២០២៣

គណិតវិទ្យា ថ្នាក់ទី ១២

លីមីត និង ភាពជាប់ អនុគមន៍

អ្នករៀបរៀង៖ ខែម ពុទ្ធី

គណៈកម្មការរៀបរៀង	លោកគ្រូ ខែម ពុទ្ធី
ត្រួតពិនិត្យខ្លឹមសារ	លោកគ្រូ ខែម ពុទ្ធី
រៀបចំខ្លឹមសារ	លោកគ្រូ ខែម ពុទ្ធី
រចនាគម្រប និងទំព័រ	លោកគ្រូ ខែម ពុទ្ធី
រូបភាព និងតារាង	លោកគ្រូ ខែម ពុទ្ធី
កែសម្រួលអក្ខរាវិរុទ្ធ	លោកគ្រូ ខែម ពុទ្ធី
គណៈកម្មការពិនិត្យចុងក្រោយ	លោកគ្រូ ខែម ពុទ្ធី

សៀវភៅនេះត្រូវបានរៀបចំឡើងសម្រាប់សិស្សានុសិស្សថ្នាក់ទី១២ ជាពិសេសសម្រាប់ត្រៀមប្រឡងសញ្ញាបត្រមធ្យមសិក្សាទុតិយភូមិ និងប្រឡងប្រកួតប្រជែងសិស្សពូកែគណិតវិទ្យា។

ព័ត៌មានសៀវភៅ

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លីមីតនៃអនុគមន៍

រៀបរៀងដោយ លោកគ្រូ ខែម ពុទ្ធី

សម្រាប់បោះពុម្ពផ្សាយ៖ បណ្ណាគារអង្គរ

ISBN 120 000 000 000

អារម្ភកថា

គណិតវិទ្យា គឺជាវិទ្យាសាស្ត្រមួយដែលមានតួនាទីយ៉ាងសំខាន់ក្នុងការអភិវឌ្ឍសមត្ថភាពត្រិះរិះពិចារណា និងការដោះស្រាយបញ្ហាក្នុងជីវភាពរស់នៅប្រចាំថ្ងៃ។ ក្នុងចំណោមមែកធាងជាច្រើននៃគណិតវិទ្យា ការសិក្សាអំពី «លីមីតនៃអនុគមន៍» គឺជាមូលដ្ឋានគ្រឹះដ៏សំខាន់បំផុតមួយនៃគណិតវិទ្យាវិភាគ (Calculus) ដែលសិស្សានុសិស្សថ្នាក់ទី១២ ចាំបាច់ត្រូវតែស្វែងយល់ឱ្យបានច្បាស់លាស់។ លីមីតមិនត្រឹមតែជាស្ពានចម្លងសម្រាប់ការសិក្សាទៅលើដេរីវេ និងអាំងតេក្រាលប៉ុណ្ណោះទេ តែថែមទាំងជាគន្លឹះក្នុងការយល់ដឹងអំពីបម្រែបម្រួលបាតុភូតផ្សេងៗផងដែរ។

ដោយយល់ឃើញពីសារៈសំខាន់នេះ និងកត់សម្គាល់ឃើញពីការលំបាករបស់សិស្សានុសិស្សមួយចំនួនក្នុងការស្វែងយល់អំពីការគណនាលីមីតរាងមិនកំណត់ ខ្ញុំបាទបានខិតខំស្រាវជ្រាវ និងចងក្រងសៀវភៅ «**លីមីតនៃអនុគមន៍**» នេះឡើង។ សៀវភៅនេះត្រូវបានរៀបចំឡើងយ៉ាងសម្រិតសម្រាំង ដោយបែងចែកជា ៤ ផ្នែកធំៗរួមមាន៖ មេរៀនសង្ខេប វិធីសាស្ត្រក្នុងការគណនា លំហាត់មានដំណោះស្រាយ និងលំហាត់ប្រតិបត្តិសម្រាប់អនុវត្ត ដើម្បីសម្រួលដល់ការសិក្សាស្រាវជ្រាវរបស់សិស្សានុសិស្សឱ្យកាន់តែមានភាពងាយស្រួល និងស៊ីជម្រៅ។

ខ្ញុំបាទសង្ឃឹមយ៉ាងមុតមាំថា សៀវភៅនេះនឹងក្លាយជាឯកសារយោងដ៏មានតម្លៃសម្រាប់ការត្រៀមប្រឡងសញ្ញាបត្រមធ្យមសិក្សាទុតិយភូមិ (បាក់ឌុប) និងជួយជំរុញទឹកចិត្តសិស្សានុសិស្សទាំងអស់ឱ្យកាន់តែមានភាពជឿជាក់លើសមត្ថភាពរបស់ខ្លួន។ ទោះបីជាខ្ញុំបាទបានខិតខំប្រឹងប្រែងយ៉ាងណាក៏ដោយ ក៏កំហុសឆ្គងដោយអចេតនាអាចនឹងកើតមាន អាស្រ័យហេតុនេះខ្ញុំបាទរង់ចាំទទួលនូវរាល់មតិរិះគន់ស្ថាបនាពីសំណាក់លោកគ្រូ អ្នកគ្រូ និងមិត្តអ្នកអានដោយក្តីសោមនស្សរីករាយបំផុត។

ថ្ងៃទី.......... ខែ.......... ឆ្នាំ២០២៦

លោកគ្រូ ខែម ពុទ្ធី

សេចក្តីដឹងគុណ

ជាបឋម ខ្ញុំបាទ **លោកគ្រូ ខែម ពុទ្ធី** សូមលំអោនកាយវាចាចិត្ត ថ្លែងអំណរគុណយ៉ាងជ្រាលជ្រៅបំផុត ជូនចំពោះលោកអ្នកមានគុណទាំងពីរ គឺមាតា និងបិតារបស់ខ្ញុំ ដែលលោកទាំងពីរបានពលិកម្មគ្រប់បែបយ៉ាង ចិញ្ចឹមបីបាច់ថែរក្សា ផ្តល់ភាពកក់ក្តៅ និងអប់រំទូន្មានប្រៀនប្រដៅខ្ញុំបាទតាំងពីកល្យាណវ័យ រហូតធ្វើឱ្យរូបខ្ញុំអាចឈរយ៉ាងរឹងមាំនៅលើវិថីជីវិតមួយនេះ។ បើគ្មានការលះបង់ និងសេចក្តីស្រឡាញ់ដ៏មហាសាលពីលោកទាំងពីរទេ នោះខ្ញុំបាទក៏ប្រហែលជាគ្មានថ្ងៃនេះឡើយ។

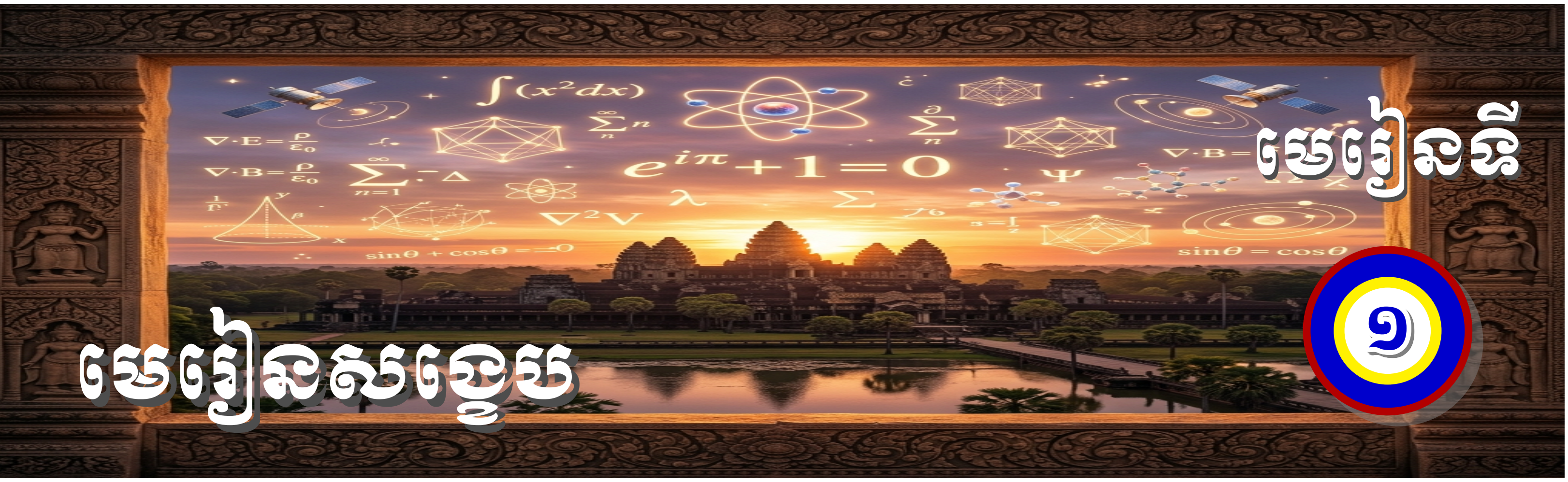
ជាបន្តបន្ទាប់ ខ្ញុំបាទសូមសម្តែងនូវសេចក្តីគោរពដឹងគុណយ៉ាងស្មោះស្ម័គ្រ ជូនចំពោះលោកគ្រូ អ្នកគ្រូ និងសាស្ត្រាចារ្យគ្រប់លំដាប់ថ្នាក់ តាំងពីកម្រិតបឋមសិក្សា រហូតដល់ថ្នាក់សាកលវិទ្យាល័យ ដែលបានខិតខំចំណាយកម្លាំងកាយចិត្ត ក្នុងការផ្ទេរចំណេះដឹង និងបង្ហាត់បង្រៀនរូបខ្ញុំឱ្យក្លាយជាធនធានមនុស្សមួយរូបប្រកបដោយសមត្ថភាព។ រាល់ដំបូន្មានដែលទទួលបានពីលោកគ្រូ អ្នកគ្រូ គឺជាប្រទីបបំភ្លឺផ្លូវដែលតែងតែចង្អុលបង្ហាញឱ្យខ្ញុំបាទដើរលើគន្លងត្រឹមត្រូវ។

សៀវភៅ «**លីមីតនៃអនុគមន៍**» មួយក្បាលនេះ មិនអាចលេចចេញជារូបរាងឡើងបានឡើយ ប្រសិនបើគ្មានការលើកទឹកចិត្តពីមនុស្សជាច្រើនជុំវិញខ្លួនខ្ញុំ។ ខ្ញុំបាទសូមអរគុណដល់មិត្តភក្តិ បងប្អូន និងសហការីទាំងអស់ ដែលតែងតែផ្តល់មតិយោបល់ ជួយកែសម្រួល និងជួយជំរុញទឹកចិត្តខ្ញុំនៅក្នុងដំណើរការនៃការចងក្រងសៀវភៅនេះជាភាសា LaTeX ។ ជាចុងបញ្ចប់ ខ្ញុំបាទសូមប្រសិទ្ធពរជ័យ សិរីសួស្តី ជ័យមង្គល ជូនដល់លោកអ្នកមានគុណ លោកគ្រូ អ្នកគ្រូ ព្រមទាំងមិត្តអ្នកសិក្សាទាំងអស់ សូមប្រកបដោយសុខភាពល្អបរិបូរណ៍ ប្រាជ្ញាភ្លឺថ្លា ជោគជ័យគ្រប់ភារកិច្ច និងសម្រេចបាននូវរាល់បំណងប្រាថ្នាក្នុងជីវិតជានិរន្តរ៍។

