neq -\dfrac{\pi}{4}+k\pi \ , k\in\mathbb{Z} \Rightarrow x\neq -\dfrac{\pi}{4}+2k\pi , k\in\mathbb{Z}$

ដូចនេះ សមីការមានមានចម្លើយ $\boxed{x=\frac{\pi}{4}+2k\pi\ ,\ k\in\mathbb{Z}}$

ខ. $\cos\left(2x+\frac{\pi}{4}\right)+\cos\left(2x-\frac{\pi}{4}\right)+4\sin x=2+\sqrt{2}\left(1-\sin x\right)$

ដោយ $\cos\left(2x+\frac{\pi}{4}\right)+\cos\left(2x-\frac{\pi}{4}\right)=2\cos\frac{2x+\frac{\pi}{4}+2x-\frac{\pi}{4}}{2}\cos\frac{2x+\frac{\pi}{4}-2x+\frac{\pi}{4}}{2}$

$$=2\cos 2x\cos\frac{\pi}{4}$$

$$=\sqrt{2}\cos 2x$$

យើងបានសមីការទៅជា $\sqrt{2}\cos 2x+4\sin x=2+\sqrt{2}\left(1-\sin x\right)$

$$\Leftrightarrow \sqrt{2}\left(1-2\sin^2 x\right)+4\sin x=2+\sqrt{2}-\sqrt{2}\sin x$$

$$\Leftrightarrow \sqrt{2}-2\sqrt{2}\sin^2 x+4\sin x-2-\sqrt{2}+\sqrt{2}\sin x=0$$

$$\Leftrightarrow 2\sqrt{2}\sin^2 x-(4+\sqrt{2})\sin x+2=0$$

$$\Leftrightarrow 2\sin^2 x-(2\sqrt{2}+1)\sin x+\sqrt{2}=0$$

$$\Delta=\left[-(2\sqrt{2}+1)\right]^2-4\cdot 2\cdot\sqrt{2}$$

$$=8+4\sqrt{2}+1-8\sqrt{2}$$

$$=9-4\sqrt{2}$$

$$=(2\sqrt{2}-1)^2$$

$$\sin x_1=\frac{(2\sqrt{2}+1)-(2\sqrt{2}-1)}{4}=\frac{1}{2}$$

$$\sin x_2=\frac{(2\sqrt{2}+1+(2\sqrt{2}-1)}{4}=\sqrt{2}>1\ (\text{មិនយក})$$

យើងបាន $\sin x=\frac{1}{2}$

$$\Rightarrow \begin{cases} x=\dfrac{\pi}{6}+2k\pi \\ x=\dfrac{5\pi}{6}+2k\pi \end{cases}, k\in\mathbb{Z}$$

ដូចនេះ សមីការមានចម្លើយ $\boxed{x=\dfrac{\pi}{6}+2k\pi\ ,\ x=\dfrac{5\pi}{6}+2k\pi\ ,\ k\in\mathbb{Z}}$

គ. $3\cot^2 x+2\sqrt{2}\sin^2 x=(2+3\sqrt{2})\cos x$

សមីការមានឫសកាលណា $\sin x\neq 0\Rightarrow x\neq k\pi\ ,\ k\in\mathbb{Z}$

ដោយ $\sin^2 x=1-\cos^2 x$ និង $\cot^2 x=\dfrac{\cos^2 x}{\sin^2 x}$

យើងបាន សមីការទៅជា $3\dfrac{\cos^2 x}{\sin^2 x}+2\sqrt{2}\sin^2 x=(2+3\sqrt{2})\cos x$

$$\Leftrightarrow 3\frac{\cos^2 x}{\sin^4 x}+2\sqrt{2}=(2+3\sqrt{2})\frac{\cos x}{\sin^2 x}$$

$$\Leftrightarrow 3\left(\frac{\cos x}{\sin^2 x}\right)^2-(2+3\sqrt{2})\left(\frac{\cos x}{\sin^2 x}\right)+2\sqrt{2}=0$$

តាង $t=\dfrac{\cos x}{\sin^2 x}$ យើងបាន $3t^2-(2+3\sqrt{2})t+2\sqrt{2}=0$

$$\begin{aligned}\Delta&=\left[-(2+3\sqrt{2})\right]^2-4\cdot 3\cdot 2\sqrt{2}\\&=4+12\sqrt{2}+18-24\sqrt{2}\\&=22-12\sqrt{2}\\&=(3\sqrt{2}-2)^2\end{aligned}$$

$$t_1=\frac{(2+3\sqrt{2})-(3\sqrt{2}-2)}{6}=\frac{2}{3}$$

$$t_2=\frac{(2+3\sqrt{2})+(3\sqrt{2}-2)}{6}=\sqrt{2}$$

តែ $t=\dfrac{\cos x}{\sin^2 x}$ យើងបាន ៖

- $t=\dfrac{2}{3}\Leftrightarrow \dfrac{\cos x}{\sin^2 x}=\dfrac{2}{3}$

$\Leftrightarrow 3\cos x=2(1-\cos^2 x)$

$\Leftrightarrow 2\cos^2 x+3\cos x-2=0$

$\Leftrightarrow (2\cos x-1)(\cos x+2)=0$

$\Leftrightarrow \left[\begin{array}{l}2\cos x-1=0\\ \cos x+2=0\end{array}\right.$

$\Leftrightarrow \left[\begin{array}{l}\cos x=\dfrac{1}{2}\\ \cos x=-2<-1(\text{មិនយក})\end{array}\right.$

$\Leftrightarrow \cos x=\cos\dfrac{\pi}{3}$

$\Rightarrow x=\pm\dfrac{\pi}{3}+2k\pi\ ,k\in\mathbb{Z}$

- $t=\sqrt{2}\Leftrightarrow \dfrac{\cos x}{\sin^2 x}=\sqrt{2}$

$\Leftrightarrow \cos x=\sqrt{2}(1-\cos^2 x)$

$\Leftrightarrow \sqrt{2}\cos^2 x+\cos x-\sqrt{2}=0$

$\Leftrightarrow \left(\sqrt{2}\cos x-1\right)\left(\cos x-\sqrt{2}\right)=0$

$\Leftrightarrow \left[\begin{array}{l}\cos x=\dfrac{\sqrt{2}}{2}\\ \cos x=\sqrt{2}>1\,(\text{មិនយក})\end{array}\right.$

$\Leftrightarrow \cos x=\cos\dfrac{\pi}{4}\ \Rightarrow x=\pm\dfrac{\pi}{4}+2k\pi\ ,k\in\mathbb{Z}$

ដូចនេះ សមីការមានចម្លើយ $\boxed{x=\pm\frac{\pi}{3}+2k\pi, x=\pm\frac{\pi}{4}+2k\pi \ , k\in\mathbb{Z}}$

ឃ. $\frac{4\sin^2 2x+6\sin^2 x-9-3\cos 2x}{\cos x}=0$

សមីការមានឫសកាលណា $\cos x\neq 0\Rightarrow x\neq\frac{\pi}{2}+k\pi\ , k\in\mathbb{Z}$

ដោយ $\sin^2 2x=1-\cos^2 2x$ , $\sin^2 x=\frac{1-\cos 2x}{2}$

យើងបានសមីការទៅជា $4(1-\cos^2 2x)+6\cdot\frac{1-\cos 2x}{2}-9-3\cos 2x=0$

$\Leftrightarrow 4-4\cos^2 2x+3-3\cos 2x-9-3\cos 2x=0$

$\Leftrightarrow -4\cos^2 2x-6\cos 2x-2=0$

$\Leftrightarrow 2\cos^2 2x+3\cos 2x+1=0$

$\Leftrightarrow (\cos 2x+1)(\cos 2x+\frac{1}{2})=0$

$\Leftrightarrow \left[\begin{array}{l}\cos 2x=-1\\ \cos 2x=-\frac{1}{2}\end{array}\right.$

$\Leftrightarrow \left[\begin{array}{l}2x=\pi+2k\pi\\ 2x=\pm\frac{2\pi}{3}+2k\pi\end{array}\right. , k\in\mathbb{Z}$

$\Rightarrow \left[\begin{array}{l}x=\frac{\pi}{2}+k\pi\\ x=\pm\frac{\pi}{3}+k\pi\end{array}\right. , k\in\mathbb{Z}$

តែ $x\neq\frac{\pi}{2}+k\pi\ , k\in\mathbb{Z}$

ដូចនេះ សមីការមានចម្លើយ $\boxed{x=\pm\frac{\pi}{3}+k\pi\ , k\in\mathbb{Z}}$

ង. $\sin 2x(\cot x+\tan 2x)=4\cos^2 x$

សមីការមានឫសកាលណា $\begin{cases}\sin x\neq 0\\ \cos 2x\neq 0\end{cases}\Rightarrow\begin{cases}x\neq k\pi\\ x\neq\dfrac{\pi}{4}+\dfrac{k\pi}{2}\end{cases},k\in\mathbb{Z}$

យើងបាន $\sin 2x(\cot x+\tan 2x)=4\cos^2 x$

$$\Leftrightarrow 2\sin x\cos x\left(\frac{\cos x}{\sin x}+\frac{\sin 2x}{\cos 2x}\right)=4\cos^2 x$$

$$\Leftrightarrow 2\cos^2 x+\frac{\sin^2 2x}{\cos 2x}=4\cos^2 x$$

$$\Leftrightarrow \frac{1-\cos^2 2x}{\cos 2x}=2\cos^2 x$$

$$\Leftrightarrow (1-\cos 2x)(1+\cos 2x)=2\cos 2x\left(\frac{1+\cos 2x}{2}\right)$$

$$\Leftrightarrow (1+\cos 2x)(1-2\cos 2x)=0$$

$$\Leftrightarrow\left[\begin{array}{l}\cos 2x=-1\\ \cos 2x=\dfrac{1}{2}\end{array}\right.$$

$$\Leftrightarrow\left[\begin{array}{l}2x=\pi+2k\pi\\ 2x=\pm\dfrac{\pi}{3}+2k\pi\end{array}\right.,k\in\mathbb{Z}$$

$$\Rightarrow\left[\begin{array}{l}x=\dfrac{\pi}{2}+k\pi\\ x=\pm\dfrac{\pi}{6}+k\pi\end{array}\right.,k\in\mathbb{Z}$$

តែ $x\neq\dfrac{\pi}{4}+\dfrac{k\pi}{2},k\in\mathbb{Z}$ នោះ $x=\dfrac{\pi}{2}+k\pi$ (មិនយក)

ដូចនេះ សមីការមានចម្លើយ $\boxed{x=\pm\dfrac{\pi}{6}+k\pi,k\in\mathbb{Z}}$

លំហាត់ទី១៣៖ ដោះស្រាយសមីការត្រីកោណមាត្រខាងក្រោម៖

ក. $\sin^4 x+\cos^4 x=\frac{7}{8}\cot\left(x+\frac{\pi}{3}\right)\cot\left(\frac{\pi}{6}-x\right)$

ខ. $5\sin x-2=3(1-\sin x)\tan^2 x$

គ. $\cos^2 3x\cos 2x-\cos^2 x=0$

ឃ. $\cos^4 x+\sin^4 x+\cos\left(x-\frac{\pi}{4}\right)\sin\left(3x-\frac{\pi}{4}\right)-\frac{3}{2}=0$

ង. $\frac{2(\sin^6 x+\cos^6 x)-\sin x\cos x}{\sqrt{2}-2\sin x}=0$

## ចំណោះស្រាយ

ក. $\sin^4 x+\cos^4 x=\frac{7}{8}\cot\left(x+\frac{\pi}{3}\right)\cot\left(\frac{\pi}{6}-x\right)$

សមីការមានឫសកាលណា $\begin{cases}\sin\left(x+\frac{\pi}{3}\right)\neq 0\\ \sin\left(\frac{\pi}{6}-x\right)\neq 0\end{cases}$

$$\Rightarrow\begin{cases}x\neq -\frac{\pi}{3}+k\pi\\ x\neq\frac{\pi}{6}+k\pi\end{cases},k\in\mathbb{Z}$$

ដោយ $\cot\left(\frac{\pi}{6}-x\right)=\tan\left[\frac{\pi}{2}-\left(\frac{\pi}{6}-x\right)\right]=\tan\left(\frac{\pi}{3}+x\right)$

នោះ $\cot\left(x+\frac{\pi}{3}\right)\cot\left(\frac{\pi}{6}-x\right)=\cot\left(\frac{\pi}{3}+x\right)\cdot\tan\left(\frac{\pi}{3}+x\right)=1$

ហើយ $\sin^4 x+\cos^4 x=\left(\sin^2 x+\cos^2 x\right)^2-2\sin^2 x\cos^2 x$

$$=1-\frac{1}{2}\sin^2 2x$$
$$=1-\frac{1}{2}\left(\frac{1-\cos 4x}{2}\right)$$
$$=\frac{4-\cos 4x}{4}$$

យើងបាន សមីការទៅជា $\frac{4-\cos 4x}{4}=\frac{7}{8}$

$$\Leftrightarrow 8-2\cos 4x=7$$
$$\Leftrightarrow \cos 4x=\frac{1}{2}=\cos\frac{\pi}{3}$$
$$\Leftrightarrow 4x=\pm\frac{\pi}{3}+2k\pi, k\in\mathbb{Z}$$
$$\Rightarrow x=\pm\frac{\pi}{12}+\frac{k\pi}{2}, k\in\mathbb{Z}$$

ដូចនេះ សមីការមានចម្លើយ $\boxed{x=\pm\frac{\pi}{12}+\frac{k\pi}{2}, k\in\mathbb{Z}}$

**លំហាត់បង្ហាត់បន្ថែម**

ក. $\sin^4 x+\cos^4 x=\frac{7}{8}\cot\left(x+\frac{\pi}{3}\right)\cot\left(\frac{\pi}{6}-x\right)$

សមីការមានឫសកាលណា $\begin{cases}\sin\left(x+\frac{\pi}{3}\right)\neq 0\\ \sin\left(\frac{\pi}{6}-x\right)\neq 0\end{cases}$

$$\Rightarrow \begin{cases} x \neq -\dfrac{\pi}{3} + k\pi \\ x \neq \dfrac{\pi}{6} + k\pi \end{cases}, k \in \mathbb{Z}$$

ដោយ $\cot\left(\dfrac{\pi}{6} - x\right) = \tan\left[\dfrac{\pi}{2} - \left(\dfrac{\pi}{6} - x\right)\right] = \tan\left(\dfrac{\pi}{3} + x\right)$

នោះ $\cot\left(x + \dfrac{\pi}{3}\right)\cot\left(\dfrac{\pi}{6} - x\right) = \cot\left(\dfrac{\pi}{3} + x\right) \cdot \tan\left(\dfrac{\pi}{3} + x\right) = 1$

ហើយ $\sin^4 x + \cos^4 x = \left(\sin^2 x + \cos^2 x\right)^2 - 2\sin^2 x \cos^2 x$

$$= 1 - \frac{1}{2}\sin^2 2x$$

យើងបានសមីការទៅជា $1 - \dfrac{1}{2}\sin^2 2x = \dfrac{7}{8}$

$$\Leftrightarrow 4\sin^2 2x = 1$$

$$\Leftrightarrow \sin^2 2x = \frac{1}{4}$$

$$\Leftrightarrow \sin 2x = \pm\frac{1}{2}$$

$$\Leftrightarrow \begin{bmatrix} \sin 2x = \dfrac{1}{2} \\ \sin 2x = -\dfrac{1}{2} \end{bmatrix}$$

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$$\Leftrightarrow \begin{cases} \begin{cases} 2x = \dfrac{\pi}{6} + 2k\pi \\ 2x = \dfrac{5\pi}{6} + 2k\pi \end{cases} \\ \begin{cases} 2x = -\dfrac{\pi}{6} + 2k\pi \\ 2x = \dfrac{7\pi}{6} + 2k\pi \end{cases} \end{cases}, k \in \mathbb{Z}$$

$$\Leftrightarrow \begin{cases} \mathbf{x = \dfrac{\pi}{12} + k\pi} \\ \mathbf{x = \dfrac{5\pi}{12} + k\pi} \\ \mathbf{x = -\dfrac{\pi}{12} + k\pi} \\ \mathbf{x = \dfrac{7\pi}{12} + k\pi} \end{cases} \mathbf{, k} \in \mathbb{Z}$$

ខ. $5\sin x - 2 = 3(1-\sin x)\tan^2 x$

សមីការមានឫសកាលណា $\cos x \neq 0 \Rightarrow x \neq \dfrac{\pi}{2} + k\pi \ , k \in \mathbb{Z}$

យើងបាន $5\sin x - 2 = 3(1-\sin x)\cdot\dfrac{\sin^2 x}{\cos^2 x}$

$$\Leftrightarrow 5\sin x - 2 = 3\frac{(1-\sin x)\sin^2 x}{(1-\sin x)(1+\sin x)}$$

$$\Leftrightarrow 5\sin x - 2 = \frac{3\sin^2 x}{1+\sin x}$$

$$\Leftrightarrow 5\sin x + 5\sin^2 x - 2 - 2\sin x = 3\sin^2 x$$

$$\Leftrightarrow 2\sin^2 x + 3\sin x - 2 = 0$$

$$\Leftrightarrow (2\sin x-1)(\sin x+2)=0$$

$$\Leftrightarrow \left[\begin{array}{l}\sin x=\dfrac{1}{2}\\ \sin x=-2<-1\ (\text{មិនយក})\end{array}\right.$$

$$\Leftrightarrow \left[\begin{array}{l}x=\dfrac{\pi}{6}+2k\pi\\ x=\dfrac{5\pi}{6}+2k\pi\end{array}\right. ,\ k\in\mathbb{Z}$$

ដូចនេះ សមីការមានចម្លើយ $\boxed{x=\dfrac{\pi}{6}+2k\pi\ ,\ x=\dfrac{5\pi}{6}+2k\pi\ ,\ k\in\mathbb{Z}}$

គ. $\cos^2 3x\cos 2x-\cos^2 x=0$

ដោយ $\cos^2 3x=\dfrac{1+\cos 6x}{2}$

យើងបាន $\dfrac{1+\cos 6x}{2}\cdot\cos 2x-\cos^2 x=0$

$$\Leftrightarrow \frac{(1+\cos 6x)\cos 2x}{2}-\frac{1+\cos 2x}{2}=0$$

$$\Leftrightarrow \cos 2x+\cos 6x\cos 2x-1-\cos 2x=0$$

$$\Leftrightarrow \cos 6x\cos 2x-1=0$$

$$\Leftrightarrow (4\cos^3 2x-3\cos 2x)\cdot\cos 2x-1=0$$

$$\Leftrightarrow 4\cos^4 2x-3\cos^2 2x-1=0$$

តាង $t=\cos^2 2x$ ដែល $0\le t\le 1$

យើងបាន $4t^2-3t-1=0$

ដោយ $a+b+c=4-3-1=0$

នោះ $t_1=1, t_2=\dfrac{c}{2}=-\dfrac{1}{4}<0$ (មិនយក)

តែ $t=\cos^2 2x$ យើងបាន $\cos^2 2x=1$

$$\Leftrightarrow \cos 2x=\pm 1$$

$$\Leftrightarrow 2x=k\pi \ , k\in\mathbb{Z}$$

$$\Rightarrow x=\frac{k\pi}{2}, k\in\mathbb{Z}$$

ដូចនេះ សមីការមានចម្លើយ $\boxed{x=\frac{k\pi}{2}, k\in\mathbb{Z}}$

ឃ. $\cos^4 x+\sin^4 x+\cos\left(x-\frac{\pi}{4}\right)\sin\left(3x-\frac{\pi}{4}\right)-\frac{3}{2}=0$

$(\cos^2x+\sin^2x)^2-2\cos^2x\sin^2x+\frac{1}{2}\left[\sin\left(x-\frac{\pi}{4}+3x-\frac{\pi}{4}\right)-\sin\left(x-\frac{\pi}{4}-3x+\frac{\pi}{4}\right)\right]-\frac{3}{2}=0$

$$\Leftrightarrow 1-\frac{1}{2}\sin^2 2x+\frac{1}{2}\sin\left(4x-\frac{\pi}{2}\right)+\frac{1}{2}\sin 2x-\frac{3}{2}=0$$

$$\Leftrightarrow -1-\sin^2 2x-\cos 4x+\sin 2x=0$$

$$\Leftrightarrow -1-\sin^2 2x-(1-2\sin^2 2x)+\sin 2x=0$$

$$\Leftrightarrow \sin^2 2x+\sin 2x-2=0$$

ដោយ $a+b+c=1+1-2=0$

នោះ $\left[\begin{array}{l}\sin 2x=1\\ \sin 2x=\frac{c}{a}=-2<-1 \text{ (មិនយក)}\end{array}\right.$

យើងបាន $\sin 2x=1\Leftrightarrow 2x=\frac{\pi}{2}+2k\pi, k\in\mathbb{Z}$

$$\Rightarrow x=\frac{\pi}{4}+k\pi \ , k\in\mathbb{Z}$$

ដូចនេះ សមីការមានចម្លើយ $\boxed{x=\frac{\pi}{4}+k\pi \ , k\in\mathbb{Z}}$

ង. $\dfrac{2(\sin^6 x+\cos^6 x)-\sin x\cos x}{\sqrt{2}-2\sin x}=0$

សមីការមានឫសកាលណា $\sqrt{2}-2\sin x\neq 0$

$$\Leftrightarrow \sin x\neq \frac{\sqrt{2}}{2}$$

$$\Rightarrow \begin{cases} x\neq \dfrac{\pi}{4}+2k\pi \\ x\neq \dfrac{3\pi}{4}+2k\pi \end{cases}, k\in\mathbb{Z}$$

យើងបានសមីការទៅជា $2(\sin^6 x+\cos^6 x)-\sin x\cos x=0$

ដោយ $\sin^6x+\cos^6x=(\sin^2x+\cos^2x)^3-3\sin^2x\cos^2x(\sin^2x+\cos^2x)$

$$=1-\frac{3}{4}\sin^2 2x$$

យើងបាន $2\left(1-\dfrac{3}{4}\sin^2 2x\right)-\sin x\cos x=0$

$\Leftrightarrow 2-\dfrac{3}{2}\sin^2 2x-\dfrac{1}{2}\sin 2x=0$

$\Leftrightarrow 3\sin^2 2x+\sin 2x-4=0$

ដោយ $a+b+c=3+1-4=0$

នោះ $\left[\begin{array}{l} \sin 2x=1 \\ \sin 2x=\dfrac{c}{a}=-\dfrac{4}{3}<-1 \text{ (មិនយក)} \end{array}\right.$

យកបាន $\sin 2x=1\Leftrightarrow 2x=\dfrac{\pi}{2}+2k\pi\ ,k\in\mathbb{Z}$

$$\Rightarrow x=\frac{\pi}{4}+k\pi\ ,k\in\mathbb{Z}$$

តែ $\begin{cases} x \neq \dfrac{\pi}{4} + 2k\pi \\ x \neq \dfrac{3\pi}{4} + 2k\pi \end{cases}, k \in \mathbb{Z}$

នោះ $x \in \left\{ \left\{ \frac{\pi}{4} + k\pi \right\} - \left\{ \frac{\pi}{4} + 2k\pi \right\} \right\} = \left\{ \frac{\pi}{4} + (2k-1)\pi \right\}, k \in \mathbb{Z}$

ដូចនេះ សមីការមានចម្លើយ $\boxed{x = \frac{\pi}{4} + (2k-1)\pi, k \in \mathbb{Z}}$

លំហាត់ទី១៤ : ដោះស្រាយសមីការខាងក្រោម

ក. $\cos 7x - \sqrt{3}\sin 7x = -\sqrt{2}$

ខ. $\sin 3x = \sqrt{2} + \cos 3x$

គ. $3\sin x + \cos x = 1$

ឃ. $\sin x + 5\cos x = 1$

ង. $\frac{1+\sqrt{3}}{2\sqrt{2}}\sin x + \frac{1-\sqrt{3}}{2\sqrt{2}}\cos x = \frac{1}{\sqrt{2}}$

## ចំណោះស្រាយ

ក. $\cos 7x - \sqrt{3}\sin 7x = -\sqrt{2}$

ដោយ $r = \sqrt{1^2 + \sqrt{3}^2} = 2$

នោះចែកអង្គទាំងពីរនឹង 2

យើងបាន $\frac{1}{2}\cos 7x - \frac{\sqrt{3}}{2}\sin 7x = -\frac{\sqrt{2}}{2}$

$$\Leftrightarrow \cos\frac{\pi}{3}\cos 7x - \sin\frac{\pi}{3}\sin x = \cos\frac{3\pi}{4}$$

$$\Leftrightarrow \cos\left(7x+\frac{\pi}{3}\right) = \cos\frac{3\pi}{4}$$

$$\Leftrightarrow \left[\begin{array}{l} 7x+\dfrac{\pi}{3} = \dfrac{3\pi}{4} + 2k\pi \\ 7x+\dfrac{\pi}{3} = -\dfrac{3\pi}{4} + 2k\pi \end{array}\right. , k \in \mathbb{Z}$$

$$\Rightarrow \left[\begin{array}{l} x = \dfrac{5\pi}{84} + \dfrac{2k\pi}{7} \\ x = -\dfrac{13\pi}{84} + \dfrac{2k\pi}{7} \end{array}\right. , k \in \mathbb{Z}$$