ថ្ងៃទី.......... ខែ.......... ឆ្នាំ២០២៦

លោកគ្រូ ខែម ពុទ្ធី

មាតិកា



១ លីមីតនៃអនុគមន៍

១.១ សញ្ញាណលីមីត

បើ x ខិតជិត a ហើយអនុគមន៍ខិតជិតតម្លៃ L ណាមួយនោះគេកំណត់សរសេរ $\lim_{x\to a} f(x)=L$
មានន័យថា អនុគមន៍ $f(x)$ មានលីមីតស្មើ L កាលណា x ខិតជិត a ។

ឧទាហរណ៍ ១

អនុគមន៍ $f(x)=x+2$។ កាលណា x ខិតជិត 3 នោះ $f(x)$ ខិតជិតតម្លៃ $3+2=5$។ ដូចនេះគេសរសេរ $\lim_{x\to 3}(x+2)=5$។

លំហាត់គំរូ ១

ចូរទាញរកតម្លៃលីមីតនៃអនុគមន៍ $f(x)=\frac{x^2-1}{x-1}$ កាលណា x ខិតជិត 1។

ដំណោះស្រាយ

យើងដឹងថា $f(x)=\frac{(x-1)(x+1)}{x-1}=x+1$ ចំពោះ $x\neq 1$។
កាលណា $x\to 1$ នោះ $f(x)\to 1+1=2$។
ដូចនេះ $\lim_{x\to 1}\frac{x^2-1}{x-1}=2$។

១.២ និយមន័យទី១

បើអនុគមន៍ $f(x)$ មានលីមីត L កាលណា x ខិតជិត c បើគ្រប់ចំនួន $\varepsilon>0$ មានចំនួន$\alpha>0$
ដែល $0<|x-c|<\alpha$ នាំឱ្យ $|f(x)-L|<\varepsilon$ ។ គេកំណត់សរសេរដោយ: $\lim_{x\to c} f(x)=L$

ឧទាហរណ៍ ២

បង្ហាញថា $\lim\limits_{x\to 2} 4x = 8$ ។

ដំណោះស្រាយ

តាមនិយមន័យ គ្រប់ $\varepsilon > 0$ មាន $\alpha > 0$ ដែល $0 < |x-2| < \alpha$ នាំឱ្យ $|4x-8| < \varepsilon$។
គេមាន $|4x-8| < \varepsilon \Rightarrow 4|x-2| < \varepsilon \Rightarrow |x-2| < \frac{\varepsilon}{4}$។
ដូចនេះដោយយក $\alpha = \frac{\varepsilon}{4}$ នោះលក្ខខណ្ឌត្រូវបានផ្ទៀងផ្ទាត់។

លំហាត់គំរូ ២

ដោយប្រើនិយមន័យបង្ហាញថា $\lim\limits_{x\to 1}(3x+2) = 5$។

ដំណោះស្រាយ

ចំពោះគ្រប់ $\varepsilon > 0$ ត្រូវរក $\alpha > 0$ ដែល $0 < |x-1| < \alpha$ នាំឱ្យ $|(3x+2)-5| < \varepsilon$។
យើងមាន $|3x-3| < \varepsilon \Leftrightarrow 3|x-1| < \varepsilon \Leftrightarrow |x-1| < \frac{\varepsilon}{3}$។
ជ្រើសរើសយក $\alpha = \frac{\varepsilon}{3}$ នោះ $|x-1| < \alpha \Rightarrow |(3x+2)-5| < \varepsilon$ ។
ដូចនេះពិតជាបាន $\lim\limits_{x\to 1}(3x+2) = 5$។

១.៣ និយមន័យទី២

បើអនុគមន៍ f ខិតទៅជិត $+\infty$ កាលណា x ខិតទៅជិត a បើចំពោះចំនួន $M > 0$ មាន $\alpha > 0$ ដែល $0 < |x-a| < \alpha$ នាំឱ្យ $f(x) > M$ ។
គេកំណត់សរសេរដោយ $\lim\limits_{x\to a} f(x) = +\infty$

ឧទាហរណ៍ ៣

បង្ហាញថា $\lim\limits_{x\to 0} \frac{1}{x^2} = +\infty$ ។

ដំណោះស្រាយ

ចំពោះគ្រប់ $M > 0$ ត្រូវរក $\alpha > 0$ ដែល $0 < |x-0| < \alpha$ នាំឱ្យ $\frac{1}{x^2} > M$។
គេមាន $\frac{1}{x^2} > M \Rightarrow x^2 < \frac{1}{M} \Rightarrow |x| < \frac{1}{\sqrt{M}}$។
ដោយយក $\alpha = \frac{1}{\sqrt{M}}$ នោះលក្ខខណ្ឌត្រូវបានផ្ទៀងផ្ទាត់។

លំហាត់គំរូ ៣

ដោយប្រើនិយមន័យបង្ហាញថា $\lim\limits_{x\to 2} \frac{1}{(x-2)^2} = +\infty$។

ដំណោះស្រាយ

ចំពោះគ្រប់ $M>0$ ត្រូវរក $\alpha>0$ ដែល $0<|x-2|<\alpha$ នាំឱ្យ $\frac{1}{(x-2)^2}>M$។
តម្រូវលក្ខខណ្ឌ $\frac{1}{(x-2)^2}>M \Leftrightarrow (x-2)^2<\frac{1}{M} \Leftrightarrow |x-2|<\frac{1}{\sqrt{M}}$។
ជ្រើសរើស $\alpha=\frac{1}{\sqrt{M}}$ នោះផ្ទៀងផ្ទាត់តាមនិយមន័យ។
ដូចនេះ $\lim\limits_{x\to 2}\frac{1}{(x-2)^2}=+\infty$។

១.៤ និយមន័យទី៣

បើអនុគមន៍ f ខិតទៅជិត $-\infty$ កាលណា x ខិតទៅជិត a បើចំពោះចំនួន $M>0$ មាន $\alpha>0$ ដែល $0<|x-a|<\alpha$ នាំឱ្យ $f(x)<-M$ ។ គេកំណត់សរសេរដោយ $\lim\limits_{x\to a}f(x)=-\infty$

ឧទាហរណ៍ ៤

បង្ហាញថា $\lim\limits_{x\to 0}\left(-\frac{1}{x^2}\right)=-\infty$ ។

ដំណោះស្រាយ

ចំពោះគ្រប់ $M>0$ ត្រូវរក $\alpha>0$ ដែល $0<|x|<\alpha$ នាំឱ្យ $-\frac{1}{x^2}<-M$។
គេមាន $-\frac{1}{x^2}<-M \Rightarrow \frac{1}{x^2}>M \Rightarrow |x|<\frac{1}{\sqrt{M}}$។
យក $\alpha=\frac{1}{\sqrt{M}}$។

លំហាត់គំរូ ៤

ដោយប្រើនិយមន័យបង្ហាញថា $\lim\limits_{x\to 1}\frac{-2}{(x-1)^2}=-\infty$។

ដំណោះស្រាយ

ចំពោះគ្រប់ $M>0$ ត្រូវរក $\alpha>0$ ដែល $0<|x-1|<\alpha$ នាំឱ្យ $\frac{-2}{(x-1)^2}<-M$។
គេមាន $\frac{-2}{(x-1)^2}<-M \Leftrightarrow \frac{2}{(x-1)^2}>M \Leftrightarrow (x-1)^2<\frac{2}{M} \Leftrightarrow |x-1|<\sqrt{\frac{2}{M}}$។
ជ្រើសរើស $\alpha=\sqrt{\frac{2}{M}}$។
ដូចនេះ $\lim\limits_{x\to 1}\frac{-2}{(x-1)^2}=-\infty$។

១.៥ និយមន័យទី៤

បើអនុគមន៍ f ខិតជិត L កាលណា x ខិតជិត $+\infty$ បើចំពោះគ្រប់ចំនួន $\varepsilon>0$ មាន $N>0$ ដែល $x>N$ នាំឱ្យ $|f(x)-L|<\varepsilon$។ គេកំណត់សរសេរដោយៈ $\lim\limits_{x\to+\infty}f(x)=L$

ឧទាហរណ៍ ៥

បង្ហាញថា $\lim\limits_{x\to+\infty}\frac{1}{x}=0$ ។

ដំណោះស្រាយ

ចំពោះគ្រប់ $\varepsilon>0$ ត្រូវរក $N>0$ ដែល $x>N$ នាំឱ្យ $\left|\frac{1}{x}-0\right|<\varepsilon$។
គេមាន $\frac{1}{|x|}<\varepsilon\Rightarrow|x|>\frac{1}{\varepsilon}$ ។ ដោយ $x\to+\infty$ យក $N=\frac{1}{\varepsilon}$។

លំហាត់គំរូ ៥

ដោយប្រើនិយមន័យបង្ហាញថា $\lim\limits_{x\to+\infty}\frac{2x+1}{x}=2$។

ដំណោះស្រាយ

ត្រូវរក $N>0$ ដែល $x>N$ នាំឱ្យ $\left|\frac{2x+1}{x}-2\right|<\varepsilon$ ចំពោះគ្រប់ $\varepsilon>0$។
គេមាន $\left|2+\frac{1}{x}-2\right|<\varepsilon\Leftrightarrow\left|\frac{1}{x}\right|<\varepsilon\Leftrightarrow x>\frac{1}{\varepsilon}$ (ដោយ $x>0$)។
យក $N=\frac{1}{\varepsilon}$ នោះវានឹងផ្ទៀងផ្ទាត់។
ដូចនេះ $\lim\limits_{x\to+\infty}\frac{2x+1}{x}=2$។

១.៦ និយមន័យទី៥

បើអនុគមន៍ f ខិតជិត L កាលណា x ខិតជិត $-\infty$ បើចំពោះគ្រប់ចំនួន $\varepsilon>0$ មាន $N>0$ ដែល $x<-N$ នាំឱ្យ $|f(x)-L|<\varepsilon$។ គេកំណត់សរសេរដោយ: $\lim\limits_{x\to-\infty}f(x)=L$

ឧទាហរណ៍ ៦

បង្ហាញថា $\lim\limits_{x\to-\infty}\frac{1}{x}=0$ ។

ដំណោះស្រាយ

ចំពោះគ្រប់ $\varepsilon>0$ ត្រូវរក $N>0$ ដែល $x<-N$ នាំឱ្យ $\left|\frac{1}{x}\right|<\varepsilon$។
គេមាន $\frac{1}{|x|}<\varepsilon\Rightarrow|x|>\frac{1}{\varepsilon}$ ។ ដោយ $x<0$ នោះ $x<-\frac{1}{\varepsilon}$។
យក $N=\frac{1}{\varepsilon}$ នោះ $x<-N$។