ដូចនេះ សមីការមានចម្លើយ

$$\boxed{x = \frac{5\pi}{84} + \frac{2k\pi}{7} \ , \ x = -\frac{13\pi}{84} + \frac{2k\pi}{7}, \quad k \in \mathbb{Z}}$$

ខ. $\sin 3x = \sqrt{2} + \cos 3x$

$$\Leftrightarrow \sin 3x - \cos 3x = \sqrt{2}$$

$$\Leftrightarrow \frac{\sqrt{2}}{2}\sin 3x - \frac{\sqrt{2}}{2}\cos 3x = 1$$

$$\Leftrightarrow \sin\left(3x-\frac{\pi}{4}\right) = 1$$

$$\Leftrightarrow 3x - \frac{\pi}{4} = \frac{\pi}{2} + 2k\pi \ , k \in \mathbb{Z}$$

$$\Rightarrow x = \frac{\pi}{4} + \frac{2k\pi}{3} \ , k \in \mathbb{Z}$$

ដូចនេះ សមីការមានចម្លើយ $\boxed{x = \dfrac{\pi}{4} + \dfrac{2k\pi}{3} \ , k \in \mathbb{Z}}$

គ. $3\sin x+\cos x=1$

**វិធីទី១**

ដោយ $r=\sqrt{3^2+1^2}=\sqrt{10}$

នោះចែកអង្គទាំងពីរនៃសមីការនឹង $\sqrt{10}$

គេបាន $\frac{3}{\sqrt{10}}\sin x+\frac{1}{\sqrt{10}}\cos x=\frac{1}{\sqrt{10}}$

តាង $\cos\alpha=\frac{1}{\sqrt{10}}$ និង $\sin\alpha=\frac{3}{\sqrt{10}}$

ដែល $\tan\alpha=\frac{\sin\alpha}{\cos\alpha}=3\Rightarrow\alpha=\arctan 3$

យើងបាន $\cos\alpha\cos x+\sin\alpha\sin x=\cos\alpha$

$$\Leftrightarrow \cos(x-\alpha)=\cos\alpha$$

$$\Leftrightarrow \begin{bmatrix} x-\alpha=\alpha+2k\pi \\ x-\alpha=-\alpha+2k\pi \end{bmatrix},\ k\in\mathbb{Z}$$

$$\Rightarrow \begin{bmatrix} x=2\alpha+2k\pi \\ x=2k\pi \end{bmatrix},\ k\in\mathbb{Z}$$

ដែល $\alpha=\arctan 3$

ដូចនេះ សមីការមានចម្លើយ

$$\boxed{x=2\arctan 3+2k\pi\ ,\ x=2k\pi\ ,k\in\mathbb{Z}}$$

**វិធីទី២**

☞ បើ $x=\pi+2k\pi$ នោះសមីការ $3\sin x+\cos x=1$

គឺ $3\sin(\pi+2k\pi)+\cos(\pi+2k\pi)=1$

$3\cdot 0-1=1$

$-1=1$ (មិនផ្ទៀងផ្ទាត់)

នោះ $x=\pi+2k\pi$ មិនមែនជាឫសនៃសមីការ $3\sin x+\cos x=1$ ទេ

☞ បើ $x\neq\pi+2k\pi$ តាង $t=\tan\frac{x}{2}$

នោះ $\cos x=\frac{1-t^2}{1+t^2}$ និង $\sin x=\frac{2t}{1+t^2}$

យើងបាន សមីការទៅជា $3\frac{2t}{1+t^2}+\frac{1-t^2}{1+t^2}=1$

$$\Leftrightarrow 6t+1-t^2=1+t^2$$
$$\Leftrightarrow 2t^2-6t=0$$
$$\Leftrightarrow 2t(t-3)=0$$
$$\Rightarrow t=0\ ,t=3$$

- $t=0\Leftrightarrow\tan\frac{x}{2}=0$

$$\Leftrightarrow\frac{x}{2}=k\pi$$
$$\Rightarrow x=2k\pi\ ,k\in\mathbb{Z}$$

- $t=3\Leftrightarrow\tan\frac{x}{2}=3$

$$\Leftrightarrow\frac{x}{2}=\arctan 3+k\pi$$
$$\Rightarrow x=2\arctan 3+2k\pi$$

ដូចនេះ សមីការមានចម្លើយ

$$\boxed{x=2\arctan 3+2k\pi\ ,\ x=2k\pi\ ,k\in\mathbb{Z}}$$

ឃ. $\sin x+5\cos x=1$

**វិធីទី១**

តាង $\tan\alpha=5\Rightarrow\alpha=\arctan 5$

យើងបានសមីការ $\sin x+\tan\alpha\cos x=1$

$$\Leftrightarrow \sin x\cos\alpha+\sin\alpha\cos x=\cos\alpha$$

$$\Leftrightarrow \sin(x+\alpha)=\sin\left(\frac{\pi}{2}-\alpha\right)$$

$$\Leftrightarrow \left[\begin{array}{l} x+\alpha=\dfrac{\pi}{2}-\alpha+2k\pi \\ x+\alpha=\pi-\dfrac{\pi}{2}+\alpha+2k\pi \end{array}\right. , k\in\mathbb{Z}$$

$$\Rightarrow \left[\begin{array}{l} x=\dfrac{\pi}{2}-2\alpha+2k\pi=\dfrac{\pi}{2}-2\arctan 5+2k\pi \\ x=\dfrac{\pi}{2}+2k\pi \end{array}\right. , k\in\mathbb{Z}$$

ដូចនេះ សមីការមានចម្លើយ

$$\boxed{x=\frac{\pi}{2}+2k\pi\ ,\ x=\frac{\pi}{2}-2\arctan 5+2k\pi\ ,\ k\in\mathbb{Z}}$$

**វិធីទី២**

☞ បើ $x=\pi+2k\pi$ យើងបាន

$$\sin(\pi+2\mathrm{k}\,\pi)+5\cos(\pi+2\mathrm{k}\,\pi)=1$$

$$0-5=1 \quad \text{(មិនផ្ទៀងផ្ទាត់)}$$

នោះ $x=\pi+2k\pi$ មិនមែនជាឫសនៃសមីការ $\sin x+5\cos x=1$ ទេ

☞ បើ $x\neq\pi+2k\pi$ តាង $t=\tan\dfrac{x}{2}$

នោះ $\cos x=\frac{1-t^2}{1+t^2}$ , $\sin x=\frac{2t}{1+t^2}$

យើងបានសមីការទៅជា $\frac{2t}{1+t^2}+5\frac{1-t^2}{1+t^2}=1$

$$\Leftrightarrow 2t+5-5t^2=1+t^2$$
$$\Leftrightarrow 3t-t-2=0$$
$$\Rightarrow t=1, t=-\frac{2}{3}$$

- $t=1\Leftrightarrow \tan\frac{x}{2}=1$

$$\Leftrightarrow \frac{x}{2}=\frac{\pi}{4}+k\pi \ , k\in\mathbb{Z}$$
$$\Rightarrow x=\frac{\pi}{2}+2k\pi \ , k\in\mathbb{Z}$$

- $t=-\frac{2}{3}\Leftrightarrow \tan\frac{x}{2}=-\frac{2}{3}$

$$\Leftrightarrow \frac{x}{2}=\arctan\left(-\frac{2}{3}\right)+k\pi \ , k\in\mathbb{Z}$$
$$\Rightarrow x=2\arctan\left(-\frac{2}{3}\right)+2k\pi \ , k\in\mathbb{Z}$$

ដូចនេះ សមីការមានចម្លើយ

$$\boxed{x=\frac{\pi}{2}+2k\pi \ , x=x=2\arctan\left(-\frac{2}{3}\right)+2k\pi \ , k\in\mathbb{Z}}$$

ង. $\frac{1+\sqrt{3}}{2\sqrt{2}}\sin x+\frac{1-\sqrt{3}}{2\sqrt{2}}\cos x=\frac{1}{\sqrt{2}}$

$$\Leftrightarrow \frac{\sqrt{2}+\sqrt{6}}{4}\sin x+\frac{\sqrt{2}-\sqrt{6}}{4}\cos x=\frac{\sqrt{2}}{2}$$

ដោយ $\cos\frac{7\pi}{12}=\cos\left(\frac{\pi}{3}+\frac{\pi}{4}\right)$

$$=\cos\frac{\pi}{3}\cos\frac{\pi}{4}-\sin\frac{\pi}{3}\sin\frac{\pi}{4}$$

$$=\frac{\sqrt{2}-\sqrt{6}}{4}$$

ហើយ $\sin\frac{7\pi}{12}=\cos\left(\frac{\pi}{3}+\frac{\pi}{4}\right)$

$$=\sin\frac{\pi}{3}\cos\frac{\pi}{4}+\cos\frac{\pi}{3}\sin\frac{\pi}{4}$$

$$=\frac{\sqrt{2}+\sqrt{6}}{4}$$

យើងបានសមីការទៅជា $\sin\frac{7\pi}{12}\sin x+\cos\frac{7\pi}{12}\cos x=\cos\frac{\pi}{4}$

$$\Leftrightarrow\cos\left(x-\frac{7\pi}{12}\right)=\cos\frac{\pi}{4}$$

$$\Leftrightarrow\begin{cases}x-\frac{7\pi}{12}=\frac{\pi}{4}+2k\pi\\ x-\frac{7\pi}{12}=-\frac{\pi}{4}+2k\pi\end{cases},k\in\mathbb{Z}$$

$$\Rightarrow\begin{cases}x=\frac{5\pi}{6}+2k\pi\\ x=\frac{\pi}{3}+2k\pi\end{cases},k\in\mathbb{Z}$$

ដូចនេះ សមីការមានចម្លើយ $\boxed{x=\frac{5\pi}{6}+2k\pi\,,\,x=\frac{\pi}{3}+2k\pi\,,\,k\in\mathbb{Z}}$

លំហាត់ទី១៥ : ដោះស្រាយសមីការខាងក្រោម ៖

ក. $8\sin x=\dfrac{\sqrt{3}}{\cos x}+\dfrac{1}{\sin x}$

ខ. $\left(\sin 2x+\sqrt{3}\cos 2x\right)^2-5=\cos\left(2x-\dfrac{\pi}{6}\right)$

គ. $2\sqrt{2}\left(\sin x+\cos x\right)\cos x=3+\cos 2x$

## ចំណោះស្រាយ

ក. $8\sin x=\dfrac{\sqrt{3}}{\cos x}+\dfrac{1}{\sin x}$

សមីការមានឫសកាលណា $\begin{cases}\cos x\neq 0\\ \sin x\neq 0\end{cases}$

$$\Leftrightarrow \sin 2x\neq 0$$

$$\Rightarrow x\neq\frac{k\pi}{2}\ ,k\in\mathbb{Z}$$

យើងបាន $8\sin x=\dfrac{\sqrt{3}}{\cos x}+\dfrac{1}{\sin x}$

$$\Leftrightarrow 8\sin^2 x\cos x=\sqrt{3}\sin x+\cos x$$

$$\Leftrightarrow 8\frac{1-\cos 2x}{2}\cos x=\sqrt{3}\sin x+\cos x$$

$$\Leftrightarrow 4\cos x-4\cos x\cos 2x=\sqrt{3}\sin x+\cos x$$

$$\Leftrightarrow -4\cdot\frac{1}{2}\left(\cos 3x+\cos x\right)=\sqrt{3}\sin x-3\cos x$$

$$\Leftrightarrow -2\cos 3x=\sqrt{3}\sin x-\cos x$$

$$\Leftrightarrow \cos 3x = \frac{1}{2}\cos x - \frac{\sqrt{3}}{2}\sin x$$

$$\Leftrightarrow \cos 3x = \cos\left(x + \frac{\pi}{3}\right)$$

$$\Leftrightarrow \begin{cases} 3x = x + \frac{\pi}{3} + 2k\pi \\ 3x = -\left(x + \frac{\pi}{3}\right) + 2k\pi \end{cases}, k \in \mathbb{Z}$$

$$\Rightarrow \begin{cases} x = \frac{\pi}{6} + k\pi \\ x = -\frac{\pi}{12} + \frac{k\pi}{2} \end{cases}, k \in \mathbb{Z}$$

ដូចនេះ សមីការមានចម្លើយ $\boxed{x = \frac{\pi}{6} + k\pi \text{ , } x = -\frac{\pi}{12} + \frac{k\pi}{2}, k \in \mathbb{Z}}$

ខ. $\left(\sin 2x + \sqrt{3}\cos 2x\right)^2 - 5 = \cos\left(2x - \frac{\pi}{6}\right)$

$$\Leftrightarrow 4\left(\frac{\sqrt{3}}{2}\cos 2x + \frac{1}{2}\sin 2x\right)^2 - 5 = \cos\left(2x - \frac{\pi}{6}\right)$$

$$\Leftrightarrow 4\cos^2\left(2x - \frac{\pi}{6}\right) - \cos\left(2x - \frac{\pi}{6}\right) - 5 = 0$$

តាង $\cos\left(2x - \frac{\pi}{6}\right) = t$ ដែល $-1 \le t \le 1$

យើងបាន $4t^2 - t - 5 = 0$

$$\Rightarrow \begin{cases} t = -1 \\ t = \frac{c}{a} = \frac{5}{4} > 1 \text{ (មិនយក)} \end{cases}$$

យើងបាន $\cos\left(2x-\frac{\pi}{6}\right)=-1$

$$\Leftrightarrow 2x-\frac{\pi}{6}=\pi+2k\pi \ , k\in\mathbb{Z}$$

$$\Rightarrow x=\frac{7\pi}{12}+k\pi \ , k\in\mathbb{Z}$$

ដូចនេះ សមីការមានចម្លើយ $\boxed{x=\frac{7\pi}{12}+k\pi \ , k\in\mathbb{Z}}$

គ. $2\sqrt{2}\left(\sin x+\cos x\right)\cos x=3+\cos 2x$

$$\Leftrightarrow 2\sqrt{2}\sin x\cos x+2\sqrt{2}\cos^2 x=3+\cos 2x$$

$$\Leftrightarrow \sqrt{2}\sin 2x+2\sqrt{2}\frac{1+\cos 2x}{2}=3+\cos 2x$$

$$\Leftrightarrow \sqrt{2}\sin 2x+(\sqrt{2}-1)\cos 2x=3-\sqrt{2}$$

ដោយ $\sqrt{2}^2+(\sqrt{2}-1)^2=2+2-2\sqrt{2}+1=5-2\sqrt{2}$

ហើយ $(3-\sqrt{2})^2=9-6\sqrt{2}+2=11-6\sqrt{2}$

នោះ $(\sqrt{2})^2+(\sqrt{2}-1)^2<(3-\sqrt{2})^2$

ដូចនេះ សមីការគ្មានឫស

លំហាត់ទី១៦ : ដោះស្រាយសមីការខាងក្រោម ៖

ក. $\sin^3 x-\sqrt{3}\cos^3 x=\sin x\cos^2 x-\sqrt{3}\sin^2 x\cos x$

ខ. $\frac{(1-2\sin x)\cos x}{(1+2\sin x)(1-\sin x)}=\sqrt{3}$

គ. $\sin x+\cos x\sin 2x+\sqrt{3}\cos 3x=2(\cos 4x+\sin^3 x)$

## ចំណោះស្រាយ

ក. $\sin^3 x-\sqrt{3}\cos^3 x=\sin x\cos^2 x-\sqrt{3}\sin^2 x\cos x$

$\Leftrightarrow \sin^2 x(\sin x+\sqrt{3}\cos x)-\cos^2 x(\sin x+\sqrt{3}\cos x)=0$

$\Leftrightarrow (\sin x+\sqrt{3}\cos x)(\sin^2 x-\cos^2 x)=0$

$\Leftrightarrow -2\cos 2x\left(\frac{1}{2}\sin x+\frac{\sqrt{3}}{2}\cos x\right)=0$

$\Leftrightarrow \cos 2x\cos\left(x-\frac{\pi}{6}\right)=0$

$\Leftrightarrow \left[\begin{array}{l}\cos 2x=0\\ \cos\left(x-\frac{\pi}{6}\right)=0\end{array}\right.$

$\Leftrightarrow \left[\begin{array}{l}2x=\frac{\pi}{2}+k\pi\\ x-\frac{\pi}{6}=\frac{\pi}{2}+k\pi\end{array}\right.,k\in\mathbb{Z}$

$\Rightarrow \left[\begin{array}{l}x=\frac{\pi}{4}+\frac{k\pi}{2}\\ x=\frac{2\pi}{3}+k\pi\end{array}\right.,k\in\mathbb{Z}$

ដូចនេះ សមីការមានចម្លើយ $\boxed{x=\frac{\pi}{4}+\frac{k\pi}{2},x=\frac{2\pi}{3}+k\pi,\mathrm{k}\in\mathbb{Z}}$

ខ. $\frac{(1-2\sin x)\cos x}{(1+2\sin x)(1-\sin x)}=\sqrt{3}$

សមីការមានឫសកាលណា $\begin{cases}1+2\sin x\neq 0\\ 1-\sin x\neq 0\end{cases}$

$$\Rightarrow \begin{cases} x \neq -\dfrac{\pi}{6} + 2k\pi \\ x \neq \dfrac{7\pi}{6} + 2k\pi \\ x \neq \dfrac{\pi}{2} + 2k\pi \end{cases}, k \in \mathbb{Z}$$

គេបាន សមីការ $(1-2\sin x)\cos x = \sqrt{3}(1+2\sin x)(1-\sin x)$

$\Leftrightarrow \cos x - 2\sin x\cos x = \sqrt{3}\left(1-\sin x+2\sin x-2\sin^2 x\right)$

$\Leftrightarrow \cos x - 2\sin x\cos x = \sqrt{3}-\sqrt{3}\sin x+2\sqrt{3}\sin x-\sqrt{3}+\sqrt{3}\cos 2x$

$\Leftrightarrow \cos x - \sqrt{3}\sin x = 2\sin x\cos x+\sqrt{3}\cos 2x$

$\Leftrightarrow \dfrac{1}{2}\cos x - \dfrac{\sqrt{3}}{2}\sin x = \dfrac{1}{2}\sin 2x+\dfrac{\sqrt{3}}{2}\cos 2x$

$\Leftrightarrow \cos\left(x+\dfrac{\pi}{3}\right) = \cos\left(2x-\dfrac{\pi}{6}\right)$ ឬ $\cos\left(2x-\dfrac{\pi}{6}\right) = \cos\left(x+\dfrac{\pi}{3}\right)$

$$\Leftrightarrow \begin{cases} 2x-\dfrac{\pi}{6} = x+\dfrac{\pi}{3}+2k\pi \\ 2x-\dfrac{\pi}{6} = -x-\dfrac{\pi}{3}+2k\pi \end{cases}, k \in \mathbb{Z}$$

$$\Rightarrow \begin{cases} x = \dfrac{\pi}{2}+2k\pi \\ x = -\dfrac{\pi}{18}+\dfrac{2k\pi}{3} \end{cases}, k \in \mathbb{Z}$$

តែ $x \neq \dfrac{\pi}{2}+2k\pi$

ដូចនេះ សមីការមានចម្លើយ $\boxed{x = -\dfrac{\pi}{18}+\dfrac{2k\pi}{3}, k \in \mathbb{Z}}$

គ. $\sin x+\cos x\sin 2x+\sqrt{3}\cos 3x=2(\cos 4x+\sin^3 x)$

$\Leftrightarrow \sin x+\frac{1}{2}\left(\sin 3x+\sin x\right)+\sqrt{3}\cos 3x=2\left(\cos 4x+\sin^3 x\right)$

$\Leftrightarrow 3\sin x+\sin 3x+2\sqrt{3}\cos 3x=4\cos 4x+4\sin^3 x$

$\Leftrightarrow 3\sin x+\sin 3x+2\sqrt{3}\cos 3x=4\cos 4x+4\left(\frac{3\sin x-\sin 3x}{4}\right)$

$\Leftrightarrow 3\sin x+\sin 3x+2\sqrt{3}\cos 3x=4\cos 4x+3\sin x-\sin 3x$

$\Leftrightarrow 2\sin 3x+2\sqrt{3}\cos 3x=4\cos 4x$

$\Leftrightarrow \frac{\sqrt{3}}{2}\cos 3x+\frac{1}{2}\sin 3x=\cos 4x$

$\Leftrightarrow \cos\left(3x-\frac{\pi}{6}\right)=\cos 4x$

$\Leftrightarrow \left[\begin{array}{l} 4x=3x-\frac{\pi}{6}+2k\pi \\ 4x=-3x+\frac{\pi}{6}+2k\pi \end{array}\right. , k\in\mathbb{Z}$

$\Rightarrow \left[\begin{array}{l} x=-\frac{\pi}{6}+2k\pi \\ x=\frac{\pi}{42}+\frac{2k\pi}{7} \end{array}\right. , k\in\mathbb{Z}$

ដូចនេះ សមីការមានចម្លើយ $\boxed{x=-\frac{\pi}{6}+2k\pi\ ,\ x=\frac{\pi}{42}+\frac{2k\pi}{7},\ k\in\mathbb{Z}}$

លំហាត់ទី១៧ : ដោះស្រាយសមីការខាងក្រោម៖

ក. $\cos x+\sin x-\sin x\cos x=1$

ខ. $(1+\sqrt{2})(\sin x+\cos x)-\sin 2x-1-\sqrt{2}=0$

គ. $5\sin 2x-12(\sin x-\cos x)+12=0$

ឃ. $\cos x\sin x=6(\sin x-\cos x-1)$

## ចំណោះស្រាយ

ក. $\cos x+\sin x-\sin x\cos x=1$

តាង $t=\cos x+\sin x=\sqrt{2}\sin\left(x+\frac{\pi}{4}\right)$ ដែល $t\in\left[-\sqrt{2},\sqrt{2}\right]$

នោះ $\sin x\cos x=\frac{t^2-1}{2}$

យើងបានសមីការទៅជា $t-\frac{t^2-1}{2}=1$

$$\Leftrightarrow 2t-t^2+1=2$$
$$\Leftrightarrow t^2-2t+1=0$$
$$\Leftrightarrow (t-1)^2=0$$
$$\Rightarrow t=1$$

យើងបាន $t=1=\sqrt{2}\sin\left(x+\frac{\pi}{4}\right)$

$$\Leftrightarrow \sin\left(x+\frac{\pi}{4}\right)=\frac{\sqrt{2}}{2}$$

$$\Leftrightarrow \left[\begin{array}{l} x+\dfrac{\pi}{4}=\dfrac{\pi}{4}+2k\pi \\ x+\dfrac{\pi}{4}=\dfrac{3\pi}{4}+2k\pi \end{array}\right. , k\in\mathbb{Z}$$

$$\Rightarrow \left[\begin{array}{l} x=2k\pi \\ x=\dfrac{\pi}{2}+2k\pi \end{array}\right. , k\in\mathbb{Z}$$

ដូចនេះ សមីការមានចម្លើយ $\boxed{x=2k\pi\ ,\ x=\dfrac{\pi}{2}+2k\pi\ ,\ k\in\mathbb{Z}}$

ខ. $(1+\sqrt{2})(\sin x+\cos x)-\sin 2x-1-\sqrt{2}=0$

តាង $t=\sin x+\cos x=\sqrt{2}\sin\left(x+\dfrac{\pi}{4}\right)$ ដែល $t\in\left[-\sqrt{2}\ ,\sqrt{2}\right]$

នោះ $\sin 2x=2\sin x\cos x=2\dfrac{t^2-1}{2}=t^2-1$

យើងបាសមីការ $(1+\sqrt{2})t-(t^2-1)-1-\sqrt{2}=0$

$$\Leftrightarrow t^2-(1+\sqrt{2})t+\sqrt{2}=0$$

ដោយ $a+b+c=1-(1+\sqrt{2})+\sqrt{2}=0$

នោះ $t_1=1, t_2=\dfrac{c}{a}=\sqrt{2}$

យើងបាន ៖ • $t=1\Leftrightarrow\sqrt{2}\sin\left(x+\dfrac{\pi}{4}\right)=1$

$$\Rightarrow \left[\begin{array}{l} x=2k\pi \\ x=\dfrac{\pi}{2}+2k\pi \end{array}\right. , k\in\mathbb{Z}$$ (តាមសម្រាយ(ក))

- $t=\sqrt{2} \Leftrightarrow \sqrt{2}\sin\left(x+\frac{\pi}{4}\right)=\sqrt{2}$

$$\Leftrightarrow \sin x\left(x+\frac{\pi}{4}\right)=1$$

$$\Leftrightarrow x+\frac{\pi}{4}=\frac{\pi}{2}+2k\pi$$

$$\Rightarrow x=\frac{\pi}{4}+2k\pi\,,\ k\in\mathbb{Z}$$

ដូចនេះ សមីការមានចម្លើយ

$$\boxed{x=2k\pi\,,\ x=\frac{\pi}{2}+2k\pi\,,\ x=\frac{\pi}{4}+2k\pi\,,\ k\in\mathbb{Z}}$$

គ. $5\sin 2x-12\left(\sin x-\cos x\right)+12=0$

តាង $\mathrm{t}=\sin x-\cos x=\sqrt{2}\sin\left(x-\frac{\pi}{4}\right)$ ដែល $t\in\left[-\sqrt{2}\,,\sqrt{2}\right]$

នោះ $t^2=\sin^2 x+\cos^2 x-2\sin x\cos x=1-\sin 2x$

$$\Rightarrow \sin 2x=1-t^2$$

យើងបានសមីការ $5(1-t^2)-12\mathrm{t}+12=0$

$$\Leftrightarrow 5t^2+12t-17=0$$

ដោយ $a+b+c=5+12-17=0$

នោះ $t_1=1\ ,t_2=-\frac{17}{5}\notin\left[-\sqrt{2}\,,\sqrt{2}\right]$ (មិនយក)

យើងបាន $\mathrm{t}=1=\sqrt{2}\sin\left(x-\frac{\pi}{4}\right)$

$$\Leftrightarrow \sin\left(x-\frac{\pi}{4}\right)=\frac{\sqrt{2}}{2}$$

$$\Leftrightarrow \left[\begin{array}{l} x-\dfrac{\pi}{4}=\dfrac{\pi}{4}+2k\pi \\ x-\dfrac{\pi}{4}=\pi-\dfrac{\pi}{4}+2k\pi \end{array}\right. , k\in\mathbb{Z}$$

$$\Rightarrow \left[\begin{array}{l} x=\dfrac{\pi}{2}+2k\pi \\ x=\pi+2k\pi \end{array}\right. , k\in\mathbb{Z}$$

ដូចនេះ សមីការមានចម្លើយ $\boxed{x=\pi+2k\pi\ ,\ x=\dfrac{\pi}{2}+2k\pi\ ,\ k\in\mathbb{Z}}$

ឃ. $\cos x\sin x=6(\sin x-\cos x-1)$

តាង $t=\sin x-\cos x=\sqrt{2}\sin\left(x-\dfrac{\pi}{4}\right)$ ដែល $t\in\left[-\sqrt{2}\ ,\sqrt{2}\right]$

$$\Rightarrow \sin x\cos x=\frac{1-t^2}{2}$$

យើងបាន សមីការ $\dfrac{1-t^2}{2}=6(t-1)$

$$\Leftrightarrow t^2+12t-13=0$$

$$\Leftrightarrow \left[\begin{array}{l} t=1 \\ t=-13\notin\left[-\sqrt{2}\ ,\sqrt{2}\right] \text{(មិនយក)} \end{array}\right.$$

យើងបាន $t=1\Leftrightarrow\sqrt{2}\sin\left(x-\dfrac{\pi}{4}\right)=1$

$$\Leftrightarrow \sin\left(x-\frac{\pi}{4}\right)=\frac{\sqrt{2}}{2}=\sin\frac{\pi}{4}$$

$$\Leftrightarrow \left[\begin{array}{l} x-\dfrac{\pi}{4}=\dfrac{\pi}{4}+2k\pi \\ x-\dfrac{\pi}{4}=\pi-\dfrac{\pi}{4}+2k\pi \end{array}\right.$$

$$\Rightarrow \begin{cases} x = \dfrac{\pi}{2} + 2k\pi \\ x = \pi + 2k\pi \end{cases}, k \in \mathbb{Z}$$

ដូចនេះ សមីការមានចម្លើយ $\boxed{x = \pi + 2k\pi \, , \, x = \dfrac{\pi}{2} + 2k\pi \, , \, k \in \mathbb{Z}}$

លំហាត់ទី១៨ : ដោះស្រាយសមីការខាងក្រោម ៖

ក. $\sin x + \cos x + \sin x \cos x - 1 = 0$

ខ. $\cos 2x + 5 = (4 - 2\cos x)(\sin x - \cos x)$

គ. $\sqrt{2}\left(\sin x + \cos x\right) = \tan x + \cot x$

ឃ. $\cos^3 x + \sin^3 x = \sin 2x + \sin x + \cos x$

ង. $2\sin^3 x - \cos 2x + \cos x = 0$

## ដំណោះស្រាយ

ក. $\sin x + \cos x + \sin x \cos x - 1 = 0$

តាង $t = \sin x + \cos x = \sqrt{2}\cos\left(x - \dfrac{\pi}{4}\right)$ ដែល $-\sqrt{2} \le t \le \sqrt{2}$

នោះ $\sin x \cos x = \dfrac{t^2 - 1}{2}$

យើងបានសមីការទៅជា $t + \dfrac{t^2 - 1}{2} - 1 = 0$

$$\Leftrightarrow t^2 + 2t - 3 = 0$$

$$\Leftrightarrow \begin{bmatrix} t=1 \\ t=-3 \notin \left[-\sqrt{2}\,,\sqrt{2}\right] \text{(មិនយក)} \end{bmatrix}$$

$$\Leftrightarrow \sqrt{2}\cos\left(x-\frac{\pi}{4}\right)=1$$

$$\Leftrightarrow \cos\left(x-\frac{\pi}{4}\right)=\frac{\sqrt{2}}{2}=\cos\frac{\pi}{4}$$

$$\Leftrightarrow \begin{bmatrix} x-\frac{\pi}{4}=\frac{\pi}{4}+2k\pi \\ x-\frac{\pi}{4}=-\frac{\pi}{4}+2k\pi \end{bmatrix}, k\in\mathbb{Z}$$

$$\Rightarrow \begin{bmatrix} x=\frac{\pi}{2}+2k\pi \\ x=2k\pi \end{bmatrix}, k\in\mathbb{Z}$$

ដូចនេះ សមីការមានចម្លើយ $\boxed{x=\frac{\pi}{2}+2k\pi, x=2k\pi\,,k\in\mathbb{Z}}$

ខ. $\cos 2x+5=(4-2\cos x)(\sin x-\cos x)$

$\Leftrightarrow \cos^2 x-\sin^2 x+5=4\sin x-4\cos x-2\sin x\cos x+2\cos^2 x$

$\Leftrightarrow -\cos^2 x-\sin^2 x+5-4(\sin x-\cos x)+2\sin x\cos x=0$

$\Leftrightarrow 4(\sin x-\cos x)-2\sin x\cos x-4=0$

តាង $t=\sin x-\cos x=\sqrt{2}\sin\left(x-\frac{\pi}{4}\right)$ ដែល $-\sqrt{2}\le t\le\sqrt{2}$

នោះ $2\sin x\cos x=1-t^2$

យើងបានសមីការទៅជា $4t-(1-t^2)-4=0$

$\Leftrightarrow t^2+4t-5=0$

$$\Leftrightarrow \left[\begin{array}{l} t=1 \\ t_2=-5\notin\left[-\sqrt{2}\,,\sqrt{2}\right] \text{ (មិនយក)} \end{array}\right.$$

$$\Leftrightarrow \sqrt{2}\sin\left(x-\frac{\pi}{4}\right)=1$$

$$\Leftrightarrow \sin\left(x-\frac{\pi}{4}\right)=\frac{\sqrt{2}}{2}$$

$$\Leftrightarrow \left[\begin{array}{l} x-\frac{\pi}{4}=\frac{\pi}{4}+2k\pi \\ x-\frac{\pi}{4}=\pi-\frac{\pi}{4}+2k\pi \end{array}\right.$$

$$\Rightarrow \left[\begin{array}{l} x=\frac{\pi}{2}+2k\pi \\ x=\pi+2k\pi \end{array}\right. , k\in\mathbb{Z}$$

ដូចនេះ សមីការមានចម្លើយ $\boxed{x=\frac{\pi}{2}+2k\pi\,,\ x=\pi+2k\pi\,,\ k\in\mathbb{Z}}$

គ. $\sqrt{2}\left(\sin x+\cos x\right)=\tan x+\cot x$

សមីការមានឫសកាលណា $\begin{cases}\sin x\neq 0\\ \cos x\neq 0\end{cases}$

$$\Leftrightarrow \sin 2x\neq 0$$

$$\Leftrightarrow x\neq\frac{\pi}{2}+k\pi\,,\ k\in\mathbb{Z}$$

សមីការអាចសរសេរ $\sqrt{2}\left(\sin x+\cos x\right)=\frac{\sin x}{\cos x}+\frac{\cos x}{\sin x}=\frac{1}{\sin x\cos x}$

$$\Leftrightarrow \sqrt{2}(\sin x+\cos x)\cdot\sin x\cos x=1$$

តាង $t=\sin x+\cos x=\sqrt{2}\sin\left(x+\frac{\pi}{4}\right)$ ដែល $-\sqrt{2}\le t\le\sqrt{2}$

នោះ $\sin x\cos x=\dfrac{t^2-1}{2}$

យើងបានសមីការ $\sqrt{2}t\cdot\dfrac{t^2-1}{2}=1$

$$\Leftrightarrow \sqrt{2}t^3-\sqrt{2}t-2=0$$
$$\Leftrightarrow t^3-t-\sqrt{2}=0$$
$$\Leftrightarrow \left(t-\sqrt{2}\right)\left(t^2+\sqrt{2}t+1\right)=0$$
$$\Leftrightarrow \left[\begin{array}{l} t-\sqrt{2}=0 \\ t^2+\sqrt{2}t+1=0 \end{array}\right.$$
$$\Leftrightarrow \left[\begin{array}{l} t=\sqrt{2} \\ \Delta=2-4=-2<0 \end{array}\right.$$
$$\Leftrightarrow \sqrt{2}\sin\left(x+\frac{\pi}{4}\right)=\sqrt{2}$$
$$\Leftrightarrow \sin\left(x+\frac{\pi}{4}\right)=1$$
$$\Leftrightarrow x+\frac{\pi}{4}=\frac{\pi}{2}+2k\pi\,,\ k\in\mathbb{Z}$$
$$\Rightarrow x=\frac{\pi}{4}+2k\pi\ \ ,\ k\in\mathbb{Z}$$

ដូចនេះ សមីការមានចម្លើយ $\boxed{x=\dfrac{\pi}{4}+2k\pi\ \ ,\ k\in\mathbb{Z}}$

ឃ. $\cos^3 x+\sin^3 x=\sin 2x+\sin x+\cos x$

$$\Leftrightarrow (\cos x+\sin x)\left(1-\sin x\cos x\right)-(\sin x+\cos x)-2\sin x\cos x=0$$
$$\Leftrightarrow -\sin x\cos x(\sin x+\cos x)-2\sin x\cos x=0$$
$$\Leftrightarrow \sin x\cos x\left(\sin x+\cos x+2\right)=0$$

$$\Leftrightarrow \left[\begin{array}{l}\sin x\cos x=0\\ \sin x+\cos x+2=0\end{array}\right.$$

$$\Leftrightarrow \left[\begin{array}{l}\sin 2x=0\\ \sin x+\cos x=-2\end{array}\right.$$

$$\Leftrightarrow \left[\begin{array}{l}2x=k\pi\\ 1^2+1^2<(-2)^2\end{array}\right.$$

$$\Rightarrow x=\frac{k\pi}{2}$$

ដូចនេះ សមីការមានចម្លើយ $\boxed{x=\frac{k\pi}{2},k\in\mathbb{Z}}$

**វិធីបម្លាំងទៀត**

យើងមានសមីការ $\cos^3 x+\sin^3 x=\sin 2x+\sin x+\cos x$

$$\Leftrightarrow (\cos x+\sin x)\left(1-\sin x\cos x\right)-(\sin x+\cos x)-2\sin x\cos x=0$$

តាង $t=\cos x+\sin x=\sqrt{2}\cos\left(x-\frac{\pi}{4}\right)$ ដែល $-\sqrt{2}\le t\le\sqrt{2}$

នោះ $\sin x\cos x=\frac{t^2-1}{2}$

យើងបានសមីការទៅជា $t\left(1-\frac{t^2-1}{2}\right)-t-2\cdot\frac{t^2-1}{2}=0$

$$\Leftrightarrow 2t-t^3+t-2t-2t^2+2=0$$

$$\Leftrightarrow t^3+2t^2-t-2=0$$

$$\Leftrightarrow (t+2)(t^2-1)=0$$

$$\Leftrightarrow \left[\begin{array}{l}t=-2\notin\left[-\sqrt{2},\sqrt{2}\right]\text{ (មិនយក)}\\ \text{t}=-1\\ \text{t}=1\end{array}\right.$$

$$\Leftrightarrow \begin{cases} \sqrt{2}\cos\left(x-\dfrac{\pi}{4}\right)=-1 \\ \sqrt{2}\cos\left(x-\dfrac{\pi}{4}\right)=1 \end{cases}$$

$$\Leftrightarrow \begin{cases} \cos\left(x+\dfrac{\pi}{4}\right)=\dfrac{\sqrt{2}}{2} \\ \cos\left(x+\dfrac{\pi}{4}\right)=-\dfrac{\sqrt{2}}{2} \end{cases}$$

$$\Leftrightarrow \begin{cases} x-\dfrac{\pi}{4}=\dfrac{\pi}{4}+2k\pi \\ x-\dfrac{\pi}{4}=-\dfrac{\pi}{4}+2k\pi \\ x-\dfrac{\pi}{4}=\dfrac{3\pi}{4}+2k\pi \\ x-\dfrac{\pi}{4}=-\dfrac{3\pi}{4}+2k\pi \end{cases}$$

$$\Leftrightarrow \begin{cases} x=\dfrac{\pi}{2}+2k\pi \\ x=2k\pi \\ x=\pi+2k\pi \\ x=-\dfrac{\pi}{2}+2k\pi \end{cases} \Leftrightarrow x=\frac{k\pi}{2},\ k\in\mathbb{Z}$$

ដូចនេះ សមីការមានចម្លើយ $\boxed{x=\dfrac{k\pi}{2},\ k\in\mathbb{Z}}$ ឬមានចម្លើយ

$$\boxed{x=\frac{\pi}{2}+2k\pi,\ x=2k\pi,\ x=\pi+2k\pi,\ x=-\frac{\pi}{2}+2k\pi}$$

ង. $2\sin^3 x-\cos 2x+\cos x=0$

$\Leftrightarrow 2\sin^3 x+2\sin^2 x-1+\cos x=0$

$\Leftrightarrow 2\sin^2 x(\sin x+1)-(1-\cos x)=0$

$\Leftrightarrow 2(1-\cos x)(1+\cos x)(1+\sin x)-(1-\cos x)=0$

$\Leftrightarrow (1-\cos x)\left[2+2(\sin x+\cos x)+2\sin x\cos x-1\right]=0$

$\Leftrightarrow (1-\cos x)\left[1+2(\sin x+\cos x)+2\sin x\cos x\right]=0$

- $1-\cos x=0\Leftrightarrow \cos x=1\Rightarrow x=2k\pi,\ k\in\mathbb{Z}$
- $1+2(\sin x+\cos x)+2\sin x\cos x=0$

តាង $t=\sin x+\cos x=\sqrt{2}\sin\left(x+\dfrac{\pi}{4}\right)$ ដែល $-\sqrt{2}\le t\le\sqrt{2}$

នោះ $2\sin x\cos x=t^2-1$

យើងបាន $1+2t+t^2-1=0$

$\Leftrightarrow t(t+2)=0$

$\Leftrightarrow \left[\begin{array}{l} t=0 \\ t=-2\notin\left[-\sqrt{2},\sqrt{2}\right] \text{(មិនយក)}\end{array}\right.$

$\Leftrightarrow \sqrt{2}\sin\left(x+\dfrac{\pi}{4}\right)=0$

$\Leftrightarrow \sin\left(x+\dfrac{\pi}{4}\right)=0$

$\Leftrightarrow x+\dfrac{\pi}{4}=k\pi,\ k\in\mathbb{Z}$

$\Rightarrow x=-\dfrac{\pi}{4}+k\pi\ ,\ k\in\mathbb{Z}$

ដូចនេះ សមីការមានចម្លើយ $\boxed{x=-\dfrac{\pi}{4}+k\pi,\ x=2k\pi,\ k\in\mathbb{Z}}$

លំហាត់ទី១៩ : ដោះស្រាយសមីការខាងក្រោម ៖

ក. $\cos^2 2x-\sqrt{3}\sin 4x=1+\sin^2 2x$

ខ. $\sin x-4\sin^3 x+\cos x=0$

គ. $\cos^2 x-\sqrt{3}\sin 2x=1+\sin^2 x$

ឃ. $\cos^3 x-4\sin^3 x-3\cos x\sin^2 x+\sin x=0$

## ចំណោះស្រាយ

ក. $\cos^2 2x-\sqrt{3}\sin 4x=1+\sin^2 2x$

$\Leftrightarrow \cos^2 2x-2\sqrt{3}\sin 2x\cos 2x=1+\sin^2 2x \quad (1)$

☞ បើ $\cos^2 2x\neq 0\Leftrightarrow 2x\neq\frac{\pi}{2}+k\pi\Rightarrow x\neq\frac{\pi}{4}+\frac{k\pi}{2}, k\in\mathbb{Z}$

ចែកអង្គទាំងពីរនៃសមីការ (1) នឹង $\cos^2 2x$ យើងបាន៖

$1-2\sqrt{3}\tan 2x=\frac{1}{\cos^2 2x}+\tan^2 2x$

$\Leftrightarrow 1-2\sqrt{3}\tan 2x=1+\tan^2 2x+\tan^2 2x$

$\Leftrightarrow 2\tan^2 2x+2\sqrt{3}\tan 2x=0$

$\Leftrightarrow 2\tan 2x\left(\tan 2x+\sqrt{3}\right)=0$

$\Leftrightarrow \left[\begin{array}{l}\tan 2x=0\\ \tan 2x=-\sqrt{3}\end{array}\right.$

$\Leftrightarrow \left[\begin{array}{l}2x=k\pi\\ 2x=-\frac{\pi}{3}+k\pi\end{array}\right., k\in\mathbb{Z}$

$$\Rightarrow\left[\begin{array}{l} x=\dfrac{k\pi}{2} \\ x=-\dfrac{\pi}{6}+\dfrac{k\pi}{2} \end{array}\right. , k\in\mathbb{Z}$$

☞ បើ $\cos x=0\Leftrightarrow x=\dfrac{\pi}{2}+k\pi \ , k\in\mathbb{Z}$

យើងបាន $\cos^2\left[2\left(\dfrac{\pi}{2}+k\pi\right)\right]-\sqrt{3}\sin\left[4\left(\dfrac{\pi}{2}+k\pi\right)\right]=1+\sin^2\left[2\left(\dfrac{\pi}{2}+k\pi\right)\right]$

$$\Leftrightarrow \cos^2\pi-\sqrt{3}\sin 2\pi=1+\sin^2\pi$$
$$\Leftrightarrow 1=1 \text{ (ផ្ទៀងផ្ទាត់)}$$

នោះ $x=\dfrac{\pi}{2}+k\pi=\dfrac{(2k+1)\pi}{2} \ , k\in\mathbb{Z}$ ជាឫសនៃសមីការ

តែ $\left\{\dfrac{(2k+1)\pi}{2}\right\}\subset\left\{\dfrac{k\pi}{2}\right\}$

ដូចនេះ សមីការមានចម្លើយ $\boxed{x=\dfrac{k\pi}{2}\ , x=-\dfrac{\pi}{6}+\dfrac{k\pi}{2}\ , k\in\mathbb{Z}}$

## វិធីបម្លាំងទៀត

យើងមាន $\cos^2 2x-\sqrt{3}\sin 4x=1+\sin^2 2x$

$$\Leftrightarrow \frac{1+\cos 4x}{2}-\sqrt{3}\sin 4x=1+\frac{1-\cos 4x}{2}$$
$$\Leftrightarrow 1+\cos 4x-2\sqrt{3}\sin 4x=2+1-\cos 4x$$
$$\Leftrightarrow \cos 4x-\sqrt{3}\sin 4x=1$$
$$\Leftrightarrow \frac{1}{2}\cos 4x-\frac{\sqrt{3}}{2}\sin 4x=\frac{1}{2}$$
$$\Leftrightarrow \cos\left(4x+\frac{\pi}{3}\right)=\cos\frac{\pi}{3}$$

$$\Leftrightarrow \left[\begin{array}{l} 4x+\dfrac{\pi}{3}=\dfrac{\pi}{3}+2k\pi \\ 4x+\dfrac{\pi}{3}=-\dfrac{\pi}{3}+2k\pi \end{array}\right. , k\in\mathbb{Z}$$

$$\Rightarrow \left[\begin{array}{l} x=\dfrac{k\pi}{2} \\ x=-\dfrac{\pi}{6}+\dfrac{k\pi}{2} \end{array}\right.$$

ដូចនេះ សមីការមានចម្លើយ $\boxed{x=\dfrac{k\pi}{2}, x=-\dfrac{\pi}{6}+\dfrac{k\pi}{2}, k\in\mathbb{Z}}$

ខ. $\sin x-4\sin^3 x+\cos x=0$ (2)

☞ បើ $\cos x\neq 0\Leftrightarrow x\neq\dfrac{\pi}{2}+k\pi$ ចែកអង្គទាំងពីរនៃ (2) នឹង $\cos^3 x$

យើងបាន $\dfrac{\sin x}{\cos^3 x}-4\dfrac{\sin^3 x}{\cos^3 x}+\dfrac{\cos x}{\cos^3 x}=0$

$\Leftrightarrow \tan x(\tan^2 x+1)-4\tan^3 x+(\tan^2 x+1)=0$

$\Leftrightarrow \tan^3 x+\tan x-4\tan^3 x+\tan^2 x+1=0$

$\Leftrightarrow 3\tan^3 x-\tan^2 x-\tan x-1=0$

$\Leftrightarrow (\tan x-1)(3\tan^2 x+2\tan x+1)=0$

$\Leftrightarrow \tan x=1$ (ព្រោះ $3\tan^2 x+2\tan x+1=0$ មាន $\Delta'=1-3<0$)

$\Rightarrow x=\dfrac{\pi}{4}+k\pi$ , $k\in\mathbb{Z}$

☞ បើ $\cos x=0\Leftrightarrow x=\dfrac{\pi}{2}+k\pi$ យើងបាន:

$$\sin\left(\frac{\pi}{2}+k\pi\right)-4\sin^3\left(\frac{\pi}{2}+k\pi\right)+\cos\left(\frac{\pi}{2}+k\pi\right)=0$$

$-1+4+0=0$

$3=0$ (មិនផ្ទៀងផ្ទាត់) នោះ $x=\frac{\pi}{2}+k\pi$ មិនមែនជាឫសសមីការទេ

ដូចនេះ សមីការមានចម្លើយ $\boxed{x=\frac{\pi}{4}+k\pi\ ,k\in\mathbb{Z}}$

គ. $\cos^2 x-\sqrt{3}\sin 2x=1+\sin^2 x$ (3)

$\Leftrightarrow \cos^2 x-2\sqrt{3}\sin x\cos x-\sin^2 x-1=0$

☞ បើ $\cos x\neq 0\Leftrightarrow x\neq\frac{\pi}{2}+k\pi$, $k\in\mathbb{Z}$ ចែកអង្គទាំងពីរនៃ (3) នឹង $\cos^2 x$

យើងបាន $1-2\sqrt{3}\tan x-\tan^2 x-\frac{1}{\cos^2 x}=0$

$\Leftrightarrow 1-2\sqrt{3}\tan x-\tan^2 x-1-\tan^2 x=0$

$\Leftrightarrow \tan^2 x+\sqrt{3}\tan x=0$

$\Leftrightarrow \tan x(\tan x+\sqrt{3})=0$

$\Leftrightarrow \left[\begin{array}{l}\tan x=0\\ \tan x=-\sqrt{3}\end{array}\right.$

$\Leftrightarrow \left[\begin{array}{l}x=k\pi\\ x=-\frac{\pi}{3}+k\pi\end{array}\right.,k\in\mathbb{Z}$

☞ បើ $\cos x=0\Leftrightarrow x=\frac{\pi}{2}+k\pi$, $k\in\mathbb{Z}$

យើងបាន $\cos^2\left(\frac{\pi}{2}+k\pi\right)-\sqrt{3}\sin\left(\pi+2k\pi\right)=1+\sin^2\left(\frac{\pi}{2}+k\pi\right)$

$0=2$ (មិនផ្ទៀងផ្ទាត់)

នោះ $x=\frac{\pi}{2}+k\pi$ មិនមែនជាឫសនៃសមីការនេះទេ

ដូចនេះ សមីការមានចម្លើយ $\boxed{x=k\pi\ ,\ x=-\frac{\pi}{3}+k\pi\ ,\ k\in\mathbb{Z}}$

ឃ. $\cos^3 x-4\sin^3 x-3\cos x\sin^2 x+\sin x=0$ (4)

ចែកអង្គទាំងពីរនៃសមីការនឹង $\cos^3 x$ ដែល $\cos x\neq 0$

$$\Leftrightarrow x\neq\frac{\pi}{2}+k\pi\ ,\ k\in\mathbb{Z}$$

យើងបាន $\cos^3 x-4\sin^3 x-3\cos x\sin^2 x+\sin x=0$ (4)

$$\Leftrightarrow 1-4\tan^3 x-3\tan^2 x+\tan x(\tan^2 x+1)=0$$
$$\Leftrightarrow -3\tan^3 x-3\tan^2 x+\tan x+1=0$$
$$\Leftrightarrow -3\tan^2 x(\tan x+1)+(\tan x+1)=0$$
$$\Leftrightarrow (\tan x+1)(-3\tan^2 x+1)=0$$
$$\Leftrightarrow \left[\begin{array}{l}\tan x+1=0\\ -3\tan^2 x+1=0\end{array}\right.$$
$$\Leftrightarrow \left[\begin{array}{l}\tan x=-1\\ \tan x=-\frac{\sqrt{3}}{3}\\ \tan x=\frac{\sqrt{3}}{3}\end{array}\right.$$
$$\Leftrightarrow \left[\begin{array}{l}x=-\frac{\pi}{4}+k\pi\\ x=-\frac{\pi}{6}+k\pi\ ,\ k\in\mathbb{Z}\\ x=\frac{\pi}{6}+k\pi\end{array}\right.$$