លំហាត់គំរូ ៦

ដោយប្រើនិយមន័យបង្ហាញថា $\lim\limits_{x\to-\infty}\frac{x+5}{x}=1$។

ដំណោះស្រាយ

មេរៀនទី 1 មេរៀនសង្ខេប

ត្រូវរក $N>0$ ដែល $x<-N$ នាំឱ្យ $\left|\frac{x+5}{x}-1\right|<\varepsilon$។
គេមាន $\left|1+\frac{5}{x}-1\right|<\varepsilon \Leftrightarrow \frac{5}{|x|}<\varepsilon \Leftrightarrow |x|>\frac{5}{\varepsilon}$។
ដោយ $x<0$ នោះ $x<-\frac{5}{\varepsilon}$។
យក $N=\frac{5}{\varepsilon}$ ជាការស្រេច។
ដូចនេះ $\lim\limits_{x\to-\infty}\frac{x+5}{x}=1$។

១.៧ និយមន័យទី៦

បើអនុគមន៍ f ខិតជិត $+\infty$ កាលណា x ខិតជិត $+\infty$ បើចំពោះគ្រប់ចំនួន $M>0$ មាន $N>0$ ដែល $x>N$ នាំឱ្យ $f(x)>M$។ គេកំណត់សរសេរដោយ: $\lim\limits_{x\to+\infty}f(x)=+\infty$

ឧទាហរណ៍ ៧

បង្ហាញថា $\lim\limits_{x\to+\infty}x^2=+\infty$ ។

ដំណោះស្រាយ

ចំពោះគ្រប់ $M>0$ ត្រូវរក $N>0$ ដែល $x>N$ នាំឱ្យ $x^2>M$។
គេមាន $x^2>M \Rightarrow x>\sqrt{M}$ (ព្រោះ $x\to+\infty$)។
យក $N=\sqrt{M}$។

លំហាត់គំរូ ៧

ដោយប្រើនិយមន័យបង្ហាញថា $\lim\limits_{x\to+\infty}(x+2)=+\infty$។

ដំណោះស្រាយ

ត្រូវរក $N>0$ ដែល $x>N$ នាំឱ្យ $x+2>M$ ចំពោះគ្រប់ $M>0$។
គេមាន $x+2>M \Leftrightarrow x>M-2$។
ប្រសិនបើ $M>2$ យក $N=M-2$។ បើ $M\leq 2$ យក $N=1$ ជាដើម។ ជាទូទៅយក $N=|M-2|+1$។
ដូចនេះ $\lim\limits_{x\to+\infty}(x+2)=+\infty$។

១.៨ និយមន័យទី៧

បើអនុគមន៍ f ខិតជិត $-\infty$ កាលណា x ខិតជិត $+\infty$ បើចំពោះគ្រប់ចំនួន $M>0$ មាន $N>0$ ដែល $x>N$ នាំឱ្យ $f(x)<-M$។ គេកំណត់សរសេរដោយ: $\lim\limits_{x\to+\infty}f(x)=-\infty$

ឧទាហរណ៍ ៨

បង្ហាញថា $\lim\limits_{x\to+\infty}(-x)=-\infty$ ។

ដំណោះស្រាយ

ចំពោះគ្រប់ $M>0$ ត្រូវរក $N>0$ ដែល $x>N$ នាំឱ្យ $-x<-M$។
គេមាន $-x<-M\Rightarrow x>M$។
យក $N=M$។

លំហាត់គំរូ ៨

ដោយប្រើនិយមន័យបង្ហាញថា $\lim\limits_{x\to+\infty}(1-2x)=-\infty$។

ដំណោះស្រាយ

ត្រូវរក $N>0$ ដែល $x>N$ នាំឱ្យ $1-2x<-M$ ចំពោះគ្រប់ $M>0$។
គេមាន $1-2x<-M\Leftrightarrow -2x<-M-1\Leftrightarrow x>\frac{M+1}{2}$។
យក $N=\frac{M+1}{2}$។
ដូចនេះ $\lim\limits_{x\to+\infty}(1-2x)=-\infty$។

១.៩ និយមន័យទី៨

បើអនុគមន៍ f ខិតជិត $-\infty$ កាលណា x ខិតជិត $-\infty$ បើចំពោះគ្រប់ចំនួន $M>0$ មាន $N>0$ ដែល $x<-N$ នាំឱ្យ $f(x)<-M$។ គេកំណត់សរសេរដោយៈ $\lim\limits_{x\to-\infty}f(x)=-\infty$

ឧទាហរណ៍ ៩

បង្ហាញថា $\lim\limits_{x\to-\infty}x=-\infty$ ។

ដំណោះស្រាយ

ចំពោះគ្រប់ $M>0$ ត្រូវរក $N>0$ ដែល $x<-N$ នាំឱ្យ $x<-M$។
នេះជារឿងងាយ ដោយគ្រាន់តែយក $N=M$ នោះនឹងបានការ។

លំហាត់គំរូ ៩

ដោយប្រើនិយមន័យបង្ហាញថា $\lim\limits_{x\to-\infty}(2x+3)=-\infty$។

ដំណោះស្រាយ

ត្រូវរក $N>0$ ដែល $x<-N$ នាំឱ្យ $2x+3<-M$ ចំពោះគ្រប់ $M>0$។
គេមាន $2x+3<-M\Leftrightarrow 2x<-M-3\Leftrightarrow x<\frac{-M-3}{2}=-\frac{M+3}{2}$។

យក $N=\frac{M+3}{2}$។
ដូចនេះ $\lim\limits_{x\to-\infty}(2x+3)=-\infty$។

១.១០ និយមន័យទី៩

បើអនុគមន៍ f ខិតជិត $+\infty$ កាលណា x ខិតជិត $-\infty$ បើចំពោះគ្រប់ចំនួន $M>0$ មាន $N>0$ ដែល $x<-N$ នាំឱ្យ $f(x)>M$។ គេកំណត់សរសេរដោយ: $\lim\limits_{x\to-\infty}f(x)=+\infty$

ឧទាហរណ៍ ១០

បង្ហាញថា $\lim\limits_{x\to-\infty}(-x)=+\infty$ ។

ដំណោះស្រាយ

ចំពោះគ្រប់ $M>0$ ត្រូវរក $N>0$ ដែល $x<-N$ នាំឱ្យ $-x>M$។
គេមាន $-x>M\Rightarrow x<-M$។
យក $N=M$។

លំហាត់គំរូ ១០

ដោយប្រើនិយមន័យបង្ហាញថា $\lim\limits_{x\to-\infty}x^2=+\infty$។

ដំណោះស្រាយ

ត្រូវរក $N>0$ ដែល $x<-N$ នាំឱ្យ $x^2>M$ ចំពោះគ្រប់ $M>0$។
គេមាន $x^2>M\Leftrightarrow|x|>\sqrt{M}$។
ដោយ $x\to-\infty$ (ពោលគឺ $x<0$) នោះ $-x>\sqrt{M}\Leftrightarrow x<-\sqrt{M}$។
យក $N=\sqrt{M}$។
ដូចនេះ $\lim\limits_{x\to-\infty}x^2=+\infty$។

១.១១ ប្រមាណវិធីលើលីមីត

បើ $\lim\limits_{x\to a}f(x)=L$ និង $\lim\limits_{x\to a}g(x)=M$ ដែល $(L,M,k,a\in\mathbb{R})$ នោះគេបានប្រមាណវិធីលើលីមីតដូចខាងក្រោម៖

១. $\lim\limits_{x\to a}k=k$

២. $\lim\limits_{x\to a}[kf(x)]=k\lim\limits_{x\to a}f(x)=k\times L$, k ជាចំនួនថេរ

៣. $\lim\limits_{x\to a}[f(x)+g(x)]=\lim\limits_{x\to a}f(x)+\lim\limits_{x\to a}g(x)=L+M$

៤. $\lim\limits_{x\to a}[f(x)-g(x)]=\lim\limits_{x\to a}f(x)-\lim\limits_{x\to a}g(x)=L-M$

៥. $\lim\limits_{x\to a}[f(x)g(x)]=\lim\limits_{x\to a}f(x)\times\lim\limits_{x\to a}g(x)=M\times L$

៦. $\lim\limits_{x\to a}\frac{f(x)}{g(x)}=\frac{\lim\limits_{x\to a}f(x)}{\lim\limits_{x\to a}g(x)}=\frac{L}{M}$, ដែល $M\neq 0$, $g(a)\neq 0$

៧. $\lim\limits_{x\to a}[f(x)]^n=[\lim\limits_{x\to a}f(x)]^n=L^n$, n ជាចំនួនគត់រ៉ឺឡាទីប វិជ្ជមាន

១.១២ ទ្រឹស្តីបទ

១. បើ $P(x)$ ជាពហុធា និង a ជាចំនួនពិត នោះគេបាន: $\lim\limits_{x\to a}P(x)=P(a)$

២. បើ $Q(x)$ ជាអនុគមន៍សនិទានដែល $Q(x)=\frac{p(x)}{r(x)}$ និង a ជាចំនួនពិតនោះគេបាន:
$\lim\limits_{x\to a}Q(x)=Q(a)=\frac{p(a)}{r(a)}$ ដែល $r(a)\neq 0$

៣. បើ $a>0$ និង $n\in\mathbb{N}$ ឬ $a\leq 0$ និង n សេស គេបាន: $\lim\limits_{x\to a}\sqrt[n]{x}=\sqrt[n]{a}=(a)^{\frac{1}{n}}$

៤. បើ $\lim\limits_{x\to a}f(x)=L$ និង $n\in\mathbb{N}$ ឬ $\lim\limits_{x\to a}f(x)\leq 0$ និង n សេស គេបាន:
$\lim\limits_{x\to a}\sqrt[n]{f(x)}=\sqrt[n]{\lim\limits_{x\to a}f(x)}=\sqrt[n]{L}=L^{\frac{1}{n}}$

៥. បើ f និង g ជាអនុគមន៍ដែល $\lim\limits_{x\to a}g(x)=L$ និង $\lim\limits_{x\to a}f(x)=L$ គេបាន:
$\lim\limits_{x\to a}f[g(x)]=f(L)$

៦. បើ a ជាចំនួនពិតនៅក្នុងដែនកំណត់នៃអនុគមន៍ត្រីកោណមាត្រគេបាន:

ក. $\lim\limits_{x\to a}\cos x=\cos a$ ខ. $\lim\limits_{x\to a}\sin x=\sin a$ គ. $\lim\limits_{x\to a}\tan x=\tan a$

ឃ. $\lim\limits_{x\to a}\cot x=\cot a$

១.១៣ លីមីតខាងឆ្វេង និង ខាងស្តាំ

១. បើអនុគមន៍ $f(x)$ ខិតជិត L កាលណា x ខិតជិតx_0 ពីខាងឆ្វេងនោះ L ជាលីមីតឆ្វេងនៃ $f(x)$ ហើយគេកំណត់សរសេរដោយ:
$\lim\limits_{x\to x_0^-}f(x)=L$ ។

២. បើអនុគមន៍ $f(x)$ ខិតជិត R កាលណា x ខិតជិតx_0 ពីខាងស្តាំនោះ R ជាលីមីតស្តាំនៃ $f(x)$ ហើយគេកំណត់សរសេរដោយ:
$\lim\limits_{x\to x_0^+}f(x)=R$ ។