ដូចនេះ សមីការមានចម្លើយ

$$x=-\frac{\pi}{4}+k\pi\ ,\ x=-\frac{\pi}{6}+k\pi\ ,\ x=\frac{\pi}{6}+k\pi\ ,\ k\in\mathbb{Z}$$

លំហាត់ទី២០ : ដោះស្រាយសមីការខាងក្រោម ៖

ក. $\sin^2 x(\tan x-2)=3(\cos 2x+\sin x\cos x)$

ខ. $\sin x\sin 2x+\sin 3x=6\cos^3 x$

គ. $3\cos^4 x-\sin^2 2x+\sin^4 x=0$

ឃ. $\sin 3x+\cos 3x+2\cos x=0$

## ចំណោះស្រាយ

ក. $\sin^2 x(\tan x-2)=3(\cos 2x+\sin x\cos x)$

$\Leftrightarrow \sin^2 x(\tan x-2)=3(\cos^2 x-\sin^2 x+\sin x\cos x)$

សមីការមានចម្លើយកាលណា $\cos x\neq 0\Leftrightarrow x\neq\frac{\pi}{2}+k\pi\ ,k\in\mathbb{Z}$

ចែកអង្គទាំងពីរនៃសមីការនឹង $\cos^2 x$ ដែល $\cos x\neq 0\Leftrightarrow x\neq\frac{\pi}{2}+k\pi$

យើងបាន $\sin^2 x(\tan x-2)=3(\cos^2 x-\sin^2 x+\sin x\cos x)$

$\Leftrightarrow \tan^2 x(\tan x-2)=3(1-\tan^2 x+\tan x)$

$\Leftrightarrow \tan^3 x-2\tan^2 x-3+3\tan^2 x-3\tan x=0$

$\Leftrightarrow \tan^3 x+\tan^2 x-3\tan x-3=0$

$\Leftrightarrow \tan^2 x(\tan x+1)-3(\tan x+1)=0$

$\Leftrightarrow (\tan x+1)(\tan^2 x-3)=0$

$$\Leftrightarrow \left[\begin{array}{l}\tan x+1=0\\ \tan^2 x-3=0\end{array}\right.$$

$$\Leftrightarrow \left[\begin{array}{l}\tan x=-1\\ \tan x=-\sqrt{3}\\ \tan x=\sqrt{3}\end{array}\right.$$

$$\Leftrightarrow \left[\begin{array}{l}x=-\dfrac{\pi}{4}+k\pi\\ x=-\dfrac{\pi}{3}+k\pi\ ,\ k\in\mathbb{Z}\\ x=\dfrac{\pi}{3}+k\pi\end{array}\right.$$

ដូចនេះ សមីការមានចម្លើយ

$$\boxed{x=-\frac{\pi}{4}+k\pi\ ,\ x=-\frac{\pi}{3}+k\pi\ ,\ x=\frac{\pi}{3}+k\pi\ ,\ k\in\mathbb{Z}}$$

ខ. $\sin x\sin 2x+\sin 3x=6\cos^3 x$

$\Leftrightarrow 2\sin^2 x\cos x+(3\sin x-4\sin^3 x)=6\cos^3 x$

ចែកអង្គទាំងពីរនឹង $\cos^3 x$ ដែល $\cos x\neq 0\Leftrightarrow x\neq\dfrac{\pi}{2}+k\pi\ ,\ k\in\mathbb{Z}$

យើងបាន $\dfrac{2\sin^2 x\cos x}{\cos^3 x}+3\dfrac{\sin x}{\cos^3 x}-4\dfrac{\sin^3 x}{\cos^3 x}=6$

$\Leftrightarrow 2\tan^2 x+3\tan x(\tan^2 x+1)-4\tan^3 x=6$

$\Leftrightarrow -\tan^3 x+2\tan^2 x+3\tan x-6=0$

$\Leftrightarrow -\tan^2 x(\tan x-2)+3(\tan x-2)=0$

$\Leftrightarrow (\tan x-2)(-\tan^2 x+3)=0$

$$\Leftrightarrow \left[\begin{array}{l} \tan x = 2 \\ \tan x = -\sqrt{3} \\ \tan x = \sqrt{3} \end{array}\right.$$

$$\Leftrightarrow \left[\begin{array}{l} x = \arctan 2 + k\pi \\ x = -\dfrac{\pi}{3} + k\pi \\ x = \dfrac{\pi}{3} + k\pi \end{array}\right. , k \in \mathbb{Z}$$

ដូចនេះ សមីការមានចម្លើយ

$$\boxed{x = -\frac{\pi}{3} + k\pi \, , \, x = \frac{\pi}{3} + k\pi \, , \, x = \arctan 2 + k\pi \, , \, k \in \mathbb{Z}}$$

គ. $3\cos^4 x - \sin^2 2x + \sin^4 x = 0$

$\Leftrightarrow 3\cos^4 x - 4\sin^2 x\cos^2 x + \sin^4 x = 0$

ចែកអង្គទាំងពីរនៃសមីការ $\cos^4 x$ ដែល $\cos x \neq 0$

យើងបាន $3 - 4\tan^2 x + \tan^4 x = 0$

$$\Leftrightarrow (\tan^2 x - 1)(\tan^2 x - 3) = 0$$

$$\Leftrightarrow \left[\begin{array}{l} \tan^2 x = 1 \\ \tan^2 x = 3 \end{array}\right.$$

$$\Leftrightarrow \left[\begin{array}{l} \tan x = \pm 1 \\ \tan x = \pm\sqrt{3} \end{array}\right.$$

$$\Rightarrow \left[\begin{array}{l} x = \pm\dfrac{\pi}{4} + k\pi \\ x = \pm\dfrac{\pi}{3} + k\pi \end{array}\right. , k \in \mathbb{Z}$$

ដូចនេះ សមីការមានចម្លើយ $\boxed{x=\pm\frac{\pi}{4}+k\pi\,,\,x=\pm\frac{\pi}{3}+k\pi\,,\,k\in\mathbb{Z}}$

ឃ. $\sin 3x+\cos 3x+2\cos x=0$

$\Leftrightarrow 3\sin x-4\sin^3 x+4\cos^3 x-3\cos x+2\cos x=0$

$\Leftrightarrow 4\cos^3 x-4\sin^3 x+3\sin x-\cos x=0$

ដោយ $\cos x=0\Leftrightarrow x=\frac{\pi}{2}+k\pi\,,k\in\mathbb{Z}$ មិនមែនជាឫសនៃសមីការ ចែកអង្គទាំងពីរនៃសមីការនឹង $\cos^3 x$ យើងបាន :

$4-4\tan^3 x+3\tan x(\tan^2 x+1)-(1+\tan^2 x)=0$

$\Leftrightarrow -\tan^3 x-\tan^2 x+3\tan x+3=0$

$\Leftrightarrow -\tan^2 x(\tan x+1)+3(\tan x+1)=0$

$\Leftrightarrow (\tan x+1)(-\tan^2 x+3)=0$

$$\Leftrightarrow\left[\begin{array}{l}\tan x=-1\\ \tan x=\pm\sqrt{3}\end{array}\right.$$

$$\Rightarrow\left[\begin{array}{l}x=-\frac{\pi}{4}+k\pi\\ x=\pm\frac{\pi}{3}+k\pi\end{array}\right.,\ k\in\mathbb{Z}$$

ដូចនេះ សមីការមានចម្លើយ $\boxed{x=-\frac{\pi}{4}+k\pi\,,\,x=\pm\frac{\pi}{3}+k\pi\,,\,k\in\mathbb{Z}}$

លំហាត់ទី២១ : ដោះស្រាយសមីការខាងក្រោម ៖

ក. $\sin x+\sin 2x+\sin 3x=\cos x+\cos 2x+\cos 3x$

ខ. $4\sin^3 x+3\cos^3 x-3\sin x-\sin^2 x\cos x=0$

គ. $\tan^2 x\cot^2 2x\cot 3x=\tan^2 x-\cot^2 2x+\cot 3x$

ឃ. $\frac{(1-\cos x)^2+(1+\cos x)^2}{4(1-\sin x)}-\tan^2 x\sin x=\frac{1}{2}(1+\sin x)+\tan^2 x$

## ដំណោះស្រាយ

ក. $\sin x+\sin 2x+\sin 3x=\cos x+\cos 2x+\cos 3x$

$\Leftrightarrow(\sin x+\sin 3x)+\sin 2x=(\cos x+\cos 3x)+\cos 2x$

$\Leftrightarrow 2\sin 2x\cos x+\sin 2x=2\cos 2x\cos x+\cos 2x$

$\Leftrightarrow \sin 2x(2\cos x+1)-\cos 2x(2\cos x+1)=0$

$\Leftrightarrow(2\cos x+1)(\sin 2x-\cos 2x)=0$

$\Leftrightarrow\left[\begin{array}{l}2\cos x+1=0\\ \sin 2x-\cos 2x=0\end{array}\right.$

$\Leftrightarrow\left[\begin{array}{l}\cos x=-\frac{1}{2}\\ \tan 2x=1\end{array}\right.$

$\Leftrightarrow\left[\begin{array}{l}x=\pm\frac{2\pi}{3}+k\pi\\ 2x=\frac{\pi}{4}+k\pi\end{array}\right.,\ k\in\mathbb{Z}$

$$\Rightarrow \begin{cases} x = \pm\dfrac{2\pi}{3} + k\pi \\ x = \dfrac{\pi}{8} + \dfrac{k\pi}{2} \end{cases}, k \in \mathbb{Z}$$

ដូចនេះ សមីការមានចម្លើយ $\boxed{x = \pm\dfrac{2\pi}{3} + k\pi \, , \, x = \dfrac{\pi}{8} + \dfrac{k\pi}{2} \, , \, k \in \mathbb{Z}}$

ខ. $4\sin^3 x + 3\cos^3 x - 3\sin x - \sin^2 x \cos x = 0$

$\Leftrightarrow \sin x(4\sin^2 x - 3) + \cos x(3\cos^2 x - \sin^2 x) = 0$

$\Leftrightarrow \sin x(4\sin^2 x - 3) + \cos x(3 - 3\sin^2 x - \sin^2 x) = 0$

$\Leftrightarrow \sin x(4\sin^2 x - 3) - \cos x(4\sin^2 x - 3) = 0$

$\Leftrightarrow (4\sin^2 x - 3)(\sin x - \cos x) = 0$

$$\Leftrightarrow \begin{cases} 4\sin^2 x - 3 = 0 \\ \sin x - \cos x = 0 \end{cases}$$

$$\Leftrightarrow \begin{cases} \sin x = \dfrac{\sqrt{3}}{2} \\ \sin x = -\dfrac{\sqrt{3}}{2} \\ \tan x = 1 \end{cases}$$

$$\Leftrightarrow \begin{cases} x = \dfrac{\pi}{3} + 2k\pi \ \vee \ x = \dfrac{2\pi}{3} + 2k\pi \, , \, k \in \mathbb{Z} \\ x = -\dfrac{\pi}{3} + 2k\pi \vee x = \dfrac{4\pi}{3} + 2k\pi \, , \, k \in \mathbb{Z} \\ x = \dfrac{\pi}{4} + k\pi \, , \, k \in \mathbb{Z} \end{cases}$$

$$\Leftrightarrow \begin{bmatrix} x=\pm\dfrac{\pi}{3}+k\pi \\ x=\dfrac{\pi}{4}+k\pi \end{bmatrix}, k\in\mathbb{Z}$$

ដូចនេះ សមីការមានចម្លើយ $\boxed{x=\pm\dfrac{\pi}{3}+k\pi,\ x=\dfrac{\pi}{4}+k\pi,\ k\in\mathbb{Z}}$

គ. $\tan^2 x\cot^2 2x\cot 3x=\tan^2 x-\cot^2 2x+\cot 3x$

សមីការមានឫសកាលណា $\begin{cases}\cos x\neq 0\\ \sin 2x\neq 0\\ \sin 3x\neq 0\end{cases}\Leftrightarrow\begin{cases}x\neq\dfrac{\pi}{2}+k\pi\\ x\neq\dfrac{k\pi}{2}\\ x\neq\dfrac{k\pi}{3}\end{cases}$

យើងបាន $\cot 3x(\tan^2 x\cot^2 2x-1)=\tan^2 x-\cot^2 2x$

$$\Leftrightarrow \cot 3x\left(\frac{\sin^2 x}{\cos^2 x}\cdot\frac{\cos^2 2x}{\sin^2 2x}-1\right)=\frac{\sin^2 x}{\cos^2 x}-\frac{\cos^2 2x}{\sin^2 2x}$$

$$\Leftrightarrow \cot 3x\left[\frac{(1-\cos 2x)(1+\cos 4x)}{(1+\cos 2x)(1-\cos 4x)}-1\right]=\frac{1-\cos 2x}{1+\cos 2x}-\frac{1+\cos 4x}{1-\cos 4x}$$

$$\Leftrightarrow \cot 3x\left[(1-\cos 2x)(1+\cos 4x)-(1+\cos 2x)(1-\cos 4x)\right]$$
$$=(1-\cos 2x)(1-\cos 4x)-(1+\cos 2x)(1+\cos 4x)$$

$\Leftrightarrow$ cot3x(1+cos4x-cos2x-cos2xcos4x-1+cos4x-cos2x+cos2xcos4x)

=1-cos4x-cos2x+cos2xcos4x-1-cos4x-cos2x-cos2xcos4x

$$\Leftrightarrow \cot 3x(2\cos 4x-2\cos 2x)=-2\cos 4x-2\cos 2x$$

$$\Leftrightarrow \frac{\cos 3x}{\sin 3x}\cdot(-4\sin 3x\sin x)=-4\cos 3x\cos x$$

$$\Leftrightarrow \cos 3x\sin x=\cos 3x\cos x$$

$$\Leftrightarrow \cos 3x(\sin x-\cos x)=0$$

$$\Leftrightarrow \left[\begin{array}{l}\cos 3x=0\\ \sin x-\cos x=0\end{array}\right.$$

$$\Leftrightarrow \left[\begin{array}{l}3x=\dfrac{\pi}{2}+k\pi\\ \tan x=1\end{array}\right.$$

$$\Rightarrow \left[\begin{array}{l}x=\dfrac{\pi}{6}+\dfrac{k\pi}{3}\\ x=\dfrac{\pi}{4}+k\pi\end{array}\right., k\in\mathbb{Z}$$

ដូចនេះ សមីការមានចម្លើយ $\boxed{x=\dfrac{\pi}{6}+\dfrac{k\pi}{3}, x=\dfrac{\pi}{4}+k\pi, k\in\mathbb{Z}}$

ឃ. $\dfrac{(1-\cos x)^2+(1+\cos x)^2}{4(1-\sin x)}-\tan^2 x\sin x=\dfrac{1}{2}(1+\sin x)+\tan^2 x$

សមីការមានឫសកាលណា $\begin{cases}\cos x\neq 0\\ 1-\sin x\neq 0\end{cases}\Leftrightarrow x\neq\dfrac{\pi}{2}+k\pi, k\in\mathbb{Z}$

$$\frac{(1-\cos x)^2+(1+\cos x)^2}{4(1-\sin x)}-\tan^2 x\sin x=\frac{1}{2}(1+\sin x)+\tan^2 x$$

$$\Leftrightarrow \frac{2(1+\cos^2 x)}{4(1-\sin x)}-\frac{\sin^3 x}{1-\sin^2 x}=\frac{1}{2}(1+\sin x)+\frac{\sin^2 x}{1-\sin^2 x}$$

$$\Leftrightarrow 2(1+\cos^2 x)(1+\sin x)-4\sin^3 x=2(1+\sin x)\cos^2 x+4\sin^2 x$$

$$\Leftrightarrow 2(1+\cos^2 x)(1+\sin x)-2(1+\sin x)\cos^2 x-4\sin^2 x(1+\sin x)=0$$

$$\Leftrightarrow (1+\sin x)(2+2\cos^2 x-2\cos^2 x-4\sin^2 x)=0$$

$$\Leftrightarrow (1+\sin x)(2-4\sin^2 x)=0$$

$$\Leftrightarrow (1+\sin x)(1-2\sin^2 x)=0$$

$$\Leftrightarrow \begin{cases} 1+\sin x=0 \\ 1-2\sin^2 x=0 \end{cases}$$

$$\Leftrightarrow \begin{cases} \sin x=-1 \\ \sin x=-\dfrac{\sqrt{2}}{2} \\ \sin x=\dfrac{\sqrt{2}}{2} \end{cases}$$

$$\Leftrightarrow \begin{cases} x=-\dfrac{\pi}{2}+k\pi \\ x=-\dfrac{\pi}{4}+2k\pi \text{ , } x=\dfrac{5\pi}{4}+2k\pi \quad ,k\in\mathbb{Z} \\ x=\dfrac{\pi}{4}+2k\pi \text{ , } x=\dfrac{3\pi}{4}+2k\pi \end{cases}$$

$$\Leftrightarrow \begin{cases} x=-\dfrac{\pi}{2}+k\pi \\ x=\pm\dfrac{\pi}{4}+k\pi \end{cases} ,k\in\mathbb{Z} \text{ ឬ } \begin{cases} x=-\dfrac{\pi}{2}+k\pi \\ x=\dfrac{\pi}{4}+\dfrac{k\pi}{2} \end{cases} ,k\in\mathbb{Z}$$

តែ $x\neq\dfrac{\pi}{2}+k\pi\Rightarrow x\neq-\dfrac{\pi}{2}+k\pi$

ដូចនេះ សមីការមានចម្លើយ $\boxed{x=\dfrac{\pi}{4}+\dfrac{k\pi}{2},k\in\mathbb{Z}}$

លំហាត់ទី២២ : ដោះស្រាយសមីការខាងក្រោម ៖

ក. $4\cos^3 x+3\sqrt{2}\sin 2x=8\cos x$

ខ. $2\sin 3x-\dfrac{1}{\sin x}=2\cos 3x+\dfrac{1}{\cos x}$

គ. $\cos x\cos\dfrac{x}{2}\cos\dfrac{3x}{2}-\sin x\sin\dfrac{x}{2}\sin\dfrac{3x}{2}=\dfrac{1}{2}$

ឃ. $\cos x+\cos 3x+2\cos 5x=0$

## ចំណោះស្រាយ

ក. $4\cos^3 x+3\sqrt{2}\sin 2x=8\cos x$

$\Leftrightarrow 4\cos^3 x+6\sqrt{2}\sin x\cos x-8\cos x=0$

$\Leftrightarrow 2\cos x(2\cos^2 x+3\sqrt{2}\sin x-4)=0$

$\Leftrightarrow 2\cos x(2-2\sin^2 x+3\sqrt{2}\sin x-4)=0$

$\Leftrightarrow \cos x(2\sin^2 x-3\sqrt{2}\sin x+2)=0$

- $\cos x=0\Leftrightarrow x=\dfrac{\pi}{2}+k\pi\ ,k\in\mathbb{Z}$
- $2\sin^2 x-3\sqrt{2}\sin x+2=0$

  $\Delta=18-16=2$

$$\left[\begin{array}{l}\sin x=\dfrac{3\sqrt{2}-\sqrt{2}}{4}=\dfrac{\sqrt{2}}{2}\\ \sin x=\dfrac{3\sqrt{2}+\sqrt{2}}{4}=\sqrt{2}\notin[-1,1]\ (\text{មិនយក})\end{array}\right.$$

• $\sin x=\frac{\sqrt{2}}{2}\Rightarrow\left[\begin{array}{l}x=\frac{\pi}{4}+2k\pi\\ x=\frac{3\pi}{4}+2k\pi\end{array}\right.,k\in\mathbb{Z}$

ដូចនេះ សមីការមានចម្លើយ

$$x=\frac{\pi}{2}+k\pi\ ,\ x=\frac{\pi}{4}+2k\pi\ ,\ x=\frac{3\pi}{4}+2k\pi,k\in\mathbb{Z}$$

ខ. $2\sin 3x-\frac{1}{\sin x}=2\cos 3x+\frac{1}{\cos x}$

សមីការមានឫសកាលណា $\left[\begin{array}{l}\sin x\neq 0\\ \cos x\neq 0\end{array}\right.\Leftrightarrow\sin 2x\neq 0\Rightarrow x\neq\frac{k\pi}{2}$

យើងបាន $2(\sin 3x-\cos 3x)-\frac{1}{\sin x}-\frac{1}{\cos x}=0$

$\Leftrightarrow 2(3\sin x-4\sin^3 x-4\cos^3 x+3\cos x)-\frac{\cos x+\sin x}{\sin x\cos x}=0$

$\Leftrightarrow 2\left[-4(\sin x+\cos x)(\sin^2 x-\sin x\cos x+\cos^2 x)+3(\sin x+\cos x)\right]-\frac{\cos x+\sin x}{\sin x\cos x}=0$

$\Leftrightarrow(\cos x+\sin x)(-8+8\sin x\cos x+6-\frac{1}{\sin x\cos x})=0$

$\Leftrightarrow(\sin x+\cos x)\left(\frac{-2\sin x\cos x+8\sin^2 x\cos^2 x-1}{\sin x\cos x}\right)=0$

$\Leftrightarrow(\sin x+\cos x)(2\sin^2 2x-\sin 2x-1)=0$

$\Leftrightarrow\left[\begin{array}{l}\sin x+\cos x=0\\ 2\sin^2 2x-\sin 2x-1=0\end{array}\right.$

$$\Leftrightarrow \begin{cases} \tan x = -1 \\ \sin 2x = 1 \\ \sin 2x = -\dfrac{1}{2} \end{cases}$$

$$\Leftrightarrow \begin{cases} x = -\dfrac{\pi}{4} + k\pi \\ 2x = \dfrac{\pi}{2} + 2k\pi \\ 2x = -\dfrac{\pi}{6} + 2k\pi \\ 2x = \dfrac{7\pi}{6} + 2k\pi \end{cases}, k \in \mathbb{Z}$$

$$\Rightarrow \begin{cases} x = -\dfrac{\pi}{4} + k\pi \\ x = \dfrac{\pi}{4} + k\pi \\ x = -\dfrac{\pi}{12} + k\pi \\ x = \dfrac{7\pi}{12} + k\pi \end{cases}, k \in \mathbb{Z}$$

ដូចនេះ សមីការមានចម្លើយ

$$\boxed{x = \pm\frac{\pi}{4} + k\pi \text{ , } x = -\frac{\pi}{12} + k\pi \text{ , } x = \frac{7\pi}{12} + k\pi \text{ , } k \in \mathbb{Z}}$$

គ. $\cos x \cos\dfrac{x}{2}\cos\dfrac{3x}{2} - \sin x \sin\dfrac{x}{2}\sin\dfrac{3x}{2} = \dfrac{1}{2}$

$$\Leftrightarrow \frac{1}{2}\cos x(\cos 2x + \cos x) + \frac{1}{2}\sin x(\cos 2x - \cos x) = \frac{1}{2}$$

$\Leftrightarrow \cos x\cos 2x+\cos^2 x+\sin x\cos 2x-\sin x\cos x=1$

$\Leftrightarrow \cos 2x(\cos x+\sin x)+\cos^2 x-1-\sin x\cos x=0$

$\Leftrightarrow \cos 2x(\cos x+\sin x)-\sin x(\sin x+\cos x)=0$

$\Leftrightarrow (\cos x+\sin x)(\cos 2x-\sin x)=0$

$$\Leftrightarrow \left[\begin{array}{l}\cos x+\sin x=0\\ \cos 2x-\sin x=0\end{array}\right.$$

$$\Leftrightarrow \left[\begin{array}{l}\cos x=-\sin x=\cos\left(\dfrac{\pi}{2}+x\right)\\ \cos 2x=\sin x=\cos\left(\dfrac{\pi}{2}-x\right)\end{array}\right.$$

$$\Leftrightarrow \left[\begin{array}{l}x=\pm\left(\dfrac{\pi}{2}+x\right)+2k\pi\\ 2x=\pm\left(\dfrac{\pi}{2}-x\right)+2k\pi\end{array}\right. ,k\in\mathbb{Z}$$

$$\Rightarrow \left[\begin{array}{l}x=-\dfrac{\pi}{4}+k\pi\\ x=\dfrac{\pi}{6}+\dfrac{2k\pi}{3}\\ x=-\dfrac{\pi}{2}+2k\pi\end{array}\right. ,k\in\mathbb{Z}$$

ដូចនេះ សមីការមានចម្លើយ

$$\boxed{x=-\frac{\pi}{4}+k\pi\,,\,x=\frac{\pi}{6}+\frac{2k\pi}{3}\,,\,x=-\frac{\pi}{2}+2k\pi\,,\,k\in\mathbb{Z}}$$

## រប្បៀបម្យ៉ាងទៀត

គ. $\cos x\cos\frac{x}{2}\cos\frac{3x}{2}-\sin x\sin\frac{x}{2}\sin\frac{3x}{2}=\frac{1}{2}$