៣. បើអនុគមន៍ $y=f(x)$ មានលីមីតត្រង់ x_0 លុះត្រាតែលីមីតឆ្វេង ស្មើនឹង លីមីតស្តាំ។

១.១៤ លីមីតអនន្តនៃអនុគមន៍អសនិទាន

! រូបមន្តសំខាន់ៗ

ក. $\lim\limits_{x\to a}\sqrt[n]{x}=\sqrt[n]{a}$

ខ. $\lim\limits_{x\to a}\sqrt[n]{f(x)}=\sqrt[n]{\lim\limits_{x\to a}f(x)}$

១.១៥ លីមីតអនន្តនៃអនុគមន៍

! រូបមន្តសំខាន់ៗ

ក. $\lim\limits_{x\to\pm\infty}ax^2=\begin{cases}+\infty & , \text{ បើ } a>0\\ -\infty & , \text{ បើ } a<0\end{cases}$

ខ. $\lim\limits_{x\to\pm\infty}\frac{a}{x}=0\ ,\ \forall a\in\mathbb{R}$

គ. $\lim\limits_{x\to\pm\infty}\frac{x}{a}=\infty\ ,\ a\neq 0$

១.១៦ លីមីតនៃអនុគមន៍ត្រីកោណមាត្រ

! រូបមន្តសំខាន់ៗ

ក. $\lim\limits_{x\to 0}\frac{\sin x}{x}=\lim\limits_{x\to 0}\frac{x}{\sin x}=1$

ខ. $\lim\limits_{x\to 0}\frac{\sin ax}{ax}=\lim\limits_{x\to 0}\frac{ax}{\sin ax}=1$

គ. $\lim\limits_{x\to 0}\frac{\sin^2 ax}{x^2}=a^2$

ឃ. $\lim\limits_{x\to 0}\frac{\tan x}{x}=\lim\limits_{x\to 0}\frac{x}{\tan x}=1$

ង. $\lim\limits_{x\to 0}\frac{\tan ax}{ax}=\lim\limits_{x\to 0}\frac{ax}{\tan ax}=1$

ច. $\lim\limits_{x\to 0}\frac{1-\cos ax}{x^2}=\frac{a^2}{2}$

ឆ. $\lim\limits_{x\to 0}\frac{1-\cos x}{x}=0$

ជ. $\lim\limits_{x\to 0}\frac{1-\cos ax}{ax}=0$

១.១៧ លីមីតនៃអនុគមន៍អិចស្ប៉ូណង់ស្យែល

! រូបមន្តសំខាន់ៗ

ក. $\lim\limits_{x\to 0}\frac{e^x-1}{x}=1$

ខ. $\lim\limits_{x\to+\infty}\frac{e^x}{x^n}=+\infty\ ,\ n>0$

គ. $\lim\limits_{x\to+\infty}\frac{x^n}{e^x}=0\ ,\ n\geq 0$

ឃ. $\lim\limits_{x\to 0}\frac{a^x-1}{x}=\ln a\quad ,\quad (a>0)$

ង. $\lim\limits_{x\to 0}(1+x)^{\frac{1}{x}}=\lim\limits_{x\to\infty}(1+\frac{1}{x})^x=e$

ច. $\lim\limits_{x\to+\infty}a^x=\begin{cases}+\infty & \text{ បើ } a>1\\ 0 & \text{ បើ } 0<a<1\end{cases}$

១.១៨ លីមីតនៃអនុគមន៍លោការីត

! រូបមន្តសំខាន់ៗ

ក. $\lim\limits_{x\to0}\frac{\ln(1+x)}{x}=1$

ខ. $\lim\limits_{x\to0^+}\ln x=-\infty$

គ. $\lim\limits_{x\to+\infty}\ln x=+\infty$

ឃ. $\lim\limits_{x\to+\infty}\frac{\ln x}{x}=0$

ង. $\lim\limits_{x\to1^+}\frac{\ln x}{x-1}=1$

ច. $\lim\limits_{x\to0^+}x\ln x=0$

ឆ. $\lim\limits_{x\to0^+}x^n\ln x=0$

ជ. $\lim\limits_{x\to+\infty}\frac{\ln x}{x^n}=0\ ,\ n>0$

ឈ. $\lim\limits_{x\to+\infty}n\ln x=+\infty\ ,\ n>0$

ញ. $\lim\limits_{x\to0}\frac{\log_a(x+1)}{x}=\log_a e$

ដ. $\lim\limits_{x\to0^+}\log_a x$

$\begin{cases}-\infty, a>0\\+\infty, 0<a<1\end{cases}$

$=$ ឋ. $\lim\limits_{x\to+\infty}x=\begin{cases}+\infty, a>1\\-\infty, 0<a<1\end{cases}$

⚠ សម្គាល់

បើគេមាន $\lim\limits_{x\to\infty}|x|$ គេបាន៖

- បើ x ខិតជិត $+\infty$ នោះគេបាន $|x|=+\infty$
- បើ x ខិតជិត $-\infty$ នោះគេបាន $|x|=-\infty$

២ ភាពជាប់នៃអនុគមន៍

២.១ ភាពជាប់ត្រង់មួយចំនុច

បើអនុគមន៍ $y=f(x)$ ជាប់ត្រង់ចំនុច $x=a$ លុះត្រាតែ

១. $f(a)$ មានន័យ , ($f(a)$ ជាចំនួនពិត)

២. $\lim\limits_{x\to a}f(x)=n$ ដែល $n\in\mathbb{R}$, (មានលីមីតជាចំនួនពិត)

៣. $\lim\limits_{x\to a}f(x)=f(a)$, (តម្លៃលីមីត ស្មើនឹង តម្លៃអនុគមន៍ត្រង់ $x=a$)

⚠ សម្គាល់

ក្នុងលក្ខខណ្ឌខាងលើ៖

- បើអនុគមន៍មិនបានបំពេញលក្ខខណ្ឌណាមួយ ក្នុងចំណោមលក្ខខណ្ឌទាំង៣ ខាងលើនោះវាមិនជាប់ត្រង់ $x=a$ ទេ បានន័យថាវាដាច់ត្រង់ $x=a$ ។
- បើអនុគមន៍ $f(x)$ ដាច់ត្រង់ $x=a$ នោះវាមិនកំណត់ត្រង់ $x=a$ ឬ លីមីតឆ្វេង មិនស្មើ លីមីតស្តាំ ក្នុងករណីនេះ

គេមិនអាចគូសរូបដោយមិនលើកដៃបានទេ ។

២.២ លក្ខណៈអនុគមន៍ជាប់

បើ a ជាចំនួនពិត ហើយ $f(x)$ និង $g(x)$ ជាប់ត្រង់ចំនុច $x=a$ នោះគេបាន:

១. $f(x)+g(x)$ ជាប់ត្រង់ $x=a$

២. $f(x)-g(x)$ ជាប់ត្រង់ $x=a$

៣. $f(x)\times g(x)$ ជាប់ត្រង់ $x=a$

៤. $\frac{f(x)}{g(x)}$ ជាប់ត្រង់ $x=a$ ដែល $g(x)\neq 0$

៥. $cf(x)$ ជាប់ត្រង់ $x=a$ ដែល c ជាចំនួនថេរ

៦. $\frac{1}{f(x)}$ ជាប់ត្រង់ $x=a$ ដែល $f(x)\neq 0$

៧. $[f(x)]^n$ ដែល n ជាចំនួនពិត ជាប់ត្រង់ $x=a$ កាលណាវាកំណត់ត្រង់ a នោះ ។

សម្គាល់

បើ
១.អនុគមន៍លីនេអ៊ែ ជាអនុគមន៍ជាប់ចំពោះគ្រប់ចំនួនពិត ។
២.អនុគមន៍ពហុធា $P(x)=a_nx^n+a_{n-1}x^{n-1}+\ldots+a_1x+a_0$ ជាអនុគមន៍ជាប់ចំពោះគ្រប់តម្លៃ x ។
៣. អនុគមន៍សនិទាន $r(x)=\frac{p(x)}{q(x)}$, $q(x)\neq 0$ ជាអនុគមន៍ជាប់ចំពោះគ្រប់តម្លៃ x ក្នុងដែនកំណត់ ។
៤.អនុគមន៍ឫសទី n, $f(x)=\sqrt[n]{x}$ ជាអនុគមន៍ជាប់ចំពោះគ្រប់តម្លៃ x ក្នុងដែនកំណត់
៥.អនុគមន៍ត្រីកោណមាត្រ $\sin x$, $\cos x$ ជាអនុគមន៍ជាប់ចំពោះគ្រប់តម្លៃ x ។
៦.អនុគមន៍ត្រីកោណមាត្រ $\tan x,\cot x$ ជាអនុគមន៍ជាប់ចំពោះគ្រប់តម្លៃ x ក្នុងដែនកំណត់។

២.៣ ភាពជាប់នៃអនុគមន៍បណ្តាក់

បើអនុគមន៍ g ជាប់ត្រង់ a ហើយអនុគមន៍ f ជាប់ត្រង់ $g(x)$ នោះគេបាន អនុគមន៍បណ្តាក់ $f\circ g(x)$ ជាប់ត្រង់ a ។

២.៤ ភាពជាប់លើចន្លោះមួយ

១. អនុគមន៍ $f(x)$ ជាប់លើចន្លោះបើក $]a,b[$ លុះត្រាតែ $f(x)$ ជាប់ចំពោះគ្រប់ $x\in]a,b[$

២. អនុគមន៍ $f(x)$ ជាប់លើចន្លោះបិទ $[a,b]$ លុះត្រាតែ:

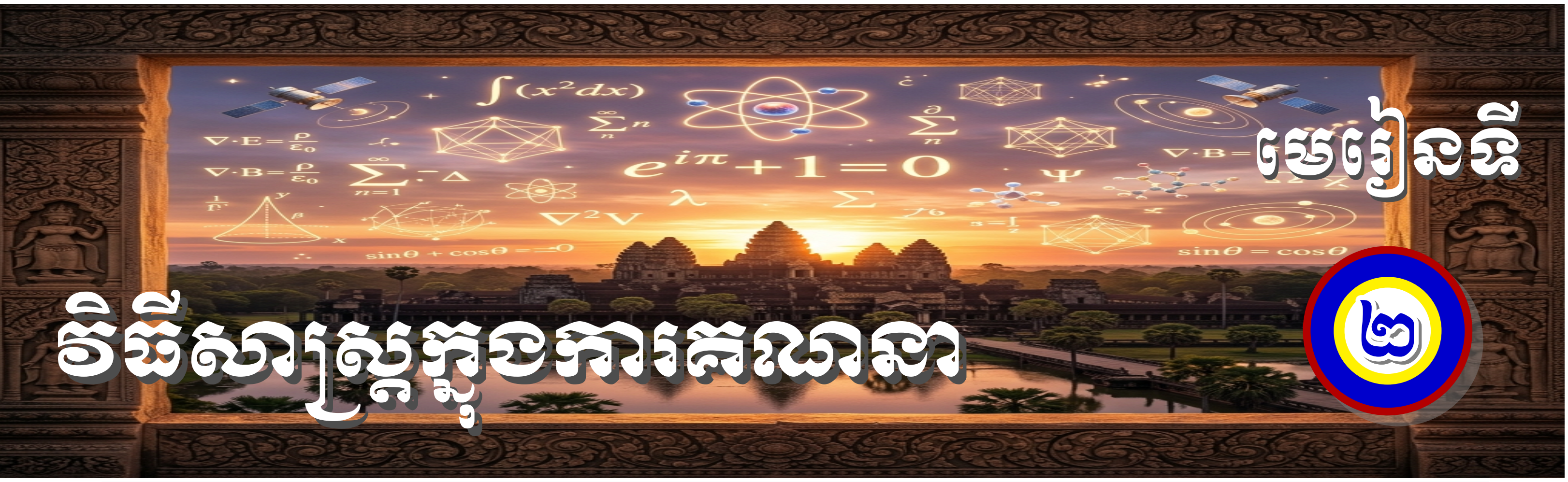
ក. $f(x)$ ជាប់លើចន្លោះបើក $]a,b[$

ខ. $f(x)$ ជាប់ឆ្វេងត្រង់ b $\left[\lim_{x\to b^-} f(x)=f(b)\right]$

គ. $f(x)$ ជាប់ស្តាំត្រង់ a $\left[\lim_{x\to a^+} f(x)=f(a)\right]$

២.៥ ទ្រឹស្តីបទតម្លៃកណ្តាល

បើ $f(x)$ ជាប់លើចន្លោះ $[a,b]$ ហើយ $f(a).f(b)<0$ នោះយ៉ាងហោចណាស់ មានចំនួន c មួយនៅចន្លោះ a និង b ដែល $f(c)=0$ ។