$\Leftrightarrow \frac{1}{2}\cos x(\cos 2x+\cos x)+\frac{1}{2}\sin x(\cos 2x-\cos x)=\frac{1}{2}$

$\Leftrightarrow \cos x\cos 2x+\cos^2 x+\sin x\cos 2x-\sin x\cos x=1$

$\Leftrightarrow \cos 2x(\cos x+\sin x)+\cos^2 x-1-\sin x\cos x=0$

$\Leftrightarrow \cos 2x(\cos x+\sin x)-\sin x(\sin x+\cos x)=0$

$\Leftrightarrow (\cos x+\sin x)(\cos 2x-\sin x)=0$

$\Leftrightarrow (\cos x+\sin x)(1-2\sin^2 x-\sin x)=0$

$$\Leftrightarrow \left[\begin{array}{l}\cos x+\sin x=0\\ -2\sin^2 x-\sin x+1=0\end{array}\right.$$

$$\Leftrightarrow \left[\begin{array}{l}\tan x=-1\\ \sin x=-1\\ \sin x=\frac{1}{2}\end{array}\right.$$

$$\Leftrightarrow \left[\begin{array}{l}x=\frac{\pi}{4}+k\pi\\ x=-\frac{\pi}{2}+2k\pi\\ x=\frac{\pi}{6}+2k\pi\\ x=\frac{5\pi}{6}+2k\pi\end{array}\right. ,k\in\mathbb{Z}$$

ឃ. $\cos x+\cos 3x+2\cos 5x=0$

$\Leftrightarrow (\cos x+\cos 5x)+(\cos 3x+\cos 5x)=0$

$\Leftrightarrow 2\cos 3x\cos 2x+2\cos 4x\cos x=0$

$\Leftrightarrow (4\cos^3 x-3\cos x)\cos 2x+(2\cos^2 2x-1)\cos x=0$

$\Leftrightarrow \cos x\left[(4\cos^2 x-3)\cos 2x+2\cos^2 2x-1\right]=0$

$\Leftrightarrow \cos x\left[\left(4\dfrac{1+\cos 2x}{2}-3\right)\cos 2x+2\cos^2 2x-1\right]=0$

$\Leftrightarrow \cos x\left(2\cos^2 2x-\cos 2x+2\cos^2 2x-1\right)=0$

$\Leftrightarrow \cos x\left(4\cos^2 2x-\cos 2x-1\right)=0$

$\Leftrightarrow \left[\begin{array}{l}\cos x=0\\ 4\cos^2 2x-\cos 2x-1=0\end{array}\right.$

$\Leftrightarrow \left[\begin{array}{l}\cos x=0\\ \cos 2x=\dfrac{1+\sqrt{17}}{8}\\ \cos x=\dfrac{1-\sqrt{17}}{8}\end{array}\right.$

$\Leftrightarrow \left[\begin{array}{l}x=\dfrac{\pi}{2}+k\pi\\ x=\pm\dfrac{1}{2}\arccos\left(\dfrac{1+\sqrt{17}}{8}\right)+k\pi\\ x=\pm\dfrac{1}{2}\arccos\left(\dfrac{1-\sqrt{17}}{8}\right)+k\pi\end{array}\right.\quad ,k\in\mathbb{Z}$

ដូចនេះសមីការមានចម្លើយ $x=\frac{\pi}{2}+k\pi$,

$x=\pm\frac{1}{2}\arccos\left(\frac{1+\sqrt{17}}{8}\right)+k\pi, x=\pm\frac{1}{2}\arccos\left(\frac{1-\sqrt{17}}{8}\right)+k\pi, k\in\mathbb{Z}$

លំហាត់ទី២៣: ដោះស្រាយសមីការខាងក្រោម ៖

ក. $\sin\frac{5x}{2}=5\cos^3 x\sin\frac{x}{2}$

ខ. $\sin^8 x+\cos^8 x=2(\sin^{10} x+\cos^{10} x)-\cos 2x$

គ. $2\cos^2\frac{6x}{5}+1=3\cos\frac{8x}{5}$

ឃ. $\sin 2x+2\tan x=3$

## ដំណោះស្រាយ

ក. $\sin\frac{5x}{2}=5\cos^3 x\sin\frac{x}{2}$

បើ $\cos\frac{x}{2}=0\Leftrightarrow x=\pi+2k\pi, k\in\mathbb{Z}$

នោះ យើងបាន $\sin\frac{5(\pi+2k\pi)}{2}=5\cos^3(\pi+2k\pi)\sin\frac{(\pi+2k\pi)}{2}$

$\pm\sin\frac{5\pi}{2}=\pm5\cos^3\pi\cdot\sin\frac{\pi}{2}$

$\pm1=\pm5$ (មិនផ្ទៀងផ្ទាត់)

នោះ $\cos\frac{x}{2}=0\Leftrightarrow x=\pi+2k\pi, k\in\mathbb{Z}$ មិនមែនជាឫសនៃសមីការ

នោះគុណទាំងពីរនៃសមីការនឹង $2\cos\frac{x}{2}$ យើងបាន:

$$2\sin\frac{5x}{2}\cos\frac{x}{2}=10\cos^3 x\sin\frac{x}{2}\cos\frac{x}{2}$$

$$\Leftrightarrow \sin 3x+\sin 2x=5\cos^3 x\sin x$$

$$\Leftrightarrow 3\sin x-4\sin^3 x+2\sin x\cos x=5\cos^3 x\sin x$$

$$\Leftrightarrow \sin x(3-4\sin^2 x+2\cos x-5\cos^3 x)=0$$

$$\Leftrightarrow \sin x(3-4+4\cos^2 x+2\cos x-5\cos^3 x)=0$$

$$\Leftrightarrow \sin x(5\cos^3 x-4\cos^2 x-2\cos x+1)=0$$

$$\Leftrightarrow 2\sin\frac{x}{2}\cos\frac{x}{2}(\cos x-1)(5\cos^2 x+\cos x-1)=0$$

$$\Leftrightarrow \left[\begin{array}{l}\sin\dfrac{x}{2}=0\\ \cos\dfrac{x}{2}=0 \text{ (មិនយក)}\\ \cos x-1=0\\ 5\cos^2 x+\cos x-1=0\end{array}\right.$$

$$\Leftrightarrow \left[\begin{array}{l}\sin\dfrac{x}{2}=0\\ \cos x=1\\ \cos x=\dfrac{-1-\sqrt{21}}{10}\\ \cos x=\dfrac{1+\sqrt{21}}{10}\end{array}\right. \Rightarrow \left[\begin{array}{l}x=2k\pi\\ x=\pm\arccos\left(\dfrac{-1-\sqrt{21}}{10}\right)+2k\pi,\ k\in\mathbb{Z}\\ x=\pm\arccos\left(\dfrac{1+\sqrt{21}}{10}\right)+2k\pi\end{array}\right.$$

ដូចនេះ សមីការមានចម្លើយ

$$\boxed{\begin{array}{l}x=2k\pi,\ x=\pm\arccos\left(\dfrac{-1-\sqrt{21}}{10}\right)+2k\pi,\\ x=\pm\arccos\left(\dfrac{1+\sqrt{21}}{10}\right)+2k\pi,\ k\in\mathbb{Z}\end{array}}$$

ខ. $\sin^8 x+\cos^8 x=2(\sin^{10} x+\cos^{10} x)-\cos 2x$

$\Leftrightarrow (\sin^8 x-2\sin^{10} x)+(\cos^8 x-2\cos^{10} x)=-\cos 2x$

$\Leftrightarrow \sin^8 x(1-2\sin^2 x)+\cos^8 x(1-2\cos^2 x)=-\cos 2x$

$\Leftrightarrow \sin^8 x\cdot\cos 2x-\cos^8 x\cdot\cos 2x+\cos 2x=0$

$\Leftrightarrow \cos 2x(\sin^8 x-\cos^8 x+1)=0$

$\Leftrightarrow \left[\begin{array}{l}\cos 2x=0\\ \sin^8 x-\cos^8 x+1=0\end{array}\right.$

- $\cos 2x=0\Rightarrow x=\dfrac{\pi}{4}+\dfrac{k\pi}{2}, k\in\mathbb{Z}$
- $\sin^8 x-\cos^8 x+1=0$

$\Leftrightarrow (\sin^4 x-\cos^4 x)(\sin^4 x+\cos^4 x)=-1$

$\Leftrightarrow (\sin^2 x-\cos^2 x)(\sin^2 x+\cos^2 x)\left[(\sin^2 x+\cos^2 x)^2-2\sin^2 x\cos^2 x\right]=-1$

$\Leftrightarrow -\cos 2x(1-\dfrac{1}{2}\sin^2 2x)=-1$

$\Leftrightarrow \cos 2x(2-1+\cos^2 2x)=2$

$\Leftrightarrow \cos 2x+\cos^3 2x=2$

$\Leftrightarrow \cos^3 2x+\cos 2x-2=0$

$\Leftrightarrow (\cos 2x-1)(\cos^2 2x+\cos 2x+2)=0$

$\Leftrightarrow \cos 2x=1$, (ព្រោះ $\cos^2 2x+\cos 2x+2=0$ មាន $\Delta=1-8<0$)

$\Rightarrow x=k\pi$

ដូចនេះ សមីការមានចម្លើយ $\boxed{x=k\pi, x=\dfrac{\pi}{4}+\dfrac{k\pi}{2}, k\in\mathbb{Z}}$

គ. $2\cos^2\dfrac{6x}{5}+1=3\cos\dfrac{8x}{5}$

$\Leftrightarrow \left(1+\cos\dfrac{12x}{5}\right)+1=3\left(2\cos^2\dfrac{4x}{5}-1\right)$

$$\Leftrightarrow 2+4\cos^3\frac{4x}{5}-3\cos\frac{4x}{5}=6\cos^2\frac{4x}{5}-3$$

$$\Leftrightarrow 4\cos^3\frac{4x}{5}-6\cos^2\frac{4x}{5}-3\cos\frac{4x}{5}+5=0$$

$$\Leftrightarrow \left(\cos\frac{4x}{5}-1\right)\left(4\cos^2\frac{4x}{5}-2\cos\frac{4x}{5}-5\right)=0$$

$$\Leftrightarrow \left[\begin{array}{l}\cos\frac{4x}{5}-1=0\\ 4\cos^2\frac{4x}{5}-2\cos\frac{4x}{5}-5=0\end{array}\right.$$

$$\Leftrightarrow \left[\begin{array}{l}\cos\frac{4x}{5}=1\\ \cos\frac{4x}{5}=\frac{1-\sqrt{21}}{4}\\ \cos\frac{4x}{5}=\frac{1+\sqrt{21}}{4}>1\ (\text{មិនយក})\end{array}\right.$$

$$\Leftrightarrow \left[\begin{array}{l}\frac{4x}{5}=2k\pi\\ \frac{4x}{5}=\pm\arccos\left(\frac{1-\sqrt{21}}{4}\right)+2k\pi\end{array}\right. , k\in\mathbb{Z}$$

$$\Rightarrow \left[\begin{array}{l}x=\frac{5k\pi}{2}\\ x=\pm\frac{5}{4}\arccos\left(\frac{1-\sqrt{21}}{4}\right)+\frac{5k\pi}{2}\end{array}\right. , k\in\mathbb{Z}$$

ដូចនេះ សមីការមានចម្លើយ

$$\boxed{x=\frac{5k\pi}{2}\ ,\ x=\pm\frac{5}{4}\arccos\left(\frac{1-\sqrt{21}}{4}\right)+\frac{5k\pi}{2}\ ,\ k\in\mathbb{Z}}$$

ឃ. $\sin 2x + 2\tan x = 3$

សមីការមានឫសកាលណា $\cos x \neq 0 \Rightarrow x = \frac{\pi}{2} + k\pi, k \in \mathbb{Z}$

ដោយ $\sin 2x = 2\sin x \cos x = \frac{2\sin x}{\frac{1}{\cos x}} = \frac{\frac{2\sin x}{\cos x}}{\frac{1}{\cos^2 x}} \frac{2\tan x}{1+\tan^2 x}$

យើងបាន $\frac{2\tan x}{1+\tan^2 x} + 2\tan x = 3$

$\Leftrightarrow 2\tan x + 2\tan x + 2\tan^3 x = 3 + 3\tan^2 x$

$\Leftrightarrow 2\tan^3 x - 3\tan^2 x + 4\tan x - 3 = 0$

$\Leftrightarrow (\tan x - 1)(2\tan^2 x - \tan x + 3) = 0$

$\Leftrightarrow \left[\begin{array}{l} \tan x - 1 = 0 \\ 2\tan^2 x - \tan x + 3 = 0 \text{ (សមីការគ្មានឫស)} \end{array}\right.$

$\Leftrightarrow \tan x = 1$

$\Leftrightarrow x = \frac{\pi}{4} + k\pi \ , k \in \mathbb{Z}$

ដូចនេះ សមីការមានចម្លើយ $\boxed{x = \frac{\pi}{4} + k\pi \ , k \in \mathbb{Z}}$

លំហាត់ទី២៥ : ដោះស្រាយសមីការខាងក្រោម ៖

ក. $2(1-\tan x)(1+\sin 2x) = 1+\tan x$

ខ. $\frac{1}{\sqrt{2}}\cot x + \frac{\sin 2x}{\sin x + \cos x} = 2\sin\left(x + \frac{\pi}{2}\right)$

គ. $8(\sin^6 x + \cos^6 x) + 3\sqrt{3}\sin 4x = 3\sqrt{3}\cos 2x - 3\sin 2x + 8$

ឃ. $\sqrt{3}\sin 2x(2\cos x + 1) + 2 = \cos 3x + \cos 2x - 3\cos x$

## ចំណោះស្រាយ

ក. $2(1-\tan x)(1+\sin 2x)=1+\tan x$

សមីការមានឫសកាលណា $\cos x\neq 0\Leftrightarrow x\neq \frac{\pi}{2}+k\pi, k\in\mathbb{Z}$

ដោយ $\sin 2x=2\sin x\cos x=\frac{2\sin x}{\frac{1}{\cos x}}=\frac{\frac{2\sin x}{\cos x}}{\frac{1}{\cos^2 x}}\frac{2\tan x}{1+\tan^2 x}$

យើងបាន $2(1-\tan x)(1+\frac{2\tan x}{1+\tan^2 x})=1+\tan x$

$\Leftrightarrow \frac{2(1-\tan x)(1+\tan x)^2}{1+\tan^2 x}=1+\tan x$

$\Leftrightarrow 2(1-\tan x)(1+\tan x)^2-(1+\tan x)(1+\tan^2 x)=0$

$\Leftrightarrow (1+\tan x)\left[2(1-\tan x)(1+\tan x)-(1+\tan^2 x)\right]=0$

$\Leftrightarrow (1+\tan x)(2-2\tan^2 x-1-\tan^2 x)=0$

$\Leftrightarrow (1+\tan x)(1-3\tan^2 x)=0$

$\Leftrightarrow \left[\begin{array}{l}\tan x+1=0\\ 1-3\tan^2 x=0\end{array}\right.$

$\Leftrightarrow \left[\begin{array}{l}\tan x=-1\\ \tan x=\pm\frac{\sqrt{3}}{3}\end{array}\right.$

$\Leftrightarrow \left[\begin{array}{l}x=-\frac{\pi}{4}+k\pi\\ x=\pm\frac{\pi}{6}+k\pi\end{array}\right., k\in\mathbb{Z}$

ដូចនេះ សមីការមានចម្លើយ $\boxed{x=-\frac{\pi}{4}+k\pi, x=\pm\frac{\pi}{6}+k\pi, k\in\mathbb{Z}}$

ខ. $\frac{1}{\sqrt{2}}\cot x+\frac{\sin 2x}{\sin x+\cos x}=2\sin\left(x+\frac{\pi}{2}\right)$

សមីការមានឫសកាលណា $\begin{cases}\sin x\neq 0\\ \sin x+\cos x\neq 0\end{cases}$

$$\Rightarrow\begin{cases}x\neq k\pi\\ x\neq -\frac{\pi}{4}+k\pi\end{cases},k\in\mathbb{Z}$$

យើងបានសមីការ $\frac{\cos x}{\sqrt{2}\sin x}+\frac{2\sin x\cos x}{\sin x+\cos x}=2\cos x$

$$\Leftrightarrow \cos x\left(\frac{1}{\sqrt{2}\sin x}+\frac{2\sin x}{\sin x+\cos x}-2\right)=0$$

$$\Leftrightarrow \cos x\left(\frac{\sin x+\cos x+2\sqrt{2}\sin^2 x-2\sqrt{2}\sin x(\sin x+\cos x)}{\sqrt{2}\sin x(\sin x+\cos x)}\right)=0$$

$$\Leftrightarrow \cos x\left(\sin x+\cos x-2\sqrt{2}\sin x\cos x\right)=0$$

$$\Leftrightarrow\begin{bmatrix}\cos x=0\\ \sin x+\cos x-2\sqrt{2}\sin x\cos x=0\end{bmatrix}$$

- $\cos x=0\Rightarrow x=\frac{\pi}{2}+k\pi\ ,k\in\mathbb{Z}$
- $\sin x+\cos x-2\sqrt{2}\sin x\cos x=0$

តាង $t=\sin x+\cos x=\sqrt{2}\cos\left(x-\frac{\pi}{4}\right)$ ដែល $-\sqrt{2}\le t\le\sqrt{2}$

នោះ $\sin x\cos x=\frac{t^2-1}{2}$

យើងបាន $t-\sqrt{2}(t^2-1)=0$

$$\sqrt{2}t^2-t-\sqrt{2}=0$$

$$\Delta=1+8=9$$

$$t_1 = \frac{1+3}{2\sqrt{2}} = \frac{4}{2\sqrt{2}} = \sqrt{2}$$

$$t_2 = \frac{1-3}{2\sqrt{2}} = \frac{-2}{2\sqrt{2}} = -\frac{1}{\sqrt{2}}$$

តែ $t = \sqrt{2}\cos\left(x - \frac{\pi}{4}\right)$ យើងបាន

- $t = \sqrt{2} \Leftrightarrow \sqrt{2}\cos\left(x - \frac{\pi}{4}\right) = \sqrt{2}$

$$\Leftrightarrow \cos\left(x - \frac{\pi}{4}\right) = 1$$

$$\Leftrightarrow x - \frac{\pi}{4} = 2k\pi$$

$$\Rightarrow x = \frac{\pi}{4} + 2k\pi \,,\, k \in \mathbb{Z}$$

- $t = -\frac{1}{\sqrt{2}} \Leftrightarrow \sqrt{2}\cos\left(x - \frac{\pi}{4}\right) = -\frac{1}{\sqrt{2}}$

$$\Leftrightarrow \cos\left(x - \frac{\pi}{4}\right) = -\frac{1}{2} = \cos\frac{2\pi}{3}$$

$$\Leftrightarrow \begin{cases} x - \frac{\pi}{4} = \frac{2\pi}{3} + 2k\pi \\ x - \frac{\pi}{4} = -\frac{2\pi}{3} + 2k\pi \end{cases}, k \in \mathbb{Z} \quad \Rightarrow \begin{cases} x = \frac{11\pi}{12} + 2k\pi \\ x = -\frac{5\pi}{12} + 2k\pi \end{cases}, k \in \mathbb{Z}$$

ដូចនេះ សមីការមានចម្លើយ

$$\boxed{x = \frac{\pi}{4} + 2k\pi \,,\, x = \frac{11\pi}{12} + 2k\pi \,,\, x = -\frac{5\pi}{12} + 2k\pi \,,\, k \in \mathbb{Z}}$$

គ. $8(\sin^6 x+\cos^6 x)+3\sqrt{3}\sin 4x=3\sqrt{3}\cos 2x-3\sin 2x+8$

- $8(\sin^6 x+\cos^6 x)=8\left[(\sin^2 x)^3+(\cos^2 x)^3\right]$

$$=8\left[(\sin^2 x+\cos^2 x)^3-3\sin^2 x\cos^2 x(\sin^2 x+\cos^2 x)\right]$$

$$=8(1-\frac{3}{4}\sin^2 2x)$$

$$=8-6\sin^2 2x$$

- $\sin 4x=2\sin 2x\cos 2x$

យើងបាន $8-6\sin^2 2x+6\sqrt{3}\sin 2x\cos 2x=3\sqrt{3}\cos 2x-3\sin 2x+8$

$$\Leftrightarrow(-6\sin^2 2x+3\sin 2x)+(6\sqrt{3}\sin 2x\cos 2x-3\sqrt{3}\cos 2x)=0$$

$$\Leftrightarrow -3\sin 2x(2\sin 2x-1)+3\sqrt{3}\cos 2x(2\sin 2x-1)=0$$

$$\Leftrightarrow(2\sin 2x-1)(-3\sin 2x+3\sqrt{3}\cos 2x)=0$$

$$\Leftrightarrow\left[\begin{array}{l}2\sin 2x-1=0\\-3\sin 2x+3\sqrt{3}\cos 2x=0\end{array}\right.$$

$$\Leftrightarrow\left[\begin{array}{l}\sin 2x=\frac{1}{2}\\\tan 2x=\sqrt{3}\end{array}\right.$$

$$\Leftrightarrow\left[\begin{array}{l}2x=\frac{\pi}{6}+2k\pi\\2x=\frac{5\pi}{6}+2k\pi\\2x=\frac{\pi}{3}+k\pi\end{array}\right.,\ k\in\mathbb{Z}$$

$$\Rightarrow \left[\begin{array}{l} x=\dfrac{\pi}{12}+k\pi \\ x=\dfrac{5\pi}{12}+k\pi \ , k\in\mathbb{Z} \\ x=\dfrac{\pi}{6}+\dfrac{k\pi}{2} \end{array}\right.$$

ដូចនេះ សមីការមានចម្លើយ

$$\boxed{x=\frac{\pi}{12}+k\pi \ , x=\frac{5\pi}{12}+k\pi \ , x=\frac{\pi}{6}+\frac{k\pi}{2}, k\in\mathbb{Z}}$$

ឃ. $\sqrt{3}\sin 2x(2\cos x+1)+2=\cos 3x+\cos 2x-3\cos x$

$\Leftrightarrow \sqrt{3}\sin 2x(2\cos x+1)+2=4\cos^3 x-3\cos x+2\cos^2 x-1-3\cos x$

$\Leftrightarrow \sqrt{3}\sin 2x(2\cos x+1)=4\cos^3 x+2\cos^2 x-6\cos x-3$

$\Leftrightarrow \sqrt{3}\sin 2x(2\cos x+1)=2\cos^2 x(2\cos x+1)-3(2\cos x+1)$

$\Leftrightarrow \sqrt{3}\sin 2x(2\cos x+1)=(2\cos x+1)(2\cos^2 x-3)$

$\Leftrightarrow (2\cos x+1)(\sqrt{3}\sin 2x-2\cos^2 x+3)=0$

$\Leftrightarrow (2\cos x+1)(\sqrt{3}\sin 2x-(1+\cos 2x)+3)=0$

$\Leftrightarrow (2\cos x+1)(\sqrt{3}\sin 2x-\cos 2x+2)=0$

$$\Leftrightarrow \left[\begin{array}{l} 2\cos x+1=0 \\ \sqrt{3}\sin 2x-\cos 2x+2=0 \end{array}\right.$$

$$\Leftrightarrow \left[\begin{array}{l} \cos x=-\dfrac{1}{2} \\ \sin\left(2x-\dfrac{\pi}{6}\right)=-1 \end{array}\right.$$

$$\Leftrightarrow \begin{cases} x = \pm\dfrac{2\pi}{3} + 2k\pi \\ 2x - \dfrac{\pi}{6} = -\dfrac{\pi}{2} + 2k\pi \end{cases}, k \in \mathbb{Z}$$

$$\Leftrightarrow \begin{cases} x = \pm\dfrac{2\pi}{3} + 2k\pi \\ x = -\dfrac{\pi}{6} + k\pi \end{cases}, k \in \mathbb{Z}$$

ដូចនេះ សមីការមានចម្លើយ $\boxed{x = \pm\dfrac{2\pi}{3} + 2k\pi \, , \, x = -\dfrac{\pi}{6} + k\pi \, , \, k \in \mathbb{Z}}$

លំហាត់ទី២៥ : ដោះស្រាយសមីការខាងក្រោម ៖

ក. $\cos x\cos 4x + \cos 2x\cos 3x = 0$

ខ. $\left|\cos x + 2\sin 2x - \cos 3x\right| = 1 + 2\sin x - \cos 2x$

គ. $2\cos^3 x + \cos 2x + \sin x = 0$

ឃ. $1 + \sin^3 2x + \cos^3 2x = \dfrac{1}{2}\sin 4x$

## ចំណោះស្រាយ

ក. $\cos x\cos 4x + \cos 2x\cos 3x = 0$

$\Leftrightarrow \cos x(2\cos^2 2x - 1) + \cos 2x(4\cos^3 x - 3\cos x) = 0$

$\Leftrightarrow \cos x\left[(2\cos^2 x - 1) + \cos 2x(4\cos^2 x - 3)\right] = 0$

$\Leftrightarrow \cos x\left[(2\cos^2 x - 1) + \cos 2x\left(4\dfrac{1+\cos 2x}{2} - 3\right)\right] = 0$

$\Leftrightarrow \cos x(2\cos^2 x - 1 + 2\cos 2x + 2\cos^2 2x - 3\cos 2x) = 0$

$\Leftrightarrow \cos x(4\cos^2 2x-\cos 2x-1)=0$

$\Leftrightarrow \left[\begin{array}{l}\cos x=0\\ 4\cos^2 2x-\cos 2x-1=0\end{array}\right.$

- $\cos x=0\Rightarrow x=\frac{\pi}{2}+k\pi$ , $k\in\mathbb{Z}$
- $4\cos^2 2x-\cos 2x-1=0$

  $\Delta=1+16=17$

$\left[\begin{array}{l}\cos 2x=\frac{1-\sqrt{17}}{8}\\ \cos 2x=\frac{1+\sqrt{17}}{8}\end{array}\right.$

$\Leftrightarrow \left[\begin{array}{l}x=\pm\frac{1}{2}\arccos\left(\frac{1-\sqrt{17}}{8}\right)+k\pi\\ x=\pm\frac{1}{2}\arccos\left(\frac{1+\sqrt{17}}{8}\right)+k\pi\end{array}\right.$ , $k\in\mathbb{Z}$

ដូចនេះ សមីការមានចម្លើយ

$$x=\frac{\pi}{2}+k\pi\ ,\ x=\pm\frac{1}{2}\arccos\left(\frac{1-\sqrt{17}}{8}\right)+k\pi,\quad x=\pm\frac{1}{2}\arccos\left(\frac{1+\sqrt{17}}{8}\right)+k\pi\ \ ,k\in\mathbb{Z}$$

ខ. $\left|\cos x+2\sin 2x-\cos 3x\right|=1+2\sin x-\cos 2x$

ដោយ $\cos x+2\sin 2x-\cos 3x=\cos x+4\sin x\cos x-(4\cos^3 x-3\cos x)$

$=4\sin x\cos x+4\cos x-4\cos^3 x$

$$= 4\cos x(\sin x + 1 - \cos^2 x)$$
$$= 4\cos x(\sin x + \sin^2 x)$$
$$= 2\sin 2x(1 + \sin x)$$

នោះ $|\cos x+2\sin 2x-\cos 3x| = |2\sin 2x(1+\sin x)| = 2(1+\sin x)|\sin 2x|$

ហើយ $1 + 2\sin x - \cos 2x = 2\sin x + 2\sin^2 x = 2\sin x(1+\sin x)$

យើងបានសមីការទៅជា $2(1+\sin x)|\sin 2x| = 2\sin x(1+\sin x)$

$$\Leftrightarrow (1+\sin x)\left(|\sin 2x| - \sin x\right) = 0$$
$$\Leftrightarrow \begin{cases} 1+\sin x = 0 \\ |\sin 2x| - \sin x = 0 \end{cases}$$

- $1+\sin x = 0 \Leftrightarrow \sin x = -1 \Rightarrow x = -\frac{\pi}{2} + 2k\pi,\ k \in \mathbb{Z}$
- $|\sin 2x| - \sin x = 0 \Leftrightarrow \sin^2 2x = \sin^2 x$

$$\Leftrightarrow 4\sin^2 x\cos^2 x - \sin^2 x = 0$$
$$\Leftrightarrow \sin^2 x\left(4\cos^2 x - 1\right) = 0$$
$$\Leftrightarrow \begin{cases} \sin^2 x = 0 \\ 4\cos^2 x - 1 = 0 \end{cases}$$
$$\Leftrightarrow \begin{cases} \sin x = 0 \\ \cos x = \frac{1}{2} \\ \cos x = -\frac{1}{2} \end{cases}$$

$$\Leftrightarrow \left[\begin{array}{l} x=k\pi \\ x=\pm\dfrac{\pi}{3}+2k\pi \\ x=\pm\dfrac{2\pi}{3}+2k\pi \end{array}\right. , k\in\mathbb{Z}$$

$$\Leftrightarrow \left[\begin{array}{l} x=k\pi \\ x=\dfrac{\pi}{3}+\dfrac{k\pi}{2} \end{array}\right. , k\in\mathbb{Z}$$

ដូចនេះ សមីការមានចម្លើយ

$$\boxed{x=-\frac{\pi}{2}+2k\pi\ ,\ x=k\pi\ ,\ x=\frac{\pi}{3}+\frac{k\pi}{2}\ ,\ k\in\mathbb{Z}}$$

គ. $2\cos^3 x+\cos 2x+\sin x=0$

$\Leftrightarrow 2\cos^3 x+2\cos^2 x-1+\sin x=0$

$\Leftrightarrow 2\cos^2 x(\cos x+1)-(1-\sin x)=0$

$\Leftrightarrow 2(1-\sin x)(1+\sin x)(1+\cos x)-(1-\sin x)=0$

$\Leftrightarrow (1-\sin x)\left[2+2(\sin x+\cos x)+2\sin x\cos x-1\right]=0$

$\Leftrightarrow (1-\sin x)\left[1+2(\sin x+\cos x)+2\sin x\cos x\right]=0$

$$\Leftrightarrow \left[\begin{array}{l} 1-\sin x=0 \\ 1+2(\sin x+\cos x)+2\sin x\cos x=0 \end{array}\right.$$

- $1-\sin x=0\Leftrightarrow \sin x=1\Rightarrow x=\dfrac{\pi}{2}+2k\pi\ ,\ k\in\mathbb{Z}$
- $1+2(\sin x+\cos x)+2\sin x\cos x=0$

តាង $t=\sin x+\cos x=\sqrt{2}\sin\left(x+\dfrac{\pi}{4}\right)$ ដែល $-\sqrt{2}\le t\le\sqrt{2}$

នោះ $2\sin x\cos x=t^2-1$

យើងបាន $1+2t+(t^2-1)=0$

$$\Leftrightarrow t^2+2t=0$$
$$\Leftrightarrow t(t+2)=0$$
$$\Leftrightarrow \begin{bmatrix} t=0 \\ t=-2\notin\left[-\sqrt{2}\,,\sqrt{2}\right] \end{bmatrix} \text{(មិនយក)}$$
$$\Leftrightarrow \sqrt{2}\sin\left(x+\frac{\pi}{4}\right)=0$$
$$\Leftrightarrow x+\frac{\pi}{4}=k\pi$$
$$\Rightarrow x=-\frac{\pi}{4}+k\pi\,,\ k\in\mathbb{Z}$$

ដូចនេះ សមីការមានចម្លើយ $\boxed{x=\frac{\pi}{2}+2k\pi\,,\ x=-\frac{\pi}{4}+k\pi\,,\ k\in\mathbb{Z}}$

ឃ. $1+\sin^3 2x+\cos^3 2x=\frac{1}{2}\sin 4x$

$\Leftrightarrow 1+(\sin 2x+\cos 2x)^3-3\sin 2x\cos 2x(\sin 2x+\cos 2x)=\sin 2x\cos x2x$

តាង $t=\sin 2x+\cos 2x=\sqrt{2}\sin\left(2x+\frac{\pi}{4}\right)$ ដែល $t\in\left[-\sqrt{2},\sqrt{2}\right]$

នោះ $\sin 2x\cos 2x=\frac{t^2-1}{2}$

យើងបានសមីការទៅជា $1+t^3-3\left(\frac{t^2-1}{2}\right)t=\frac{t^2-1}{2}$

$$\Leftrightarrow 2+2t^3-3t^3+3t=t^2-1$$
$$\Leftrightarrow t^3+t^2-3t-3=0$$
$$\Leftrightarrow t^2(t+1)-3(t+1)=0$$

$$\Leftrightarrow (t+1)(t^2-3)=0$$

$$\Leftrightarrow \begin{bmatrix} t=-1 \\ t=\pm\sqrt{3}\notin\left[-\sqrt{2}\,,\sqrt{2}\right] \text{(មិនយក)} \end{bmatrix}$$

$$\Leftrightarrow \sqrt{2}\sin\left(2x+\frac{\pi}{4}\right)=-1$$

$$\Leftrightarrow \sin\left(2x+\frac{\pi}{4}\right)=-\frac{\sqrt{2}}{2}$$

$$\Leftrightarrow \begin{bmatrix} 2x+\frac{\pi}{4}=-\frac{\pi}{4}+2k\pi \\ 2x+\frac{\pi}{4}=\frac{5\pi}{4}+2k\pi \end{bmatrix}, k\in\mathbb{Z}$$

$$\Rightarrow \begin{bmatrix} x=-\frac{\pi}{4}+k\pi \\ x=\frac{\pi}{2}+k\pi \end{bmatrix}, k\in\mathbb{Z}$$

ដូចនេះ សមីការមានចម្លើយ $\boxed{x=-\frac{\pi}{4}+k\pi\,,\ x=\frac{\pi}{2}+k\pi\,,\ k\in\mathbb{Z}}$

លំហាត់ទី២៦ : ដោះស្រាយសមីការខាងក្រោម ៖

ក. $\cos^2 3x\cos 2x-\cos^2 x=0$

ខ. $1+\sin x+\cos x+\sin 2x+\cos 2x=0$

គ. $\cos 3x+\cos 2x-\cos x-1=0$

ឃ. $\dfrac{(1+\sin x+\cos 2x)\sin\left(x+\frac{\pi}{4}\right)}{1+\tan x}=\dfrac{1}{\sqrt{2}}\cos x$

## ចំណោះស្រាយ

ក. $\cos^2 3x\cos 2x-\cos^2 x=0$

$\Leftrightarrow (4\cos^3 x-3\cos^2 x)^2\cos 2x-\cos^2 x=0$

$\Leftrightarrow \cos^2 x(4\cos^2 x-3)^2\cos 2x-\cos^2 x=0$

$\Leftrightarrow \cos^2 x\left[(4\cos^2 x-3)^2\cos 2x-1\right]=0$

$\Leftrightarrow \cos^2 x\left[(2+2\cos 2x-3)^2\cos 2x-1\right]=0$

$\Leftrightarrow \cos^2 x\left[(2\cos 2x-1)^2\cos 2x-1\right]=0$

$\Leftrightarrow \cos^2 x\left[(4\cos^2 2x-4\cos 2x+1)\cos 2x-1\right]=0$

$\Leftrightarrow \cos^2 x\left(4\cos^3 2x-4\cos^2 2x+\cos 2x-1\right)=0$

$\Leftrightarrow \cos^2 x\left[4\cos^2 2x(\cos 2x-1)+(\cos 2x-1)\right]=0$

$\Leftrightarrow \cos^2 x(\cos 2x-1)(4\cos^2 2x+1)=0$

$\Leftrightarrow \begin{bmatrix}\cos^2 x=0\\ \cos 2x-1=0\end{bmatrix}$ , $(4\cos^2 2x+1>0\ \forall x\in\mathbb{R})$

$\Leftrightarrow \begin{bmatrix}\cos x=0\\ \cos 2x=1\end{bmatrix}$

$\Leftrightarrow \begin{bmatrix}x=\dfrac{\pi}{2}+k\pi\\ x=k\pi\end{bmatrix}\Leftrightarrow x=\dfrac{k\pi}{2}\ ,k\in\mathbb{Z}$

ដូចនេះ សមីការមានចម្លើយ $\boxed{x=\dfrac{k\pi}{2}\ ,k\in\mathbb{Z}}$

ខ. $1+\sin x+\cos x+\sin 2x+\cos 2x=0$

$\Leftrightarrow 1+\sin x+\cos x+2\sin x\cos x+2\cos^2 x-1=0$

$\Leftrightarrow (\sin x+\cos x)+2\cos x(\sin x+\cos x)=0$

$\Leftrightarrow (\sin x+\cos x)(1+2\cos x)=0$

$\Leftrightarrow \left[\begin{array}{l}\sin x+\cos x=0\\1+2\cos x=0\end{array}\right.$

$\Leftrightarrow \left[\begin{array}{l}\tan x=-1\\\cos x=-\dfrac{1}{2}\end{array}\right.$

$\Leftrightarrow \left[\begin{array}{l}x=-\dfrac{\pi}{4}+k\pi\\x=\pm\dfrac{2\pi}{3}+2k\pi\end{array}\right.,\ k\in\mathbb{Z}$

ដូចនេះ សមីការមានចម្លើយ $\boxed{x=-\dfrac{\pi}{4}+k\pi\ ,\ x=\pm\dfrac{2\pi}{3}+2k\pi\ ,\ k\in\mathbb{Z}}$

គ. $\cos 3x+\cos 2x-\cos x-1=0$

$\Leftrightarrow (4\cos^3 x-3\cos x)+(2\cos^2 x-1)-\cos x-1=0$

$\Leftrightarrow 4\cos^3 x+2\cos^2 x-4\cos x-2=0$

$\Leftrightarrow 2\cos^2 x(2\cos x+1)-2(2\cos x+1)=0$

$\Leftrightarrow (2\cos x+1)(2\cos^2 x-2)=0$

$\Leftrightarrow \left[\begin{array}{l}2\cos x+1=0\\2\cos^2 x-2=0\end{array}\right.$

$\Leftrightarrow \left[\begin{array}{l}\cos x=-\dfrac{1}{2}\\\cos x=\pm 1\end{array}\right.$

$\Leftrightarrow \left[\begin{array}{l}x=\pm\dfrac{2\pi}{3}+2k\pi\\x=k\pi\end{array}\right.,\ k\in\mathbb{Z}$

ដូចនេះ សមីការមានចម្លើយ $\boxed{x=\pm\frac{2\pi}{3}+2k\pi\ ,\ x=k\pi\ ,\ k\in\mathbb{Z}}$

ឃ. $\dfrac{(1+\sin x+\cos 2x)\sin\left(x+\dfrac{\pi}{4}\right)}{1+\tan x}=\dfrac{1}{\sqrt{2}}\cos x$

សមីការមានឫសកាលណា $\begin{cases}\cos x\neq 0\\ 1+\tan x\neq 0 \text{ ឬ } \cos x+\sin x\neq 0\end{cases}$

$$\Rightarrow\begin{cases}x\neq\dfrac{\pi}{2}+k\pi\\ x\neq-\dfrac{\pi}{4}+k\pi\end{cases},\ k\in\mathbb{Z}$$

យើងបាន $\dfrac{(1+\sin x+\cos 2x)\left(\dfrac{\sqrt{2}}{2}\sin x+\dfrac{\sqrt{2}}{2}\cos x\right)}{1+\dfrac{\sin x}{\cos x}}=\dfrac{1}{\sqrt{2}}\cos x$

$$\Leftrightarrow\frac{\sqrt{2}}{2}\cdot\frac{\cos x(1+\sin x+\cos 2x)(\sin x+\cos x)}{\sin x+\cos x}=\frac{\sqrt{2}}{2}\cos x$$

$$\Leftrightarrow\cos x(1+\sin x+\cos 2x)=\cos x$$

$$\Leftrightarrow 1+\sin x+1-2\sin^2 x=1\quad(\text{ព្រោះ }\cos x\neq 0)$$

$$\Leftrightarrow 2\sin^2 x-\sin x-1=0$$

$$\Leftrightarrow\begin{cases}\sin x=1\\ \sin x=-\dfrac{1}{2}\end{cases}$$

$$\Leftrightarrow\begin{cases}x=\dfrac{\pi}{2}+2k\pi\ \ (\text{មិនយក})\\ x=-\dfrac{\pi}{6}+2k\pi\\ x=\dfrac{7\pi}{6}+2k\pi\end{cases},\ k\in\mathbb{Z}$$

ដូចនេះ សមីការមានចម្លើយ $x=-\frac{\pi}{6}+2k\pi\,,\,x=\frac{7\pi}{6}+2k\pi\,,\,k\in\mathbb{Z}$

**លំហាត់ទី២៧ : ដោះស្រាយសមីការខាងក្រោម ៖**

ក. $\sin^3 x+\cos^3 x+\sin^3 x\cot x+\cos^3 x\tan x=\sqrt{2\sin 2x}$

ខ. $\sqrt{5\cos x-\cos 2x}+2\sin x=0$

គ. $2\sin\left(3x+\frac{\pi}{4}\right)=\sqrt{1+8\sin 2x\cos^2 2x}$

ឃ. $\sqrt{\cos 2x}+\sqrt{1+\sin 2x}=2\sqrt{\sin x+\cos x}$

**ចងចាំ**

- $\sqrt{A}=\sqrt{B}\Leftrightarrow A=B\geq 0$
- $\sqrt{A}=B\Leftrightarrow\begin{cases}B\geq 0\\A=B^2\end{cases}$
- $\sqrt{A}+\sqrt{B}=\sqrt{C}\Leftrightarrow\begin{cases}A\geq 0\\B\geq 0\\A+B+2\sqrt{AB}=C\end{cases}$
- $\sqrt{AB}=\begin{cases}\sqrt{A}\cdot\sqrt{B}\text{ បើ }A\geq 0\text{ និង }B\geq 0\\\sqrt{-A}\cdot\sqrt{-B}\text{ បើ }A\leq 0\text{ និង }B\leq 0\end{cases}$

**ចំណោះស្រាយ**

ក. $\sin^3 x+\cos^3 x+\sin^3 x\cot x+\cos^3 x\tan x=\sqrt{2\sin 2x}$

សមីការមានឫសកាលណា $\begin{cases}\sin x\neq 0\\\cos x\neq 0\\\sin 2x\geq 0\end{cases}\Leftrightarrow\begin{cases}\sin 2x\neq 0\\\sin 2x\geq 0\end{cases}$

$$\Leftrightarrow \sin 2x > 0$$

យើងបាន $\sin^3 x + \cos^3 x + \sin^2 x \cos x + \cos^2 x \sin x = \sqrt{2\sin 2x}$

$$\Leftrightarrow \sin x(\sin^2 x + \cos^2 x) + \cos x(\sin^2 x + \cos^2 x) = \sqrt{2\sin 2x}$$

$$\Leftrightarrow \sin x + \cos x = \sqrt{4\sin x \cos x}$$

$$\Leftrightarrow \begin{cases} \sin^2 x + \cos^2 + 2\sin x \cos x = 4\sin x \cos x \\ \sin\left(x + \dfrac{\pi}{4}\right) \geq 0 \end{cases}$$

$$\Leftrightarrow \begin{cases} 1 = 2\sin x \cos x \\ \sin\left(x + \dfrac{\pi}{4}\right) \geq 0 \end{cases}$$

$$\Leftrightarrow \begin{cases} \sin 2x = 1 \\ \sin\left(x + \dfrac{\pi}{4}\right) \geq 0 \end{cases}$$

$$\Leftrightarrow \begin{cases} x = \dfrac{\pi}{4} + k\pi \\ \sin\left(x + \dfrac{\pi}{4}\right) \geq 0 \end{cases}, k \in \mathbb{Z}$$

$$\Leftrightarrow \begin{cases} \left[\begin{array}{l} x = \dfrac{\pi}{4} + 2k\pi \\ x = \dfrac{5\pi}{4} + 2k\pi \end{array}\right. \\ \sin\left(x + \dfrac{\pi}{4}\right) \geq 0 \end{cases}$$

$$\Leftrightarrow x = \frac{\pi}{4} + 2k\pi, k \in \mathbb{Z}$$

ដូចនេះ សមីការមានចម្លើយ $\boxed{x = \dfrac{\pi}{4} + 2k\pi, k \in \mathbb{Z}}$

ខ. $\sqrt{5\cos x-\cos 2x}+2\sin x=0$

$\Leftrightarrow \sqrt{5\cos x-\cos 2x}=-2\sin x$

$\Leftrightarrow \begin{cases} -2\sin x\geq 0 \\ 5\cos x-\cos 2x=4\sin^2 x \end{cases}$

$\Leftrightarrow \begin{cases} \sin x\leq 0 \\ 5\cos x-2\cos^2 x+1-4+4\cos^2 x=0 \end{cases}$

$\Leftrightarrow \begin{cases} \sin x\leq 0 \\ 2\cos^2 x+5\cos x-3=0 \end{cases}$

$\Leftrightarrow \begin{cases} \sin x\leq 0 \\ \left[\begin{array}{l} \cos x=\dfrac{1}{2} \\ \cos x=-3<-1 \text{ (មិនយក)} \end{array}\right. \end{cases}$

$\Leftrightarrow \begin{cases} \sin x\leq 0 \\ x=\pm\dfrac{\pi}{3}+2k\pi \end{cases}, k\in\mathbb{Z}$

$\Leftrightarrow x=-\dfrac{\pi}{3}+2k\pi, k\in\mathbb{Z}$

ដូចនេះ សមីការមានចម្លើយ $\boxed{x=-\dfrac{\pi}{3}+2k\pi, k\in\mathbb{Z}}$

គ. $2\sin\left(3x+\dfrac{\pi}{4}\right)=\sqrt{1+8\sin 2x\cos^2 2x}$

$\Leftrightarrow \begin{cases} \sin\left(3x+\dfrac{\pi}{4}\right)\geq 0 \\ 4\sin^2\left(3x+\dfrac{\pi}{4}\right)=1+8\sin 2x\cos^2 2x \end{cases}$

$$\Leftrightarrow \begin{cases} \sin\left(3x+\dfrac{\pi}{4}\right) \geq 0 \\ 2-2\cos\left(6x+\dfrac{\pi}{2}\right) = 1+4\sin 4x\cos 2x \end{cases}$$

$$\Leftrightarrow \begin{cases} \sin\left(3x+\dfrac{\pi}{4}\right) \geq 0 \\ 2+2\sin 6x = 1+2(\sin 6x+\sin 2x) \end{cases}$$

$$\Leftrightarrow \begin{cases} \sin\left(3x+\dfrac{\pi}{4}\right) \geq 0 \\ \sin 2x = \dfrac{1}{2} \end{cases}$$

$$\Leftrightarrow \begin{cases} \sin\left(3x+\dfrac{\pi}{4}\right) \geq 0 \\ \left[\begin{array}{l} x = \dfrac{\pi}{12}+k\pi \\ x = \dfrac{5\pi}{12}+k\pi \end{array}\right. \end{cases}$$

$$\Leftrightarrow \begin{cases} \sin\left(3x+\dfrac{\pi}{4}\right) \geq 0 \\ \left[\begin{array}{l} x = \dfrac{\pi}{12}+2k\pi \\ x = \pi+\dfrac{\pi}{12}+2k\pi = \dfrac{13\pi}{12}+2k\pi \\ x = \dfrac{5\pi}{12}+2k\pi \\ x = \pi+\dfrac{5\pi}{12}+2k\pi = \dfrac{17\pi}{12}+2k\pi \end{array}\right. \end{cases}$$

$$\Leftrightarrow \begin{cases} x=\dfrac{\pi}{12}+2k\pi \\ x=\dfrac{17\pi}{12}+2k\pi \end{cases}, k\in\mathbb{Z}$$

ដូចនេះ សមីការមានចម្លើយ $\boxed{x=\dfrac{\pi}{12}+2k\pi\ ,\ x=\dfrac{17\pi}{12}+2k\pi\ ,\ k\in\mathbb{Z}}$

ឃ. $\sqrt{\cos 2x}+\sqrt{1+\sin 2x}=2\sqrt{\sin x+\cos x}$

សមីការមានឫសកាលណា $\begin{cases} \cos 2x\geq 0 \\ \sin x+\cos x\geq 0 \end{cases}$

$$\Leftrightarrow \begin{cases} \cos^2 x-\sin^2 x\geq 0 \\ \sin x+\cos x\geq 0 \end{cases} \Leftrightarrow \begin{cases} \cos x-\sin x\geq 0 \\ \cos x+\sin x\geq 0 \end{cases}$$

ដោយ $\cos 2x=\cos^2 x-\sin^2 x=(\cos x-\sin x)(\cos x+\sin x)$

ហើយ $1+\sin 2x=\sin^2 x+2\sin x\cos x+\cos^2 x=(\sin x+\cos x)^2$

យើងបានសមីការខាងលើអាចសរសេរ:

$$\sqrt{(\cos x-\sin x)(\cos x+\sin x)}+\sqrt{(\sin x+\cos x)^2}=2\sqrt{\sin x+\cos x}$$

$$\Leftrightarrow \sqrt{\sin x+\cos x}\left(\sqrt{\cos x-\sin x}+\sqrt{\sin x+\cos x}-2\right)=0$$

$$\Leftrightarrow \left[\begin{array}{l} \sqrt{\sin x+\cos x}=0 \\ \sqrt{\cos x-\sin x}+\sqrt{\sin x+\cos x}-2=0 \end{array}\right.$$

- $\cos x+\sin x=0\Leftrightarrow \tan x=-1\Leftrightarrow x=-\dfrac{\pi}{4}+k\pi\ ,\ k\in\mathbb{Z}$

$$\Leftrightarrow \left[\begin{array}{l} x=-\dfrac{\pi}{4}+2k\pi \\ x=\dfrac{3\pi}{4}+2k\pi \end{array}\right., k\in\mathbb{Z}$$

$$\Leftrightarrow x=-\frac{\pi}{4}+2k\pi\,,\,k\in\mathbb{Z}$$

- $\sqrt{\cos x-\sin x}+\sqrt{\sin x+\cos x}-2=0$

$\Leftrightarrow \cos x-\sin x+2\sqrt{\cos^2 x-\sin^2 x}+\sin x+\cos x=4$

$\Leftrightarrow \sqrt{2\cos^2 x-1}=2-\cos x$

$\Leftrightarrow 2\cos^2 x-1=4-4\cos x+\cos^2 x$

$\Leftrightarrow \cos^2 x+4\cos x-5=0$

$\Leftrightarrow \left[\begin{array}{l}\cos x=1\\ \cos x=-5<-1\,(\text{មិនយក})\end{array}\right.$