មេរៀនទី ២
វិធីសាស្ត្រក្នុងការគណនា

១ ការគណនាលីមីតដោយប្រើនិយមន័យ

១.១ ករណីទី១

⚠ សម្គាល់

បើអនុគមន៍ $f(x)$ មានលីមីត L កាលណា x ខិតជិត c បើគ្រប់ចំនួនមានចំនួន$\alpha>0$ មាន $\varepsilon>0$ ដែល$0<|x-c|<\alpha$ នាំឱ្យ $|f(x)-L|<\varepsilon$ ។
គេកំណត់សរសេរដោយៈ $\lim\limits_{x\to c} f(x)=L$ ។

លំហាត់គំរូ ១

ដោយប្រើនិយមន័យបង្ហាញថា

ក. $\lim\limits_{x\to 2}(2001x-2000)=2002$

ខ. $\lim\limits_{x\to 2}(2x-1)=3$

គ. $\lim\limits_{x\to 4}(x^3-x^2-x)=-4$

ឃ. $\lim\limits_{x\to 3}\frac{x+1}{x-1}=2$

ដំណោះស្រាយ

ដោយប្រើនិយមន័យបង្ហាញថា

ក. $\lim\limits_{x\to 2}(2001x-2000)=2002$
យើងត្រូវបង្ហាញថាគ្រប់ចំនួន $\varepsilon>0$ យើងបាន $|f(x)-L|<\varepsilon$
យើងមាន $|f(x)-L|<\varepsilon$

យើងបាន$|f(x)-L|<\varepsilon \Leftrightarrow |(2001x-2000)-2002|<\varepsilon$

$$\Leftrightarrow |2001x-4002|<\varepsilon$$
$$\Leftrightarrow |2001(x-2)|<\varepsilon$$
$$\Leftrightarrow |x-2|<\frac{\varepsilon}{2001}=\alpha \quad ;\text{យក}\ \alpha=\frac{\varepsilon}{2001}$$

ដោយ$\forall\ \varepsilon>0$, $\exists\ \alpha=\frac{\varepsilon}{2001}>0$ ដែល $|x-2|<\alpha$ នាំឱ្យ $|f(x)-L|<\varepsilon$

ដូចនេះ $\boxed{\lim\limits_{x\to 2}(2001x-2000)=2002}$

ខ. $\lim\limits_{x\to 2}(2x-1)=3$

យើងត្រូវបង្ហាញថាគ្រប់ចំនួន $\varepsilon>0$ យើងបាន $|f(x)-L|<\varepsilon$

យើងមាន $|f(x)-L|<\varepsilon$

យើងបាន$|f(x)-L|<\varepsilon \Leftrightarrow |(2x-1)-3|<\varepsilon$

$$\Leftrightarrow |2x-4|<\varepsilon$$
$$\Leftrightarrow |2(x-2)|<\varepsilon$$
$$\Leftrightarrow |x-2|<\frac{\varepsilon}{2}=\alpha \quad ;\text{យក}\,\alpha=\frac{\varepsilon}{2}$$

ដោយ $\forall\varepsilon>0$ $\exists$ $\alpha=\frac{\varepsilon}{2}>0$, ដែល $|x-2|<\alpha$ នាំឱ្យ $|f(x)-L|<\alpha$

ដូចនេះ $\boxed{\lim\limits_{x\to 2}(2x-1)=3}$

គ. $\lim\limits_{x\to 4}(x^3-x^2-x)=-4$

យើងត្រូវបង្ហាញថាគ្រប់ចំនួន $\varepsilon>0$ យើងបាន $|f(x)-L|<\varepsilon$

យើងមាន $|f(x)-L|<\varepsilon$ យើងបាន

$$|f(x)-L|<\varepsilon \Leftrightarrow |(x^3-4x^2-x)+4|<\varepsilon$$
$$\Leftrightarrow |x^3-4x^2-x+4|<\varepsilon$$
$$\Leftrightarrow |x^2(x-4)-(x-4)|<\varepsilon$$
$$\Leftrightarrow |(x-4)(x^2-1)|<\varepsilon$$

តែ$x\to 4$យើងបាន$3<x<5 \Leftrightarrow 9<x^2<25$

$$\Leftrightarrow 9-1<x^2-1<25-1$$
$$\Leftrightarrow 8<x^2-1<24$$
$$\Leftrightarrow 8<|x^2-1|<24$$
$$\Leftrightarrow |x^2-1|<24$$
$$\Leftrightarrow |x-4||x^2-1|<|x-4|24$$

ដើម្បីបាន$|x-4||x^2-1|<\varepsilon$ យើងគ្រាន់តែយក

$|x-4|24<\varepsilon$ ឬ $|x-4|<\frac{\varepsilon}{24}=\alpha$;យក$\alpha=\frac{\varepsilon}{24}$

ដោយ $\forall\varepsilon>0$, $\exists$ $\alpha=\frac{\varepsilon}{24}>0$ ដែល $|x-4|<\varepsilon$ នាំឱ្យ $|f(x)-L|<\varepsilon$

ដូចនេះ $\boxed{\lim\limits_{x\to 4}(x^3-x^2-x)=-4}$

ឃ. $\lim\limits_{x\to 3}\frac{x+1}{x-1}=2$

យើងត្រូវបង្ហាញថាគ្រប់ចំនួន $\varepsilon>0$ យើងបាន $|f(x)-L|<\varepsilon$

យើងមាន $|f(x)-L|<\varepsilon$

យើងបាន : $|f(x)-L|<\varepsilon \Leftrightarrow \left|\frac{x+1}{x-1}-2\right|<\varepsilon$

$$\Leftrightarrow \left|\frac{(x+1)-2(x-1)}{x-1}\right|<\varepsilon$$
$$\Leftrightarrow \left|\frac{x+1-2x+2}{x-1}\right|<\varepsilon$$
$$\Leftrightarrow \left|\frac{-x+3}{x-1}\right|<\varepsilon$$
$$\Leftrightarrow \frac{|x-3|}{|x-1|}$$
$$\Leftrightarrow |x-3|=\varepsilon|x-1|(*)$$

ដោយ$x\to 3$យើងបាន : $2<x<4 \Leftrightarrow 2-1<x-1<4-1$

$$\Leftrightarrow 1<x-1<3$$
$$\Leftrightarrow 1<|x-1|<3$$
$$\Leftrightarrow |x-1|<3$$
$$\Leftrightarrow \varepsilon|x-1|<3\varepsilon(**)$$

តាម $(*)$ និង $(**)$ យើងបាន $|x-3|<3\varepsilon$

ដោយ $\forall\,\varepsilon>0$, $\exists\,\alpha=3\varepsilon$, ដែល $|x-3|<\varepsilon$ នាំឱ្យ $|f(x)-L|<\varepsilon$

ដូចនេះ $\boxed{\lim\limits_{x\to 3}\frac{x+1}{x-1}=2}$

១.២ ករណីទី២

សម្គាល់

បើអនុគមន៍f ខិតទៅជិត$+\infty$ កាលណាx ខិតទៅជិត a បើចំពោះចំនួន $M>0$
មាន$\alpha>0$ ដែល $0<|x-a|<\alpha$ នាំឱ្យ $f(x)>M$ ។
គេកំណត់សរសេរដោយ $\lim\limits_{x\to a}f(x)=+\infty$ ។

លំហាត់គំរូ ២

ដោយប្រើនិយមន័យបង្ហាញថា

ក. $\lim\limits_{x\to 1^+}\frac{7x+2}{x-1}=+\infty$ ខ. $\lim\limits_{x\to 1^-}\frac{x+8}{1-x}=+\infty$

មេរៀនទី ២ • វិធីសាស្ត្រក្នុងការគណនា

ដំណោះស្រាយ

ដោយប្រើនិយមន័យបង្ហាញថា

ក. $\lim\limits_{x\to1^+}\frac{7x+2}{x-1}=+\infty$

តាមនិយមន័យ: $\forall M>0\ ,\ f(x)>M$

$$f(x)>M\Leftrightarrow\frac{7x+2}{x-1}>M$$
$$\Leftrightarrow 7+\frac{9}{x-1}>M$$
$$\Leftrightarrow\frac{9}{x-1}>M-7$$
$$\Leftrightarrow x-1<\frac{9}{M-7}$$

ដោយ $x\to1^+$ នោះយើងបាន $x-1>0$ នាំឱ្យ $0<x-1<\frac{9}{M-7}$, ដែល $\alpha=\frac{9}{M-7}>0$

យើងបាន $\forall M>0\ ,\ \exists\ \alpha=\frac{9}{M}$ ដែល$|x-1|<\alpha$នាំឱ្យ$|f(x)|>M$

ដូចនេះ $\boxed{\lim\limits_{x\to1^+}\frac{7x+2}{x-1}=+\infty}$

ខ. $\lim\limits_{x\to1^-}\frac{x+8}{1-x}=+\infty$

តាមនិយមន័យ: $\forall M>0\ ,\ f(x)>M$

$$f(x)>M\Leftrightarrow\frac{x+8}{1-x}>M$$
$$\Leftrightarrow -1+\frac{9}{1-x}>M$$
$$\Leftrightarrow\frac{9}{1-x}>M+1$$
$$\Leftrightarrow x-1<\frac{9}{M+1}$$

ដោយ $x\to1^-$ នោះយើងបាន $1-x>0$ នាំឱ្យ $0<1-x<\frac{9}{M+1}$, ដែល $\alpha=\frac{9}{M+1}>0$

យើងបាន $\forall M>0\ ,\ \exists\ \alpha=\frac{9}{M+1}$ ដែល$|x-1|<\alpha$នាំឱ្យ$|f(x)|>M$

ដូចនេះ $\boxed{\lim\limits_{x\to1^-}\frac{x+8}{1-x}=+\infty}$

១.៣ ករណីទី៣

សម្គាល់

បើអនុគមន៍f ខិតទៅជិត$-\infty$ កាលណាx ខិតទៅជិត a បើចំពោះចំនួន $M>0$ មាន$\alpha>0$ ដែល $0<|x-a|<\alpha$ នាំឱ្យ $f(x)<-M$ ។ គេកំណត់សរសេរដោយ $\lim\limits_{x\to a}f(x)=-\infty$ ។

លំហាត់គំរូ ៣

ដោយប្រើនិយមន័យបង្ហាញថា

ក. $\lim\limits_{x\to0^-}\frac{x+1}{x}=-\infty$

ខ. $\lim\limits_{x\to2^+}\frac{3x+3}{2-x}=-\infty$

ដំណោះស្រាយ

ដោយប្រើនិយមន័យបង្ហាញថា

ក. $\lim\limits_{x\to0^-}\frac{x+1}{x}=-\infty$

តាមនិយមន័យ: $\forall M>0\ ,\ f(x)<-M$

$$f(x)<-M\Leftrightarrow\frac{x+1}{x}<-M$$
$$\Leftrightarrow 1+\frac{1}{x}<-M$$
$$\Leftrightarrow\frac{1}{x}<-M-1$$
$$\Leftrightarrow\frac{1}{|x|}<|-M-1|$$
$$\Leftrightarrow|x|<\frac{1}{M+1}$$

យក $\alpha = \frac{1}{M+1} > 0$
យើងបាន $\forall M > 0$,$\exists \alpha = \frac{1}{M+1}$ ដែល $|x| < \alpha$ នាំឱ្យ $|f(x)| < -M$
ដូចនេះ $\boxed{\lim\limits_{x\to 0^-} \frac{x+1}{x} = -\infty}$

ខ. $\lim\limits_{x\to 2^+} \frac{3x+3}{2-x} = -\infty$ តាមនិយមន័យ: $\forall M > 0$, $f(x) < -M$

$$f(x) < -M \Leftrightarrow \frac{3x+3}{2-x} < -M$$
$$\Leftrightarrow -3 - \frac{9}{x-2} < -M$$

ដើម្បីឲ្យ $f(x) < -M$, $\forall M > 0$; យើងគ្រាន់តែយក $-\frac{9}{x-2} < -M$
យើងបាន
$\left|-\frac{9}{x-2}\right| < |-M| \Leftrightarrow |x-2| < \frac{9}{M}$ យក $\alpha = \frac{9}{M}$
យើងបាន $\forall M > 0$ $\exists \alpha = \frac{9}{M}$ ដែល $|x-2| < \alpha$ នាំឱ្យ $f(x) < -M$
ដូចនេះ $\boxed{\lim\limits_{x\to 2^+} \frac{3x+3}{2-x} = -\infty}$

១.៤ ករណីទី៤

សម្គាល់

បើអនុគមន៍ f ខិតជិត L កាលណា x ខិតជិត $+\infty$ បើចំពោះគ្រប់ចំនួន $\varepsilon > 0$ មាន $N > 0$ ដែល $x > N$ នាំឱ្យ $|f(x) - L| < \varepsilon$។ គេកំណត់សរសេរដោយ: $\lim\limits_{x\to +\infty} f(x) = L$ ។

លំហាត់គំរូ ៤

ដោយប្រើនិយមន័យបង្ហាញថា

ក. $\lim\limits_{x\to +\infty} \frac{1}{x^2} = 0$

ខ. $\lim\limits_{x\to +\infty} \frac{2x+4}{x} = 2$

ដំណោះស្រាយ

ក. $\lim\limits_{x\to +\infty} \frac{1}{x^2} = 0$
តាមនិយមន័យ $\forall \varepsilon > 0$, $|f(x) - L| < \varepsilon$
យើងបាន $|f(x) - L| < \varepsilon \Leftrightarrow \left|\frac{1}{x^2} - 0\right| < \varepsilon$

$$\Leftrightarrow \left|\frac{1}{x^2}\right| < \varepsilon$$
$$\Leftrightarrow \frac{1}{|x^2|} < \varepsilon$$
$$\Leftrightarrow x^2 > \frac{1}{\varepsilon}$$
$$\Leftrightarrow |x| > \sqrt{\frac{1}{\varepsilon}}$$

ដោយ $x \to +\infty$ យើងបាន $|x| = x$ នាំឱ្យ $x > \sqrt{\frac{1}{\varepsilon}}$ យើងយក $N = \sqrt{\frac{1}{\varepsilon}} > 0$
យើងបាន $\forall \varepsilon > 0$, $\exists N = \sqrt{\frac{1}{\varepsilon}}$ ដែល $x > N$ នាំឱ្យ $|f(x) - L| < \varepsilon$

ដូចនេះ $\boxed{\lim\limits_{x\to +\infty} \frac{1}{x^2} = 0}$

ខ. $\lim\limits_{x\to +\infty} \frac{2x+4}{x} = 2$
តាមនីយមន័យ $\forall \varepsilon > 0$, $|f(x) - L| < \varepsilon$
យើងបាន

$$|f(x) - L| < \varepsilon \Leftrightarrow \left|\frac{2x+4}{x} - 2\right| < \varepsilon$$
$$\Leftrightarrow \left|\frac{2x+4-2x}{x}\right| < \varepsilon$$
$$\Leftrightarrow \frac{4}{x} < \varepsilon$$
$$\Leftrightarrow |x| > \frac{4}{\varepsilon}$$

ដោយ $x \to +\infty$ យើងបាន $|x| = x$ នាំឱ្យ $x > \sqrt{\frac{4}{\varepsilon}}$ យើងយក $N = \sqrt{\frac{4}{\varepsilon}} > 0$
យើងបាន $\forall \varepsilon > 0$, $\exists N = \sqrt{\frac{4}{\varepsilon}}$ ដែល $x > N$ នាំឱ្យ $|f(x) - L| < \varepsilon$

ដូចនេះ $\boxed{\lim\limits_{x\to +\infty} \frac{2x+4}{x} = 2}$

១.៥ ករណីទី៥

⚠ សម្គាល់

បើអនុគមន៍ f ខិតជិត L កាលណា x ខិតជិត $-\infty$ បើចំពោះគ្រប់ចំនួន $\varepsilon>0$ មាន $N>0$ ដែល $x<-N$ នាំឱ្យ $|f(x)-L|<\varepsilon$។ គេកំណត់សរសេរដោយៈ $\lim\limits_{x\to-\infty} f(x)=L$ ។

លំហាត់គំរូ ៥

ដោយប្រើនិយមន័យបង្ហាញថា

ក. $\lim\limits_{x\to-\infty}\frac{1}{x}=0$ ខ. $\lim\limits_{x\to-\infty}\frac{3x-1}{x-1}=3$

ដំណោះស្រាយ

ដោយប្រើនិយមន័យបង្ហាញថា

ក. $\lim\limits_{x\to-\infty}\frac{1}{x}=0$

តាមនិយមន័យ $\forall\varepsilon>0$, $|f(x)-L|<\varepsilon$

យើងបាន$|f(x)-L|<\varepsilon=\left|\frac{1}{x}-0\right|<\varepsilon$

$$\Leftrightarrow \left|\frac{1}{x}\right|<\varepsilon$$

$$\Leftrightarrow \frac{1}{|x|}<\varepsilon$$

$$\Leftrightarrow |x|<\frac{1}{\varepsilon}$$

☒☒☒ $x\to-\infty$ យើងបាន $|x|=-x$ នាំឱ្យ $x<-\frac{1}{\varepsilon}$ យើងយក $N=\frac{1}{\varepsilon}>0$

យើងបាន$\forall\ \varepsilon>0, \exists=\frac{1}{\varepsilon}>0$ ដែល $x<-N$ នាំឱ្យ $|f(x)-L|<\varepsilon$

ដូចនេះ $\boxed{\lim\limits_{x\to-\infty}\frac{1}{x}=0}$

ខ. $\lim\limits_{x\to-\infty}\frac{3x-1}{x-1}=3$

តាមនិយមន័យ $\forall\varepsilon>0$, $|f(x)-L|<\varepsilon$

យើងបាន

$$|f(x)-L|<\varepsilon \Leftrightarrow \left|\frac{3x-1}{x-1}-3\right|<\varepsilon$$

$$\Leftrightarrow \left|\frac{(3x-1)-3(x-1)}{x-1}\right|<\varepsilon$$

$$\Leftrightarrow \left|\frac{3x-1-3x+3}{x-1}\right|<\varepsilon$$

$$\Leftrightarrow \left|\frac{4}{x-1}\right|<\varepsilon$$

$$\Leftrightarrow |x-1|>\frac{2}{\varepsilon}$$

ដោយ $x\to-\infty$ យើងបាន $|x-1|=-(x-1)$ នាំឱ្យ $x-1<-\frac{2}{\varepsilon}$

យើងយក $N=\frac{2}{\varepsilon}>0$

យើងបាន$\forall\ \varepsilon>0$, $\exists N=\frac{2}{\varepsilon}>0$ ដែល $x>-N$ នាំឱ្យ $|f(x)-L|<\varepsilon$

ដូចនេះ $\boxed{\lim\limits_{x\to-\infty}\frac{3x-1}{x-1}=3}$

១.៦ ករណីទី៦

⚠ សម្គាល់

បើអនុគមន៍ f ខិតជិត $+\infty$ កាលណា x ខិតជិត $+\infty$ បើចំពោះគ្រប់ចំនួន $M>0$ មាន $N>0$ ដែល $x>N$ នាំឱ្យ $f(x)>M$។

គេកំណត់សរសេរដោយៈ $\lim\limits_{x\to+\infty} f(x) = +\infty$ ។

លំហាត់គំរូ ៦

ដោយប្រើនិយមន័យបង្ហាញថា

ក. $\lim\limits_{x\to+\infty} \frac{x+2x^2}{3x+5} = +\infty$ ខ. $\lim\limits_{x\to+\infty} \frac{5x^2+4x-1}{x+2} = +\infty$

ដំណោះស្រាយ

ដោយប្រើនិយមន័យបង្ហាញថា

ក. $\lim\limits_{x\to+\infty} \frac{x+2x^2}{3x+5} = +\infty$

តាមនិយមន័យ $\forall M>0$, $f(x)>0$

យើងបាន $f(x)>M \Leftrightarrow \dfrac{x+2x^2}{3x+5}>M$

$\Leftrightarrow \dfrac{x+2x^2}{3x+5}>M$

$\Leftrightarrow \dfrac{2}{3}x-\dfrac{7}{9}+\dfrac{\frac{35}{9}}{3x+5}>M$

ដោយ $x\to+\infty$ នោះយើងបាន $\frac{\frac{35}{9}}{3x+5}\to 0$

យើងបាន $\frac{2}{3}x-\frac{7}{9}+\frac{\frac{35}{9}}{3x+5}>M \Leftrightarrow \frac{2}{3}>M+\frac{7}{9}$

$\Rightarrow x>\frac{2}{3}\left(M+\frac{7}{9}\right)$ យើងយក $N=\frac{2}{3}\left(M+\frac{7}{9}\right)>0$

យើង បាន $\forall\ M>0$, $\exists N=\frac{2}{3}\left(M+\frac{7}{9}\right)$ ដែល $x>N$ នាំឱ្យ $f(x)>M$