$\Leftrightarrow x=2k\pi$

ដូចនេះ សមីការមានចម្លើយ $\boxed{x=-\frac{\pi}{4}+2k\pi\,,x=2k\pi\,,k\in\mathbb{Z}}$

លំហាត់ទី២៨ : ដោះស្រាយសមីការខាងក្រោម ៖

ក. $\sin^{13}x+\cos^{14}x=1$

ខ. $\sin^{5}x+\cos^{5}x=1$

## ចំណោះស្រាយ

ក. $\sin^{13}x+\cos^{14}x=1$

ដោយ $1=\cos^2 x+\sin^2 x$

យើងបាន $\sin^{13}x+\cos^{14}x=\cos^2 x+\sin^2 x$

$\Leftrightarrow \cos^2 x(\cos^{12}x-1)=\sin^2 x(1-\sin^{13}x)\quad (*)$

ដោយ $\cos^{12}x-1\le 0\Rightarrow \cos^2 x\left(\cos^{12}x-1\right)\le 0$

និង $1-\sin^{13}x\geq 0\Rightarrow \sin^2 x(1-\sin^{13}x)\geq 0$

នោះ (*) ធ្វៀងផ្ទាត់លុះត្រាតែ $\begin{cases}\cos^2 x(\cos^{12}x-1)=0\\ \sin^2 x(1-\sin^{13}x)=0\end{cases}$

$$\Leftrightarrow \begin{cases}\left[\begin{array}{l}\cos^2 x=0\\ \cos^{12}x-1=0\end{array}\right.\\ \left[\begin{array}{l}\sin^2 x=0\\ 1-\sin^{13}x=0\end{array}\right.\end{cases}$$

$$\Leftrightarrow \begin{cases}\left[\begin{array}{l}\cos x=0\\ \cos x=\pm 1\end{array}\right.\\ \left[\begin{array}{l}\sin x=0\\ \sin x=1\end{array}\right.\end{cases}$$

$$\Leftrightarrow \begin{cases}\left[\begin{array}{l}x=\dfrac{\pi}{2}+k\pi\\ x=k\pi\end{array}\right.\\ \left[\begin{array}{l}x=k\pi\\ x=\dfrac{\pi}{2}+2k\pi\end{array}\right.\end{cases}, k\in\mathbb{Z}$$

$$\Leftrightarrow \left[\begin{array}{l}x=\dfrac{\pi}{2}+2k\pi\\ x=k\pi\end{array}\right., k\in\mathbb{Z}$$

ដូចនេះ សមីការមានចម្លើយ $\boxed{x=\dfrac{\pi}{2}+2k\pi\ ,\ x=k\pi\ ,\ k\in\mathbb{Z}}$

ខ. $\sin^5 x+\cos^5 x=1$

$\Leftrightarrow \sin^5 x+\cos^5 x=\cos^2 x+\sin^2 x$

$\Leftrightarrow \cos^2 x(\cos^3 x-1)=\sin^2 x(1-\sin^3 x)$ (**)

ដោយ $\begin{cases}\cos^2 x \geq 0 \\ \cos^3 x - 1 \leq 0\end{cases} \Rightarrow \cos^2 x(\cos^3 x - 1) \leq 0$

និង $\begin{cases}\sin^2 x \geq 0 \\ 1 - \sin^3 x \geq 0\end{cases} \Rightarrow \sin^2 x(1 - \sin^3 x) \geq 0$

នោះ (**) ធ្វើឯងផ្ទាត់លុះត្រា $\begin{cases}\cos^2 x(\cos^3 x - 1) = 0 \\ \sin^2 x(1 - \sin^3 x) = 0\end{cases}$

$$\Leftrightarrow \begin{cases}\left[\begin{array}{l}\cos^2 x = 0 \\ \cos^3 x - 1 = 0\end{array}\right. \\ \left[\begin{array}{l}\sin^2 x = 0 \\ 1 - \sin^3 x = 0\end{array}\right.\end{cases}$$

$$\Leftrightarrow \begin{cases}\left[\begin{array}{l}\cos x = 0 \\ \cos x = 1\end{array}\right. \\ \left[\begin{array}{l}\sin x = 0 \\ \sin x = 1\end{array}\right.\end{cases}$$

$$\Leftrightarrow \begin{cases}\left[\begin{array}{l}x = \dfrac{\pi}{2} + k\pi \\ x = 2k\pi\end{array}\right. \\ \left[\begin{array}{l}x = k\pi \\ x = \dfrac{\pi}{2} + 2k\pi\end{array}\right.\end{cases}$$

$$\Leftrightarrow \left[\begin{array}{l}x = \dfrac{\pi}{2} + 2k\pi \\ x = 2k\pi\end{array}\right., k \in \mathbb{Z}$$

ដូចនេះ សមីការមានចម្លើយ $\boxed{x = \dfrac{\pi}{2} + 2k\pi \text{ , } x = 2k\pi \text{ , } k \in \mathbb{Z}}$

លំហាត់ទី២៩ : ដោះស្រាយសមីការខាងក្រោម ៖

ក. $\frac{\sin^{10}x+\cos^{10}x}{4}=\frac{\sin^6 x+\cos^6 x}{4\cos^2 2x+\sin^2 2x}$

ខ. $\cos^5 x+\sin^7 x+\frac{1}{2}(\cos^3 x+\sin^5 x)\sin 2x=\cos x+\sin x$

## ចំណោះស្រាយ

ក. $\frac{\sin^{10}x+\cos^{10}x}{4}=\frac{\sin^6 x+\cos^6 x}{4\cos^2 2x+\sin^2 2x}$

ដោយ $\sin^6x+\cos^6x=(\sin^2x+\cos^2x)^3-3\sin^2x\cos^2x(\sin^2x+\cos^2x)$

$$=1-\frac{3}{4}\sin^2 2x$$

ហើយ $4\cos^2 2x+\sin^2 2x=4-4\sin^2 2x+\sin^2 2x=4\left(1-\frac{3}{4}\sin^2 2x\right)$

យើងបាន $\frac{\sin^{10}x+\cos^{10}x}{4}=\frac{1-\frac{3}{4}\sin^2 2x}{4\left(1-\frac{3}{4}\sin^2 2x\right)}$

$$\Leftrightarrow \sin^{10}x+\cos^{10}x=1$$

$$\Leftrightarrow \sin^{10}x+\cos^{10}x=\cos^2 x+\sin^2 x$$

$$\Leftrightarrow \sin^2 x\left(\sin^8 x-1\right)=\cos^2 x\left(1-\cos^8 x\right) \quad (1)$$

ដោយ $\begin{cases}\sin^2 x\le 0\\ \sin^8 x-1\le 0\end{cases}\Rightarrow \sin^2 x\left(\sin^8 x-1\right)\le 0$

និង $\begin{cases}\cos^2 x\ge 0\\ 1-\cos^8 x\ge 0\end{cases}\Rightarrow \cos^2 x\left(1-\cos^8 x\right)\ge 0$

នោះ សមីការ (1) ធ្វើឱ្យផ្ទាត់លុះត្រាតែ $\begin{cases}\sin^2 x\left(\sin^8 x-1\right)=0\\ \cos^2 x\left(1-\cos^8 x\right)=0\end{cases}$

$$\Leftrightarrow \begin{cases}\left[\begin{array}{l}\sin^2 x=0\\ \sin^8 x-1=0\end{array}\right.\\ \left[\begin{array}{l}\cos^2 x=0\\ 1-\cos^8 x=0\end{array}\right.\end{cases}$$

$$\Leftrightarrow \begin{cases}\left[\begin{array}{l}\sin x=0\\ \sin x=\pm 1\end{array}\right.\\ \left[\begin{array}{l}\cos x=0\\ \cos x=\pm 1\end{array}\right.\end{cases}$$

$$\Leftrightarrow \begin{cases}\left[\begin{array}{l}x=k\pi\\ x=\dfrac{\pi}{2}+k\pi\end{array}\right.\\ \left[\begin{array}{l}x=\dfrac{\pi}{2}+k\pi\\ x=k\pi\end{array}\right.\end{cases}, k\in\mathbb{Z}$$

$$\Leftrightarrow \left[\begin{array}{l}x=k\pi\\ x=\dfrac{\pi}{2}+k\pi\end{array}\right., k\in\mathbb{Z}$$

ដូចនេះ សមីការមានចម្លើយ $\boxed{x=k\pi\,,\,x=\dfrac{\pi}{2}+k\pi\ \ ,k\in\mathbb{Z}}$

ខ. $\cos^5 x+\sin^7 x+\dfrac{1}{2}(\cos^3 x+\sin^5 x)\sin 2x=\cos x+\sin x$

$\Leftrightarrow \cos^5 x+\sin^7 x+\sin x\cos x(\cos^3 x+\sin^5 x)=\cos x+\sin x$

$\Leftrightarrow \cos^5 x+\sin^7 x+\sin x\cos^4 x+\cos x\sin^6 x=\cos x+\sin x$

$\Leftrightarrow \cos^4 x(\cos x+\sin x)+\sin^6 x(\cos x+\sin x)=\cos x+\sin x$

$\Leftrightarrow (\cos x+\sin x)\left(\cos^4 x+\sin^6 x-1\right)=0$

$\Leftrightarrow \left[\begin{array}{l}\cos x+\sin x=0\\ \cos^4 x+\sin^6 x=1\end{array}\right.$

- $\cos x+\sin x=0 \Leftrightarrow \tan x=-1 \Rightarrow x=-\dfrac{\pi}{4}+k\pi\ ,\ k\in\mathbb{Z}$
- $\cos^4 x+\sin^6 x=1=\cos^2 x+\sin^2 x$

$\Leftrightarrow \cos^2 x(\cos^2 x-1)=\sin^2 x(1-\sin^4 x)$

$\Leftrightarrow -\cos^2 x\sin^2 x=\sin^2 x(1-\sin^2 x)(1+\sin^2 x)$

$\Leftrightarrow -\sin^2 x\cos^2 x=\sin^2 x\cos^2 x(1+\sin^2 x)$

$\Leftrightarrow \sin^2 x\cos^2 x\left(2+\sin^2 x\right)=0$

$\Leftrightarrow \sin^2 x\cos^2 x=0$

$\Leftrightarrow \dfrac{1}{4}\sin^2 2x=0$

$\Leftrightarrow \sin 2x=0$

$\Leftrightarrow 2x=k\pi$

$\Rightarrow x=\dfrac{k\pi}{2}$

ដូចនេះ សមីការមានចម្លើយ $\boxed{x=-\dfrac{\pi}{4}+k\pi\ ,\ x=\dfrac{k\pi}{2}\ ,\ k\in\mathbb{Z}}$

☞ យើងអាចដោះស្រាយសមីការ $\cos^4 x+\sin^6 x=1=\cos^2 x+\sin^2 x$ តាមរបៀបម្យ៉ាងទៀតដូចខាងក្រោម៖

$\cos^4 x+\sin^6 x=1=\cos^2 x+\sin^2 x$

$\Leftrightarrow \cos^2 x(\cos^2 x-1)=\sin^2 x(1-\sin^4 x)$ (#)

ដោយ $\begin{cases}\cos^2 x \geq 0\\ \cos^2 x - 1 \leq 0\end{cases} \Rightarrow \cos^2 x(\cos^2 x - 1) \leq 0$

និង $\begin{cases}\sin^2 x \geq 0\\ 1 - \sin^4 x \geq 0\end{cases} \Rightarrow \sin^2 x(1 - \sin^4 x) \geq 0$

នោះ (#) ផ្ទៀងផ្ទាត់លុះត្រាតែ $\begin{cases}\cos^2 x(\cos^2 x - 1) = 0\\ \sin^2 x(1 - \sin^4 x) = 0\end{cases}$

$$\Leftrightarrow \begin{cases}\left[\begin{array}{l}\cos^2 x = 0\\ \cos^2 x - 1 = 0\end{array}\right.\\ \left[\begin{array}{l}\sin^2 x = 0\\ 1 - \sin^4 x = 0\end{array}\right.\end{cases}$$

$$\Leftrightarrow \begin{cases}\left[\begin{array}{l}\cos x = 0\\ \cos x = \pm 1\end{array}\right.\\ \left[\begin{array}{l}\sin x = 0\\ \sin x = \pm 1\end{array}\right.\end{cases}$$

$$\Leftrightarrow \begin{cases}\left[\begin{array}{l}x = \dfrac{\pi}{2} + k\pi = \dfrac{(2k+1)\pi}{2}\\ x = k\pi = \dfrac{2k\pi}{2}\end{array}\right.\\ \left[\begin{array}{l}x = k\pi = \dfrac{2k\pi}{2}\\ x = \dfrac{\pi}{2} + k\pi = \dfrac{(2k+1)\pi}{2}\end{array}\right.\end{cases} , k \in \mathbb{Z}$$

$\Leftrightarrow x = \dfrac{\ell\pi}{2}, \ell \in \mathbb{Z}$ ដែល $\ell = \{2k\} \cup \{2k+1\}$

លំហាត់ទី៣០ : ដោះស្រាយសមីការខាងក្រោម ៖

ក. $3\left(1+\dfrac{\cos 2x}{\cos^2 x}\right)^4+4\tan^6 x=7$

ខ. $(2\cos 3x+6\cos x+1)^3=162\cos x-27$

## ដំណោះស្រាយ

ក. $3\left(1+\dfrac{\cos 2x}{\cos^2 x}\right)^4+4\tan^6 x=7$

សមីការមានឫសកាលណា $\cos x\neq 0\Leftrightarrow x\neq\dfrac{\pi}{2}+k\pi,\ k\in\mathbb{Z}$

តាង $\begin{cases}u=1+\dfrac{\cos 2x}{\cos^2 x}\\ v=\tan^2 x\end{cases}$ នោះ $3u^4+4v^3=7$ (1)

ហើយ $u+v=1+\dfrac{\cos 2x}{\cos^2 x}+\tan^2 x=1+\dfrac{\cos^2 x-\sin^2 x+\sin^2 x}{\cos^2 x}=2$

នោះ $v=2-u$ យកជំនួសក្នុង (1)

យើងបាន $3u^4+4(2-u)^3=7$

$\Leftrightarrow 3u^4+4(8-12u+6u^2-u^3)=7$

$\Leftrightarrow 3u^4-4u^3+24u^2-48u+25=0$

$\Leftrightarrow (u-1)^2(3u^2+2u+25)=0$

$\Leftrightarrow\left[\begin{array}{l}(u-1)^2=0\\ 3u^2+2u+25=0\ (*)\end{array}\right.$

$\Leftrightarrow\left[\begin{array}{l}u=1\\ \Delta'=1-75=-74<0\Rightarrow \text{សមីការ}(*)\text{គ្មានឫស}\end{array}\right.$

- $u=1\Leftrightarrow 1+\dfrac{\cos 2x}{\cos^2 x}=1$

$\Leftrightarrow \cos 2x=0$

$\Leftrightarrow 2x=\dfrac{\pi}{2}+k\pi$

$\Rightarrow x=\dfrac{\pi}{4}+\dfrac{k\pi}{2},\ k\in\mathbb{Z}$

ដូចនេះ សមីការមានចម្លើយ $\boxed{x=\dfrac{\pi}{4}+\dfrac{k\pi}{2},\ k\in\mathbb{Z}}$

ខ. $(2\cos 3x+6\cos x+1)^3=162\cos x-27$

$\Leftrightarrow\left[2(4\cos^3 x-3\cos x)+6\cos x+1\right]^3=27\left(6\cos x-1\right)$

$\Leftrightarrow\left[(2\cos x)^3+1\right]^3=27\left[3(2\cos x)-1\right]$

$\Leftrightarrow(2\cos x)^3+1=3\sqrt[3]{3(2\cos x)-1}$

តាង $t=2\cos x$ ដែល $|t|\le 2$ យើងបាន $t^3+1=3\sqrt[3]{3t-1}$

តាង $u=\sqrt[3]{3t-1}\Rightarrow u^3=3t-1$ ហើយ $t^3=3\sqrt[3]{3t-1}-1=3u-1$

យើងបាន $\begin{cases}t^3=3u-1\\u^3=3t-1\end{cases}$

$\Leftrightarrow t^3-u^3=3(u-t)$

$\Leftrightarrow (t-u)(t^2+tu+u^2)+3(t-u)=0$

$\Leftrightarrow (t-u)(t^2+tu+u^2+3)=0$

ដោយ $t^2+tu+u^2+3>0$

យើងបាន $t-u=0\Rightarrow t=u$

$$\Leftrightarrow t^3 = u^3$$
$$\Leftrightarrow t^3 - u^3 = 0$$
$$\Leftrightarrow t^3 - (3t-1) = 0$$
$$\Leftrightarrow t^3 - 3t + 1 = 0$$
$$\Leftrightarrow (2\cos x)^3 - 6\cos x + 1 = 0$$
$$\Leftrightarrow 8\cos^3 x - 6\cos x + 1 = 0$$
$$\Leftrightarrow 2(4\cos^4 x - 3\cos x) + 1 = 0$$
$$\Leftrightarrow 2\cos 3x + 1 = 0$$
$$\Leftrightarrow \cos 3x = -\frac{1}{2} = \cos\frac{2\pi}{3}$$
$$\Leftrightarrow 3x = \pm\frac{2\pi}{3} + 2k\pi\,,\ k \in \mathbb{Z}$$
$$\Rightarrow x = \pm\frac{2\pi}{9} + \frac{2k\pi}{3}\,,\ k \in \mathbb{Z}$$

ដូចនេះ សមីការមានចម្លើយ $\boxed{x = \pm\frac{2\pi}{9} + \frac{2k\pi}{3}\,,\ k \in \mathbb{Z}}$

📖 សូមរង់ចាំអានស្សៀវតៅ **ចំឆ្នូនកុំឆ្លិច** ដែលជាស្នាដៃរបស់ខ្ញុំជាបន្តទៀត

✍ អ្នករៀបរៀង **សុខ ពិសិដ្ឋ** ( គ្រូបង្រៀនគណិតវិទ្យានៃវិទ្យាល័យកំពង់ត្រែបែក )

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## លំហាត់អនុវត្តន៍

១. ដោះស្រាយសមីការត្រីកោណមាត្រខាងក្រោម៖

ក. $\tan 2x = \frac{\sqrt{3}}{3}$ ចំ. $x = \frac{\pi}{12} + \frac{k\pi}{2}, k \in \mathbb{Z}$

ខ. $\sin 10x = 1$ ចំ. $x = \frac{\pi}{20} + \frac{k\pi}{5}, k \in \mathbb{Z}$

គ. $4\cos^2 x - 3 = 0$ ចំ. $\begin{cases} x = \pm\frac{\pi}{6} + 2k\pi \\ x = \pm\frac{5\pi}{6} + 2k\pi \end{cases}, k \in \mathbb{Z}$

ឃ. $3\sin x - 2 = 0$ ចំ. $\begin{cases} x = \arcsin\frac{2}{3} + 2k\pi \\ x = \pi - \arcsin\frac{2}{3} + 2k\pi \end{cases}, k \in \mathbb{Z}$

ង. $1 + \sin 2x = \cos 2x + 2\sin^2 x$ ចំ. $x = \frac{k\pi}{2}, k \in \mathbb{Z}$

២. ដោះស្រាយសមីការត្រីកោណមាត្រខាងក្រោម៖

ក. $\sin\left(\frac{2x}{3} - \frac{\pi}{3}\right) = 0$ ចំ. $x = \frac{\pi}{2} + \frac{3k\pi}{2}, k \in \mathbb{Z}$

ខ. $1 + \cos\left(2x - \frac{\pi}{6}\right) = 0$ ចំ. $x = \frac{7\pi}{12} + k\pi, k \in \mathbb{Z}$

គ. $\tan\left(\frac{x}{2} + \frac{\pi}{3}\right) = \frac{\sqrt{3}}{3}$ ចំ. $x = -\frac{\pi}{3} + 2k\pi, k \in \mathbb{Z}$

ឃ. $\sin^2(x+1) = 1$ ចំ. $\begin{cases} x = \frac{\pi}{2} + 2k\pi - 1 \\ x = -\frac{\pi}{2} + 2k\pi - 1 \end{cases}, k \in \mathbb{Z}$

ង. $2\cos\left[\frac{\pi}{6}\left(\sin x-1+\frac{\sqrt{2}}{2}\right)\right]=\sqrt{3}$ ចំ. $x=k\pi+(-1)^{n+1}\frac{\pi}{4}$

៣. ដោះស្រាយសមីការត្រីកោណមាត្រខាងក្រោម :

ក. $\sin x=\sin\left(\frac{4x}{5}+3\pi\right)$ ចំ. $\begin{cases}x=15\pi+10k\pi\\x=-\frac{10\pi}{9}+\frac{10k\pi}{9}\end{cases},k\in\mathbb{Z}$

ខ. $\cos(2x+\pi)=-\sin x$ ចំ. $\begin{cases}x=\frac{\pi}{6}+\frac{2k\pi}{3}\\x=-\frac{\pi}{2}+2k\pi\end{cases},k\in\mathbb{Z}$

គ. $\sin x=-\cos 2x$ ចំ. $\begin{cases}x=\frac{\pi}{2}+2k\pi\\x=-\frac{\pi}{6}+\frac{2k\pi}{3}\end{cases},k\in\mathbb{Z}$

ឃ. $\tan x+\tan(2x+\frac{\pi}{3})=0$ ចំ. $x=-\frac{\pi}{9}+\frac{k\pi}{3},k\in\mathbb{Z}$

ង. $\cos 5x-\sin(3x+\pi)=0$ ចំ. $\begin{cases}x=\frac{\pi}{4}+k\pi\\x=-\frac{\pi}{16}+\frac{k\pi}{4}\end{cases},k\in\mathbb{Z}$

៤. ដោះស្រាយសមីការត្រីកោណមាត្រខាងក្រោម :

ក. $4\sin x\cos x\cos 2x=1$ ចំ. $x=\frac{\pi}{8}+\frac{k\pi}{2},k\in\mathbb{Z}$

ខ. $\cos x\cos 2x\cos 4x\cos 8x\cos 16x\cos 32x=\frac{1}{64\sin x}$

ចំ. $x=\frac{\pi}{128}+\frac{k\pi}{32},k\in\mathbb{Z}$

គ. $\tan x+\dfrac{\cos x}{1+\sin x}=1$ ចំ. $\begin{cases} x=-\dfrac{\pi}{2}+2k\pi \\ x=2k\pi \end{cases}, k\in\mathbb{Z}$

ឃ. $\cos 3x-\cos 2x-\cos x=\sin 2x-\sin x-1$

ចំ. $\begin{cases} x=-\dfrac{\pi}{6}+2k\pi \\ x=\dfrac{7\pi}{6}+2k\pi \\ x=2k\pi \\ x=\dfrac{\pi}{3}+\dfrac{2k\pi}{3} \end{cases} k,\in\mathbb{Z}$

ង. $(1-\tan x)(1+\sin 2x)=1+\tan x$

ចំ. $\begin{cases} x=2k\pi \\ x=-\dfrac{\pi}{4}+2k\pi \end{cases}, k\in\mathbb{Z}$

ច. $\cos 10x+2\cos^2 4x+6\cos 3x\cos x=\cos x+8\cos x\cos^3 x$

ចំ. $x=2k\pi \ , k\in\mathbb{Z}$

ឆ. $4\sin^3 x+3\cos^2 x-3\sin x-\sin^2 x\cos x=0$

ចំ. $\begin{cases} x=\pm\dfrac{\pi}{3}+k\pi \\ x=\dfrac{\pi}{4}+k\pi \end{cases}, k\in\mathbb{Z}$

ជ. $\sin^6 x+\cos^6 x=2(\sin^8 x+\cos^8 x)$

ចំ. $x=\dfrac{\pi}{4}+\dfrac{k\pi}{2} \ , k\in\mathbb{Z}$

ឈ. $\tan^2 x-\tan x\tan 3x=2$

ចំ. $x=\frac{\pi}{4}+\frac{k\pi}{2}$ , $k\in\mathbb{Z}$

៥. ដោះស្រាយសមីការត្រីកោណមាត្រខាងក្រោម :

ក. $\sin 2x=1$ , $x\in\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$ ចំ. $x=\frac{\pi}{4}$

ខ. $\cos x+\cos 3x=0$ , $x\in\left[0,\frac{\pi}{2}\right]$ ចំ. $x\in\left\{\frac{\pi}{4},\frac{\pi}{2}\right\}$

៦. ដោះស្រាយសមីការត្រីកោណមាត្រខាងក្រោម :

ក. $2+3\cos x=2\tan\frac{x}{2}$ ចំ. $x=\frac{\pi}{2}+2k\pi$ , $k\in\mathbb{Z}$

ខ. $\frac{\cos x-2\sin x\cos x}{2\cos^2 x+\sin x-1}=\sqrt{3}$ ចំ. $x=-\frac{\pi}{18}+\frac{2k\pi}{3}$ , $k\in\mathbb{Z}$

គ. $\sin x+\sin 2x+\sin 3x=\cos x+\cos 2x+\cos 3x$

ចំ. $\begin{cases}x=\pm\frac{2\pi}{3}+2k\pi\\ x=\frac{\pi}{8}+\frac{k\pi}{2}\end{cases}$ , $k\in\mathbb{Z}$