ដូចនេះ $\boxed{\lim\limits_{x\to+\infty} \frac{x+2x^2}{3x+5} = +\infty}$

ខ. $\lim\limits_{x\to+\infty} \frac{5x^2+4x-1}{x+2} = +\infty$

តាមនិយមន័យ $\forall M>0$, $f(x)>0$

យើងបាន

$f(x)>M \Leftrightarrow \dfrac{5x^2+4x-1}{x+2}>M$

$\Leftrightarrow 5x-6+\dfrac{11}{x+2}>M$

ដោយ $x\to+\infty$ នោះយើងបាន $\frac{11}{x+2}\to 0$

យើងបាន $5x-6+\frac{11}{x+2}>M \Leftrightarrow 5x-6>M \Leftrightarrow 5x>M+6$

$\Rightarrow x>\frac{M+6}{5}$ យក $N=\frac{M+6}{5}>0$

យើងបាន $\forall M>0$, $\exists N=\frac{M+6}{5}$ ដែល $x>N$នាំឱ្យ $f(x)>0$

ដូចនេះ $\boxed{\lim\limits_{x\to+\infty} \frac{5x^2+4x-1}{x+2} = +\infty}$

១.៧ ករណីទី៧

⚠ សម្គាល់

បើអនុគមន៍ f ខិតជិត $-\infty$ កាលណា x ខិតជិត $+\infty$ បើចំពោះគ្រប់ចំនួន $M>0$ មាន $N>0$ ដែល $x>N$ នាំឱ្យ $f(x)<-M$។
គេកំណត់សរសេរដោយៈ $\lim\limits_{x\to+\infty} f(x) = -\infty$ ។

លំហាត់គំរូ ៧

ដោយប្រើនិយមន័យបង្ហាញថា

ក. $\lim\limits_{x\to+\infty} (-x^2) = -\infty$ ខ. $\lim\limits_{x\to+\infty} \frac{x^3-x+3}{1-x^2} = -\infty$

ដំណោះស្រាយ

ដោយប្រើនិយមន័យបង្ហាញថា

ក. $\lim_{x\to+\infty}(-x^2)=-\infty$

តាមនិយមន័យ $\forall M>0$, $f(x)<-M$

$f(x)<-M \Leftrightarrow -x^2<-M$

$\Leftrightarrow x^2>M$

$\Leftrightarrow |x|>\sqrt{M}$

ដោយ $x\to+\infty$ នោះ $|x|=x \Rightarrow x>\sqrt{M}$ យើងយក $N=\sqrt{M}>0$

យើងបាន$\forall M>0$, $\exists N=\sqrt{M}$, ដែល $x>M$ នាំឱ្យ $f(x)<-M$

ដូចនេះ $\boxed{\lim_{x\to+\infty}(-x^2)=-\infty}$

ខ. $\lim_{x\to+\infty}\frac{x^3-x+3}{1-x^2}=-\infty$

តាមនិយមន័យ $\forall M>0$, $f(x)<-M$

$f(x)<-M \Leftrightarrow \dfrac{x^3-x+3}{1-x^2}<-M$

$\Leftrightarrow -x+\dfrac{3}{1-x^2}<-M$

ដោយ $x\to+\infty$ នោះយើងបាន $\frac{3}{1-x^2}\to 0$

យើងបាន $-x+\frac{3}{1-x^2}<-M \Leftrightarrow -x<-M$

$\Rightarrow x>M$ យក $N=M>0$

យើងបាន $\forall M>0$, $\exists N=M$ ដែល $x>N$ នាំឱ្យ $f(x)<-M$

ដូចនេះ $\boxed{\lim_{x\to+\infty}\frac{x^3-x+3}{1-x^2}=-\infty}$

១.៨ ករណីទី៨

សម្គាល់

បើអនុគមន៍ f ខិតជិត $-\infty$ កាលណា x ខិតជិត $-\infty$ បើចំពោះគ្រប់ចំនួន $M>0$ មាន $N>0$ ដែល $x<-N$ នាំឱ្យ $f(x)<-M$។ គេកំណត់សរសេរដោយ: $\lim_{x\to-\infty}f(x)=-\infty$ ។

លំហាត់គំរូ ៨

ដោយប្រើនិយមន័យបង្ហាញថា

ក. $\lim_{x\to-\infty}(1-x^2)=-\infty$

ខ. $\lim_{x\to-\infty}\frac{x^2+3}{x}=-\infty$

ដំណោះស្រាយ

ដោយប្រើនិយមន័យបង្ហាញថា

ក. $\lim_{x\to-\infty}(1-x^2)=-\infty$

តាមនិយមន័យ $\forall M>0$, $f(x)<-M$

$f(x)<-M \Leftrightarrow 1-x^2<-M$

$\Leftrightarrow x^2>M+1$

$\Leftrightarrow |x|>\sqrt{M+1}$

ដោយ $x\to-\infty$ នោះ $|x|=-x$

យើងបាន $-x>\sqrt{M+1} \Leftrightarrow x<-\sqrt{M+1}$ យក $N=\sqrt{M+1}>0$

យើង បាន $\forall M>0$, $\exists N=\sqrt{M+1}$ ដែល $x<-N$ នាំ ឱ្យ $f(x)<-M$

ដូចនេះ $\boxed{\lim_{x\to-\infty}(1-x^2)=-\infty}$

ខ. $\lim_{x\to-\infty}\frac{x^2+3}{x}=-\infty$

តាមនិយមន័យ $\forall M>0$, $f(x)<-M$

$f(x)<-M \Leftrightarrow \dfrac{x^2+3}{x}<-M$

$\Leftrightarrow x+\dfrac{3}{x}<-M$

ដោយ $x\to-\infty \Leftrightarrow \frac{3}{x}\to 0$ យើងបាន $x<-M$ យក$N=M>0$

យើងបាន $\forall M>0$, $\exists N=M$ ដែល $x>N$ នាំឱ្យ $f(x)<-M$

ដូចនេះ $\boxed{\lim_{x\to-\infty}\frac{x^2+3}{x}=-\infty}$

១.៩ ករណីទី៩

សម្គាល់

បើអនុគមន៍ f ខិតជិត $+\infty$ កាលណា x ខិតជិត $-\infty$ បើចំពោះគ្រប់ចំនួន $M>0$ មាន $N>0$ ដែល $x<-N$ នាំឱ្យ $f(x)>M$ ។
គេកំណត់សរសេរដោយៈ $\lim\limits_{x\to-\infty} f(x)=+\infty$ ។

លំហាត់គំរូ ៩

ដោយប្រើនិយមន័យបង្ហាញថា

ក. $\lim\limits_{x\to-\infty}(x^2+2x+3)=+\infty$ ខ. $\lim\limits_{x\to-\infty}\frac{-x^2+6x}{2x+1}=+\infty$

ដំណោះស្រាយ

ដោយប្រើនិយមន័យបង្ហាញថា

ក. $\lim\limits_{x\to-\infty}(x^2+2x+3)=+\infty$

តាមនិយមន័យ $\forall M>0$, $f(x)>M$

$$f(x)>M \Leftrightarrow x^2+2x+3>M$$
$$\Leftrightarrow (x+1)^2+2>M$$
$$\Leftrightarrow (x+1)^2>M$$
$$\Leftrightarrow |x+1|>\sqrt{M}$$

ដោយ $x\to-\infty$ នោះ $|x+1|=-(x+1)$

យើងបាន $|x+1|>\sqrt{M}\Leftrightarrow x+1<-\sqrt{M}$

$\Rightarrow x<-(\sqrt{M}+1)$ យក $N=\sqrt{M}+1$

យើង បាន $\forall M>0$, $\exists N=(\sqrt{M}+1)$ ដែល $x<-N$ នាំឱ្យ $f(x)<-M$

ដូចនេះៈ $\boxed{\lim\limits_{x\to-\infty}(x^2+2x+3)=+\infty}$

ខ. $\lim\limits_{x\to-\infty}\frac{-x^2+6x}{2x+1}=+\infty$

$$f(x)>M\Leftrightarrow \frac{-x^2+6x}{2x+1}>M$$
$$\Leftrightarrow \frac{-x^2+6x}{2x+1}>M$$
$$\Leftrightarrow -\frac{1}{2}x+\frac{13}{4}-\frac{\frac{13}{4}}{2x+1}>M$$

ដោយ $x\to-\infty$ នោះយើងបាន $\frac{\frac{13}{4}}{2x+1}\to 0$

ដើម្បីបាន $f(x)>M$ យើងគ្រាន់តែយក $-\frac{1}{2}x>M\Rightarrow x<2M$ យក $N=2M>0$

យើងបាន $\forall M>0$, $\exists N=2M$, ដែល $x<-N$ នាំឱ្យ $f(x)>M$

ដូចនេះៈ $\boxed{\lim\limits_{x\to-\infty}\frac{-x^2+6x}{2x+1}=+\infty}$

២ របៀបគណនាលីមីត តាមការដាក់ជាផលគុណកត្តា

២.១ របៀប គណនា លីមីត តាម ការ ដាក់ជា ផលគុណ កត្តា រាងមិនកំណត់ $\frac{0}{0}$

សម្គាល់

• ប្រសិនបើលីមីតមានរាង $\lim_{x\to a}\frac{f(x)}{g(x)}$ ដែល $f(x)$ និង $g(x)$ ខិតជិត 0 នោះលីមីតរាងមិនកំណត់ $\frac{0}{0}$ ។ ក្នុងករណីនេះដើម្បីគណនា គេត្រូវបំបែកភាគយក និង ភាគបែងជាផលគុណកត្តាសិន បន្ទាប់មកសម្រួលកត្តាដែលដូចគ្នា រួចគណនាលីមីតថ្មីដែលនៅសល់។
• បើសិនជាសម្រួលហើយនៅតែរាងមិនកំណត់ $\frac{0}{0}$ ទៀតត្រូវដាក់ជាកត្តាដើម្បីសម្រួលបន្តទៀត រួចគណនាលីមីតថ្មីដែលនៅសល់។

រូបមន្តណែនាំសម្រាប់គណនាលំហាត់

សម្គាល់

ដើម្បីងាយស្រួលក្នុងការគណនាត្រូវចាំរូបមន្តមួយចំនួនដូចខាងក្រោម៖

1. $a^2-b^2=(a+b)(a-b)$
2. $a^3-b^3=(a-b)(a^2+ab+b^2)$
3. $a^3+b^3=(a+b)(a^2-ab+b^3)$
4. $a^4-b^4=(a^2-b^2)(a^2+b^2)=(a-b)(a+b)(a^2+b^2)$
5. $a^4+b^4=(a^2-ib^2)(a^2+ib^2)$
6. $a^5-b^5=(a-b)(a^4+a^3b+a^2b^2+ab^3+b^4)$
7. $a^5+b^5=(a+b)(a^4-a^3b+a^2b^2-ab^3+b^4)$

ក្នុងករណី n ជាចំនួនសេស

8. $a^n+b^n=(a+b)(a^{n-1}-a^{n-2}b+a^{n-3}b^2-...-ab^{n-2}+b^{n-1})$

ក្នុងករណី n ជាចំនួនគូ

9. $a^n-b^n=(a-b)(a^{n-1}+a^{n-2}b+a^{n-3}b^2+...+ab^{n-2}+b^{n-1})$
10. $a^n+b^n=(a+b)(a^{n-1}-a^{n-2}b+a^{n-3}b^2-...+ab^{n-2}-b^{n-1})$

លំហាត់គំរូ ១០

គណនាលីមីតខាងក្រោម៖

ក. $\lim_{x\to -1}\frac{x^5+1}{x^3+1}$ ខ. $\lim_{x\to 1}\frac{x^4-4x+3}{x^3-x^2-x+1}$ គ. $\lim_{x\to 2}\frac{x^6-x^2-60}{x^2-4}$