ឃ. $\sin^2 3x-\cos^2 4x=\sin^2 5x-\cos^2 6x$

ចំ. $\begin{cases}x=\frac{\pi}{2}+k\pi\\ x=-\frac{k\pi}{2}\\ x=\frac{k\pi}{9}\end{cases}$ , $k\in\mathbb{Z}$

ង. $\sin^2\left(\frac{x}{2}-\frac{\pi}{4}\right)\tan^2 x-\cos^2\frac{x}{2}=0$

ចំ. $\begin{cases} x=\pi+2k\pi \\ x=-\dfrac{\pi}{4}+k\pi \end{cases}, k\in\mathbb{Z}$

៧. ដោះស្រាយសមីការខាងក្រោម៖

ក. $\tan^2 x+(\sqrt{3}-1)\tan x-\sqrt{3}=0$

ខ. $\cos^2 x+4\sin x-4=0$

គ. $1+\cos 4x=\cos 2x$

ឃ. $2\cos 2x+\cos x=1$

ង. $2\sin^2 2x-2(2-\sqrt{2})\sin x\cos x+\sqrt{2}=0$

ច. $\cos 2x+(10-\sqrt{3})\sin x+5\sqrt{3}-1=0$

ឆ. $\cos^2\left(\dfrac{\pi}{2}+2x\right)-\cos^2 2x-3\cos\left(\dfrac{\pi}{2}-2x\right)+2=0$

ចម្លើយ

ក. $\begin{cases} x=-\dfrac{\pi}{4}+k\pi \\ x=\dfrac{\pi}{3}+k\pi \end{cases}, k\in\mathbb{Z}$

ខ. $x=k\pi\ , k\in\mathbb{Z}$

គ. $\begin{cases} x=\dfrac{\pi}{4}+\dfrac{k\pi}{2} \\ x=\pm\dfrac{\pi}{6}+k\pi \end{cases}, k\in\mathbb{Z}$

ឃ. $\begin{cases} x=\pi+2k\pi \\ x=\pm\arccos\left(\dfrac{3}{4}\right)+2k\pi \end{cases}, k\in\mathbb{Z}$

ង. $\begin{cases} x=\dfrac{\pi}{4}+k\pi \\ x=\dfrac{\pi}{8}+k\pi \\ x=\dfrac{3\pi}{8}+k\pi \end{cases}, k\in\mathbb{Z}$

ច. $\begin{cases} x=-\dfrac{\pi}{3}+2k\pi \\ x=\dfrac{4\pi}{3}+2k\pi \end{cases}, k\in\mathbb{Z}$

ឆ. $\begin{cases} x=\dfrac{\pi}{4}+k\pi \\ x=\dfrac{\pi}{6}+2k\pi \\ x=\dfrac{5\pi}{6}+2k\pi \end{cases}, k\in\mathbb{Z}$

៨. ដោះស្រាយសមីការខាងក្រោម

ក. $5(1+\cos x)=2+\sin^4 x-\cos^4 x$

ខ. $\sin^4 x+\cos^4 x=\dfrac{2}{3}\sin^2\left(x+\dfrac{\pi}{4}\right)+\dfrac{1}{3}$

គ. $\dfrac{\sin^4 x+\cos^4 x}{5\sin 2x}=\dfrac{1}{2}\cot 2x-\dfrac{1}{8\sin 2x}$

ឃ. $\sin^4 x+\cos^4 x=-\dfrac{5}{12}\cos\left(x-\dfrac{\pi}{4}\right)\sin\left(3x-\dfrac{\pi}{4}\right)+\dfrac{21}{24}$

## ចម្លើយ

ក. $x=\pi+2k\pi\ , k\in\mathbb{Z}$

ខ. $\begin{cases} x=\arcsin\left(\dfrac{1-\sqrt{7}}{2}\right)+2k\pi \\ x=\pi-\arcsin\left(\dfrac{1-\sqrt{7}}{2}\right)+2k\pi \end{cases}, k\in\mathbb{Z}$

គ. $x=\pm\dfrac{\pi}{6}+k\pi \ , k\in\mathbb{Z}$

ឃ. $\begin{cases} x=\dfrac{\pi}{12}+k\pi \\ x=\dfrac{5\pi}{12}+k\pi \end{cases}, k\in\mathbb{Z}$

## ៩. ដោះស្រាយសមីការខាងក្រោម

ក. $48-\dfrac{1}{\cos^4 x}-\dfrac{2}{\sin^2 x}\left(1+\cot 2x\cot x\right)=0$

ខ. $\dfrac{\sin^4 2x+\cos^4 2x}{\tan\left(\dfrac{\pi}{4}-x\right)\tan\left(\dfrac{\pi}{4}+x\right)}=\cos^4 4x$

គ. $\tan^3\left(x-\dfrac{\pi}{4}\right)=\tan x-1$

ឃ. $\cot x=\tan x+\dfrac{2\cos 4x}{\sin 2x}$

ង. $\sin^4 x+\cos^4 x=2\sin x\cos x-\dfrac{1}{2}\cos^2 2x$

ច. $\sin^4 x+\cos^4 x=\dfrac{5}{2}\cos 2x-\dfrac{5}{8}$

ឆ. $\dfrac{2(\sin^4 x+\cos^4 x)-4\sin x\cos x+1}{\sqrt{2}-2\sin x}=0$

## ចម្លើយ

ក. $x=\frac{\pi}{8}+\frac{k\pi}{4}\ ,k\in\mathbb{Z}$

ខ. $x=\frac{k\pi}{4}$ ដែល $k=0,\pm2,\pm4,\pm6,...$

គ. $\left[\begin{array}{l}x=\frac{\pi}{4}+k\pi\\x=k\pi\end{array}\right.\ ,k\in\mathbb{Z}$

ឃ. $x=\pm\frac{2\pi}{3}+k\pi\ ,k\in\mathbb{Z}$

ង. $x=\frac{\pi}{4}+k\pi\ ,k\in\mathbb{Z}$

ច. $\sin^4x+\cos^4x=\frac{5}{2}\cos2x-\frac{5}{8}$

$\Leftrightarrow1-\frac{1}{2}\sin^2 2x=\frac{5}{2}\cos2x-\frac{5}{8}$

$\Leftrightarrow8-4\sin^2 2x=20\cos2x-5$

$\Leftrightarrow-4\left(1-\cos^2 2x\right)-20\cos2x+13=0$

$\Leftrightarrow4\cos^2 2x-20\cos2x+9=0$

$\Delta'=100-36=64$

$\Leftrightarrow\left[\begin{array}{l}\cos2x=\frac{10-8}{4}=\frac{1}{2}\\\cos2x=\frac{10+8}{4}=\frac{9}{2}>1\text{ (មិនយក)}\end{array}\right.$

$\Leftrightarrow\cos2x=\cos\frac{\pi}{3}$

$\Rightarrow x=\pm\frac{\pi}{6}+k\pi\ ,k\in\mathbb{Z}$

ឆ. $x=\frac{\pi}{4}+k\pi$ , $k=\pm1,\pm3,\pm5,\cdots$

90. ដោះស្រាយសមីការខាងក្រោម៖

ក. $\sin5x+\cos5x=\sqrt{2}$

ខ. $\sqrt{3}\cos3x+\sin3x=\sqrt{2}$

គ. $2\sin x-5\cos x=5$

ឃ. $\cos\left(x+\frac{\pi}{2}\right)+\sqrt{3}\cos(-x)=\sqrt{3}$

ចម្លើយ

ក. $x=\frac{\pi}{20}+\frac{2k\pi}{5}$ , $k\in\mathbb{Z}$

ខ. $\left[\begin{array}{l} x=-\frac{\pi}{36}+\frac{2k\pi}{3} \\ x=-\frac{7\pi}{36}+\frac{2k\pi}{3} \end{array}\right.$ , $k\in\mathbb{Z}$

គ. $x=2\arctan\left(\frac{5}{2}\right)+2k\pi$ , $k\in\mathbb{Z}$

ឃ. $\left[\begin{array}{l} x=2k\pi \\ x=\frac{5\pi}{3}+2k\pi \end{array}\right.$ , $k\in\mathbb{Z}$

91. ដោះស្រាយសមីការខាងក្រោម ៖

ក. $4\sin x\cos3x+2\sin2x=\sqrt{3}\sin x+\cos x$

ខ. $2\sqrt{3}\cot x-\frac{1}{\sin x}=1+\frac{\sqrt{3}\cot x}{\sin x}-\cot^2 x$

គ. $\cos2x-3\sin x+\cos x-2=0$

ចម្លើយ

ក. $\begin{cases} x=\dfrac{\pi}{18}+\dfrac{2k\pi}{3} \\ x=\dfrac{\pi}{6}+\dfrac{2k\pi}{5} \end{cases}, k\in\mathbb{Z}$

ខ. $\begin{cases} x=\dfrac{\pi}{6}+2k\pi \\ x=\dfrac{\pi}{6}+\dfrac{2k\pi}{3} \end{cases}, k\in\mathbb{Z}$

គ. $\begin{cases} x=2k\pi \\ x=\dfrac{3\pi}{2}+2k\pi \end{cases}, k\in\mathbb{Z}$

១២. ដោះស្រាយសមីការខាងក្រោម៖

ក. $\dfrac{(2-\sqrt{3})\cos x-2\sin^2\left(\dfrac{x}{2}-\dfrac{\pi}{4}\right)}{2\cos x-1}=1$

ខ. $8\sin^2 2x\cos 2x=\sqrt{3}\sin 2x+\cos 2x$

គ. $\left(\sin\dfrac{x}{2}+\cos\dfrac{x}{2}\right)^2+\sqrt{3}\cos x=2$

ចម្លើយ

ក. $x=\dfrac{2\pi}{3}+2k\pi \ , k\in\mathbb{Z}$

ខ. $\begin{cases} x=-\dfrac{\pi}{24}+\dfrac{2k\pi}{7} \\ x=-\dfrac{5\pi}{12}+\dfrac{k\pi}{2} \end{cases}, k\in\mathbb{Z}$

គ. $\begin{cases} x=-\dfrac{\pi}{6}+2k\pi \\ x=\dfrac{\pi}{2}+2k\pi \end{cases}, k\in\mathbb{Z}$

១៣. ដោះស្រាយសមីការខាងក្រោម ៖

ក. $-1+\sin^3 x+\cos^3 x=\dfrac{3}{2}\sin 2x$

ខ. $1+\tan x=2\sqrt{2}\sin x$

គ. $\cos x\sin x+\left|\cos x+\sin x\right|=1$

ចម្លើយ

ក. $\begin{cases} x=2k\pi \\ x=\dfrac{\pi}{2}+2k\pi \\ x=\arctan\left(\dfrac{-2+\sqrt{3}}{\sqrt{2}}\right)-\dfrac{\pi}{4}+2k\pi \\ x=\dfrac{3\pi}{4}-\arcsin\left(\dfrac{-2+\sqrt{3}}{\sqrt{2}}\right)+2k\pi \end{cases}, k\in\mathbb{Z}$

ខ. $\begin{cases} x=\dfrac{\pi}{4}+2k\pi \\ x=-\dfrac{\pi}{12}+2k\pi \\ x=\dfrac{7\pi}{12}+2k\pi \end{cases}, k\in\mathbb{Z}$

គ. $x=\dfrac{k\pi}{2}, k\in\mathbb{Z}$

១៤. ដោះស្រាយសមីការខាងក្រោម ៖

ក. $\cos^3 x+\sin^3 x=\sin 2x+\sin x+\cos x$

ខ. $\cos^3 x-\sin^3 x=-1$

គ. $1+\cos^3 x-\sin^3 x=\sin 2x$

ឃ. $\sin 2x+\sqrt{2}\sin\left(x-\dfrac{\pi}{4}\right)=1$

ង. $2\cos 2x+\sin^2 x\cos x+\sin x\cos^2 x=2(\sin x+\cos x)$

ចម្លើយ

ក. $x=\pi+k\pi\ ,k\in\mathbb{Z}$

ខ. $\left[\begin{array}{l} x=\dfrac{\pi}{2}+2k\pi \\ x=\pi+2k\pi \end{array}\right. ,k\in\mathbb{Z}$

គ. $x=\dfrac{\pi}{4}+k\pi\ ,k\in\mathbb{Z}$

ឃ. $\left[\begin{array}{l} x=\dfrac{\pi}{4}+k\pi \\ x=\dfrac{\pi}{2}+2k\pi \\ x=\pi+2k\pi \end{array}\right. ,k\in\mathbb{Z}$

ង. $x=-\dfrac{\pi}{4}+k\pi\ ,k\in\mathbb{Z}$

១៥. ដោះស្រាយសមីការខាងក្រោម ៖

ក. $\cos^2 x-\sqrt{3}\sin 2x=1+\sin^2 x$

ខ. $\sin 2x+2\tan x=3$

គ. $\sin^2 x+\sqrt{3}\sin x\cos x+2\cos^2 x=\dfrac{3+\sqrt{2}}{2}$

ឃ. $2\cos^2 x+\cos 2x+\sin x=0$

ង. $4(\sin^3 x+\cos^3 x)=\cos x+3\sin x$

ច. $4\sqrt{3}\sin x\cos x+4\cos^2 x=2\sin^2 x+\dfrac{5}{2}$

ឆ. $\sin^2 x-3\sin x\cos x+1=0$

## ចម្លើយ

ក. $\begin{cases} x=k\pi \\ x=-\dfrac{\pi}{3}+k\pi \end{cases}, k\in\mathbb{Z}$

ខ. $x=\dfrac{\pi}{4}+k\pi, k\in\mathbb{Z}$

គ. $\begin{cases} x=\arctan\left(\dfrac{\sqrt{3}+\sqrt{2}}{1+\sqrt{2}}\right)+k\pi \\ x=\arctan\left(\dfrac{\sqrt{3}-\sqrt{2}}{1+\sqrt{2}}\right)+k\pi \end{cases}, k\in\mathbb{Z}$

ឃ. $\begin{cases} x=-\dfrac{\pi}{45}+k\pi \\ x=\arctan 3+k\pi \end{cases}, k\in\mathbb{Z}$

ង. $\begin{cases} x=\dfrac{\pi}{4}+k\pi \\ x=\pm\dfrac{\pi}{3}+k\pi \end{cases}, k\in\mathbb{Z}$

ច. $\begin{cases} x=\dfrac{\pi}{4}+k\pi \\ x=k\pi \end{cases}, k\in\mathbb{Z}$

ឆ. $\left[\begin{array}{l} x=\dfrac{\pi}{4}+k\pi \\ x=\arctan\dfrac{1}{2}+k\pi \end{array}\right. , k\in\mathbb{Z}$

១៦. ដោះស្រាយសមីការខាងក្រោម៖

ក. $\sin^4 x+\cos^4 x=2\sin 2x\cos 2x-\dfrac{1}{2}\cos^2 2x$

ខ. $\sin^4 x+\cos^4 x=-\dfrac{5}{4}\sin 2x\cos 2x-\dfrac{1}{4}\cos^2 2x+\dfrac{3}{2}$

គ. $6\sin x-2\cos^3 x=\dfrac{5\sin 4x\cos x}{2\cos 2x}$

ឃ. $\tan x\sin^2 x-2\sin^2 x=3(\cos 2x+\sin x\cos x)$

ង. $2\sqrt{2}\cos^3\left(x-\dfrac{\pi}{4}\right)-3\cos x-\sin x=0$

ច. $\sqrt{2}\sin^3\left(x+\dfrac{\pi}{4}\right)=2\sin x$

ឆ. $8\cos^3\left(x+\dfrac{\pi}{3}\right)=\cos 3x$

ចម្លើយ

ក. $\left[\begin{array}{l} x=\dfrac{\pi}{8}+\dfrac{k\pi}{2} \\ x=\dfrac{1}{2}\arctan(-3)+\dfrac{k\pi}{2} \end{array}\right. , k\in\mathbb{Z}$

ខ. $\left[\begin{array}{l} x=\dfrac{\pi}{8}+\dfrac{k\pi}{2} \\ x=\dfrac{1}{2}\arctan\left(\dfrac{1}{5}\right)+\dfrac{k\pi}{2} \end{array}\right. , k\in\mathbb{Z}$

គ. សមីការគ្មានចម្លើយ

ឃ. $\begin{cases} x=-\dfrac{\pi}{4}+k\pi \\ x=\pm\dfrac{\pi}{3}+k\pi \end{cases}, k\in\mathbb{Z}$

ង. $\begin{cases} x=\dfrac{\pi}{2}+k\pi \\ x=\dfrac{\pi}{4}+k\pi \end{cases}, k\in\mathbb{Z}$

ច. $x=\dfrac{\pi}{4}+k\pi \ , k\in\mathbb{Z}$

ឆ. $\begin{cases} x=k\pi \\ x=\dfrac{\pi}{3}+k\pi \\ x=\dfrac{\pi}{6}+k\pi \end{cases}, k\in\mathbb{Z}$

១៧. ដោះស្រាយសមីការខាងក្រោម ៖

ក. $2\sin 2x-\cos 2x=7\sin x+2\cos x-4$

ខ. $9\sin x+6\cos x-3\sin 2x+\cos 2x=8$

គ. $\sin 2x-\cos 2x=3\sin x+\cos x-2$

ឃ. $\tan x-\sin 2x-\cos 2x+2\left(2\cos x-\dfrac{1}{\cos x}\right)=0$

ង. $\sin^3 x+\cos^3 x=\sin x-\cos x$

ចម្លើយ

ក. $\begin{cases} x=\dfrac{\pi}{6}+2k\pi \\ x=\dfrac{5\pi}{6}+2k\pi \end{cases}, k\in\mathbb{Z}$

ខ. $x=\dfrac{\pi}{2}+2k\pi\ , k\in\mathbb{Z}$

គ. $\begin{cases} x=\dfrac{\pi}{6}+2k\pi \\ x=\dfrac{5\pi}{6}=2k\pi \\ x=2k\pi \\ x=\dfrac{\pi}{2}+2k\pi \end{cases}, k\in\mathbb{Z}$

ឃ. $x=\dfrac{\pi}{4}+\dfrac{k\pi}{2}\ , k\in\mathbb{Z}$

ង. $x=\dfrac{\pi}{2}+k\pi\ , k\in\mathbb{Z}$

១៨. ដោះស្រាយសមីការខាងក្រោម ៖

ក. $\sin x+\sin^2 x+\sin^3 x+\sin^4 x=\cos x+\cos^2 x+\cos^3 x+\cos^4 x$

ខ. $3\tan^3 x-\tan x+\dfrac{3(1+\sin x)}{\cos^2 x}=8\cos^2\left(\dfrac{\pi}{4}-\dfrac{x}{2}\right)$

គ. $3(\cot x-\cos x)-5(\tan x-\sin x)=2$

ឃ. $\tan^2 x(1-\sin^3 x)+\cos^3 x-1=0$

ចម្លើយ

ក. $\begin{cases} x=\dfrac{\pi}{4}+k\pi \\ x=\pi+2k\pi \\ x=-\dfrac{\pi}{2}+2k\pi \end{cases}, k\in\mathbb{Z}$

ខ. $\begin{cases} x=\pm\dfrac{\pi}{6}+k\pi \\ x=\arcsin\left(\dfrac{\sqrt{2}-1}{\sqrt{2}}\right)-\dfrac{\pi}{4}+2k\pi \\ x=\dfrac{3\pi}{4}-\arcsin\left(\dfrac{\sqrt{2}-1}{\sqrt{2}}\right)+2k\pi \end{cases}, k\in\mathbb{Z}$

គ. $\begin{cases} x=\arcsin\left(\dfrac{1-\sqrt{2}}{2}\right)-\dfrac{\pi}{4}+2k\pi \\ x=\dfrac{3\pi}{4}-\arcsin\left(\dfrac{1-\sqrt{2}}{2}=2k\pi\right) \\ x=\arctan\left(\dfrac{3}{5}\right)+k\pi \end{cases}, k\in\mathbb{Z}$

ឃ. $\begin{cases} x=\dfrac{\pi}{4}=k\pi \\ x=2k\pi \\ x=\pm\arccos\left(\dfrac{\sqrt{2}-1}{\sqrt{2}}\right)+\dfrac{\pi}{4}+2k\pi \end{cases}, k\in\mathbb{Z}$

**១៩.** ដោះស្រាយសមីការខាងក្រោម ៖

ក. $2\sin^3 x-\sin x=2\cos^3 x-\cos x+\cos 2x$

ខ. $\cos x(\cos 4x+2)+\cos 2x\cos 3x=0$

គ. $1+\cot 2x=\dfrac{1-\cos 2x}{\sin^2 2x}$

ឃ. $\cos^3 x+\cos^2 x+2\sin x-2=0$

ង. $4\cos^2 x-3\cos 3x=\cos x+\sin 2x+\cos 2x$

ច. $1+\sin x+\cos 3x=\cos x+\sin 2x+\cos 2x$

## ចម្លើយ

ក. $\begin{cases} x=\pm\dfrac{\pi}{4}+2k\pi \\ x=\pm\dfrac{3\pi}{4}+2k\pi \\ x=2k\pi \end{cases}, k\in\mathbb{Z}$

ខ. $x=\dfrac{\pi}{2}+k\pi \ , k\in\mathbb{Z}$

គ. $\begin{cases} x=-\dfrac{\pi}{4}+2k\pi \\ x=k\pi \end{cases}, k\in\mathbb{Z}$

ឃ. $\begin{cases} x=\dfrac{\pi}{2}+2k\pi \\ x=2k\pi \end{cases}, k\in\mathbb{Z}$

ង. $\begin{cases} x=\pi+2k\pi \\ x=\pm\dfrac{\pi}{3}+2k\pi \\ x=\pm\dfrac{2\pi}{3}=2k\pi \end{cases}, k\in\mathbb{Z}$

ច. $\begin{cases} x = k\pi \\ x = \dfrac{\pi}{4} + k\pi \end{cases}, k \in \mathbb{Z}$

## ២០. ដោះស្រាយសមីការខាងក្រោម ៖

ក. $\sin 2x + \sin x - \dfrac{1}{2\sin x} - \dfrac{1}{\sin 2x} = 2\cot 2x$

ខ. $\sin\left(5x - \dfrac{\pi}{4}\right) - \cos\left(\dfrac{x}{2} - \dfrac{\pi}{4}\right) = \sqrt{2}\cos\dfrac{3x}{2}$

គ. $\sin 2x + \cos 2x + 3\sin x - \cos x - 2 = 0$

ឃ. $\tan^4 x + 1 = \dfrac{(2 - \sin^2 2x)\sin 3x}{\cos^4 x}$

ង. $3\cos^4 x - 4\sin^2 x\cos^2 x + \sin^4 x = 0$

ច. $\sin 2x = 1 + \sqrt{2}\cos x + \cos 2x$

### ចម្លើយ

ក. $x = \dfrac{\pi}{4} + \dfrac{k\pi}{2}, k \in \mathbb{Z}$

ខ. $\begin{cases} x = \dfrac{\pi}{3} + \dfrac{2k\pi}{3} \\ x = \dfrac{\pi}{4} + k\pi \\ x = \dfrac{\pi}{2} + k\pi \end{cases}, k \in \mathbb{Z}$

គ. $\begin{cases} x=\dfrac{\pi}{6}+2k\pi \\ x=\dfrac{5\pi}{6}+2k\pi \\ x=-\pi+2k\pi \\ x=\dfrac{\pi}{2}+2k\pi \end{cases}, k\in\mathbb{Z}$

ឃ. $\begin{cases} x=\dfrac{\pi}{18}+\dfrac{2k\pi}{3} \\ x=\dfrac{17\pi}{18}+\dfrac{2k\pi}{3} \end{cases}, k\in\mathbb{Z}$

ង. $\begin{cases} x=\dfrac{\pi}{4}+k\pi \\ x=\pm\dfrac{\pi}{3}+2k\pi \\ x=\pm\dfrac{2\pi}{3}+2k\pi \end{cases}, k\in\mathbb{Z}$ ច. $\begin{cases} x=\dfrac{\pi}{2}+k\pi \\ x=\dfrac{5\pi}{12}+2k\pi \\ x=\dfrac{13\pi}{12}+2k\pi \end{cases}, k\in\mathbb{Z}$

២១. ដោះស្រាយសមីការខាងក្រោម

ក. $\sqrt{1+\sin x}+\cos x=0$

ខ. $\left(\sqrt{1-\cos x}+\sqrt{\cos x}\right)\cos 2x=\dfrac{1}{2}\sin 4x$

គ. $\dfrac{\sqrt{1-\sin 2x}+\sqrt{1+\sin 2x}}{\sin x}=4\cos x$

ឃ. $\sin x+\sqrt{3}\cos x+\sqrt{\sin x+\sqrt{3}\cos x}=2$

ចម្លើយ

ក. $\begin{cases} x=\pi+2k\pi \\ x=-\dfrac{\pi}{2}+2k\pi \end{cases}, k\in\mathbb{Z}$

ខ. $x=\pm\frac{\pi}{4}+2k\pi\,,k\in\mathbb{Z}$

គ. $\left[\begin{array}{l} x=\frac{\pi}{6}+k\pi \\ x=\frac{\pi}{3}+k\pi \end{array}\right.,k\in\mathbb{Z}$

ឃ. $x=-\frac{\pi}{6}+2k\pi\vee x=\frac{\pi}{2}+2k\pi\ \ ,k\in\mathbb{Z}$