ឃ. $\lim_{x\to 3}\frac{x^2-5x+6}{x^2-9}$

ដំណោះស្រាយ

គណនាលីមីតខាងក្រោម៖

ក. $\lim_{x\to -1}\frac{x^5+1}{x^3+1}$, រាងមិនកំណត់ $\frac{0}{0}$

តាមរូបមន្ត a^n+b^n យើងមាន

- $x^5+1=(x+1)(x^4-x^3+x^2-x+1)$
- $x^3+1=(x+1)(x^2-x+1)$

យើងបាន
$$\lim_{x\to -1}\frac{x^5+1}{x^3+1}=\lim_{x\to -1}\frac{\cancel{(x+1)}(x^4-x^3+x^2-x+1)}{\cancel{(x+1)}(x^2-x+1)}$$
$$=\lim_{x\to -1}\frac{x^4-x^3+x^2-x+1}{x^2-x+1}$$
$$=\frac{(-1)^4-(-1)^3+(-1)^2-(-1)+1}{(-1)^2-(-1)+1}$$
$$=\frac{5}{3}$$

ដូចនេះ $\boxed{\lim\limits_{x\to -1}\frac{x^5+1}{x^3+1}=\frac{5}{3}}$

ខ. $\lim\limits_{x\to 1}\frac{x^4-4x+3}{x^3-x^2-x+1}$, រាងមិនកំណត់ $\frac{0}{0}$

$$=\lim_{x\to 1}\frac{(x^4-x)-3(x-1)}{x^2(x-1)-(x-1)}$$
$$=\lim_{x\to 1}\frac{x(x-1)(x^2+x+1)-3(x-1)}{(x-1)(x^2-1)}$$
$$=\lim_{x\to 1}\frac{\cancel{(x-1)}[x(x^2+x+1)-3]}{\cancel{(x-1)}(x^2-1)}$$
$$=\lim_{x\to 1}\frac{x^3+x^2+x-3}{x^2-1}$$, (លីមីតថ្មីនៅតែរាងមិនកំណត់ $\frac{0}{0}$)
$$=\lim_{x\to 1}\frac{(x^3-1)+(x^2-1)+(x-1)}{(x-1)(x+1)}$$
$$=\lim_{x\to 1}\frac{\cancel{(x-1)}(x^2+x+1)+(x-1)(x+1)+(x-1)}{\cancel{(x-1)}(x+1)}$$
$$=\lim_{x\to 1}\frac{(x-1)[(x^2+x+1)+(x+1)+1]}{(x-1)(x+1)}$$
$$=\lim_{x\to 1}\frac{(x^2+x+1)+(x+1)+1]}{x+1}$$
$$=\lim_{x\to 1}\frac{(1^2+1+1)+(1+1)+1}{1+1}=3$$

ដូចនេះ $\boxed{\lim\limits_{x\to 1}\frac{x^4-4x+3}{x^3-x^2-x+1}=3}$

គ. $\lim\limits_{x\to 2}\frac{x^6-x^2-60}{x^2-4}$, រាងមិនកំណត់ $\frac{0}{0}$

$$\lim_{x\to 2}\frac{\cancel{(x^2-4)}(x^4+4x^2+15)}{\cancel{x^2-4}}$$
$$\lim_{x\to 2}x^4+4x^2+15=16+16+15=47$$

ដូចនេះ $\boxed{\lim\limits_{x\to 2}\frac{x^6-x^2-60}{x^2-4}=47}$

ឃ. $\lim\limits_{x\to 3}\frac{x^2-5x+6}{x^2-9}$, រាងមិនកំណត់ $\frac{0}{0}$

យើងមាន

- $x^2-5x+6=(x-3)(x-2)$ និង $(x^2-9)=(x-3)(x+3)$

យើងបាន $\lim\limits_{x\to 3}\frac{x^2-5x+6}{x^2-9}=\lim\limits_{x\to 3}\frac{\cancel{(x-3)}(x-2)}{\cancel{(x-3)}(x+3)}$

$$=\lim_{x\to 3}\frac{x-2}{x+3}$$
$$=\frac{3-2}{3+3}=\frac{1}{6}$$

ដូចនេះ $\boxed{\lim\limits_{x\to 3}\frac{x^2-5x+6}{x^2-9}=\frac{1}{6}}$

៣ របៀបគណនាលីមីតនៃអនុគមន៍អសនិទាន

៣.១ របៀបគណនាលីមីតនៃអនុគមន៍អសនិទាន រាង $\sqrt{A}-B$

សម្គាល់

ប្រសិនបើកន្សោមមានរាង $\frac{\sqrt{A}-B}{F(x)}$ ឬ $\frac{G(x)}{\sqrt{A}-B}$ យើងត្រូវគុណនឹងកន្សោមឆ្លាស់គឺ $\sqrt{A}+B$ យើងបាន$(\sqrt{A}-B)(\sqrt{A}+B)=A-B^2$ ឬ $\sqrt{A}-B=\frac{A-B^2}{\sqrt{A}+B}$

លំហាត់គំរូ ១១

គណនាលីមីតខាងក្រោម៖

ក. $\lim\limits_{x\to 2}\frac{\sqrt{x^3+1}-3}{x^2-2x}$

ខ. $\lim\limits_{x\to 2}\frac{\sqrt{x^2-3}-1}{\sqrt{x+2}-2}$

គ. $\lim\limits_{x\to 1}\frac{x+1-\sqrt{x^2+3}}{\sqrt{x}-1}$

ឃ. $\lim\limits_{x\to 3}\frac{x^2-9}{x-1-\sqrt{x+1}}$

ដំណោះស្រាយ

គណនាលីមីតខាងក្រោម៖

ក. $\lim\limits_{x\to 2}\frac{\sqrt{x^3+1}-3}{x^2-2x}$

យើងមាន៖

- $x^2-2x=x(x-2)$
- $\sqrt{x^3+1}-3=\sqrt{x^3+1}-3\times\frac{\sqrt{x^3+1}+3}{\sqrt{x^3+1}+3}$

$$=\frac{x^3+1-3^2}{\sqrt{x^3+1}+3}$$
$$=\frac{x^3-8}{\sqrt{x^3+1}+3}$$
$$=\frac{(x-2)(x^2+2x+4)}{\sqrt{x^3+1}+3}$$

យើងបាន $\lim\limits_{x\to 2}\frac{\sqrt{x^3+1}-3}{x^2-2x}=\lim\limits_{x\to 2}\frac{\cancel{(x-2)}(x^2+2x+4)}{x\cancel{(x-2)}(\sqrt{x^3+1}+3)}$

$$=\lim_{x\to 2}\frac{x^2+2x+4}{x(\sqrt{x^3+1}+3)}$$
$$=\frac{2^2+2\times 2+4}{2(\sqrt{2^3+1}+3)}$$
$$=1$$

ដូចនេះ៖ $\boxed{\lim\limits_{x\to 2}\frac{\sqrt{x^3+1}-3}{x^2-2x}=1}$

ខ. $\lim\limits_{x\to 2}\frac{\sqrt{x^2-3}-1}{\sqrt{x+2}-2}$, (គុណកន្សោមឆ្លាស់អង្គទាំងពីរ)

យើងបាន $\frac{\sqrt{x^2-3}-1}{\sqrt{x+2}-2}=\frac{\sqrt{x^2-3}-1}{\sqrt{x+2}-2}\times\frac{\sqrt{x^2-3}+1}{\sqrt{x^2-3}+1}\times\frac{\sqrt{x+2}+2}{\sqrt{x+2}+2}$

$$=\frac{(x^2-3-1)(\sqrt{x+2}+2)}{(x+2-4)(\sqrt{x^2-3}+1)}$$
$$=\frac{(x^2-4)(\sqrt{x+2}+2)}{(x-2)(\sqrt{x^2-3}+1)}$$
$$=\frac{\cancel{(x-2)}(x+2)(\sqrt{x+2}+2)}{\cancel{(x-2)}(\sqrt{x^2-3}+1)}$$
$$=\frac{(x+2)(\sqrt{x+2}+2)}{(\sqrt{x^2-3}+1)}$$

យើងបាន $\lim\limits_{x\to 2}\frac{\sqrt{x^2-3}-1}{\sqrt{x+2}-2}=\lim\limits_{x\to 2}\frac{(x+2)(\sqrt{x+2}+2)}{(\sqrt{x^2-3}+1)}$

$$=\frac{(2+2)(\sqrt{2+2}+2)}{(\sqrt{2^2-3}+1)}$$
$$=\frac{4\times 4}{2}=8$$

ដូចនេះ៖ $\boxed{\lim\limits_{x\to 2}\frac{\sqrt{x^2-3}-1}{\sqrt{x+2}-2}=8}$

គ. $\lim\limits_{x\to 1}\frac{x+1-\sqrt{x^2+3}}{\sqrt{x}-1}$, (គុណកន្សោមឆ្លាស់អង្គទាំងពីរ) យើងមាន៖

$$\frac{x+1-\sqrt{x^2+3}}{\sqrt{x}-1}=\frac{x+1-\sqrt{x^2+3}}{\sqrt{x}-1}\times\frac{x+1+\sqrt{x^2+3}}{x+1+\sqrt{x^2+3}}\times\frac{\sqrt{x}+1}{\sqrt{x}+1}$$
$$=\frac{[(x+1)^2-(x^2+3)](\sqrt{x}+1)}{(x-1)(x+1+\sqrt{x^2+3})}$$
$$=\frac{2(x-1)(\sqrt{x}+1)}{(x-1)(x+1+\sqrt{x^2+3})}$$
$$=\frac{2\cancel{(x-1)}(\sqrt{x}+1)}{\cancel{(x-1)}(x+1+\sqrt{x^2+3})}$$
$$=\frac{2(\sqrt{x}+1)}{x+1+\sqrt{x^2+3}}$$
$$=\frac{2(\sqrt{1}+1)}{1+1+\sqrt{1^2+3}}$$
$$=1$$

ដូចនេះ៖ $\boxed{\lim\limits_{x\to 1}\frac{x+1-\sqrt{x^2+3}}{\sqrt{x}-1}=1}$

ឃ. $\lim\limits_{x\to 3}\frac{x^2-9}{x-1-\sqrt{x+1}}$, (គុណកន្សោមឆ្លាស់អង្គទាំងពីរ)

យើងបាន $\frac{x^2-9}{x-1-\sqrt{x+1}}=\frac{x^2-9}{x-1-\sqrt{x+1}}\times\frac{x-1+\sqrt{x+1}}{x-1+\sqrt{x+1}}$

$$=\frac{(x^2-9)(x-1+\sqrt{x+1})}{(x-1)^2-(x+1)}$$
$$=\frac{\cancel{(x-3)}(x+3)(x-1+\sqrt{x+1})}{x\cancel{(x-3)}}$$
$$=\frac{(x+3)(x-1+\sqrt{x+1})}{x}$$
$$=\frac{(3+3)(3-1+\sqrt{3+1})}{3}$$
$$=8$$

ដូចនេះ៖ $\boxed{\lim\limits_{x\to 3}\frac{x^2-9}{x-1-\sqrt{x+1}}=8}$

៣.២ របៀបគណនាលីមីតនៃអនុគមន៍អសនិទាន រាង $A-\sqrt{B}$

សម្គាល់

ប្រសិនបើកន្សោមរាង $\frac{A-\sqrt{B}}{F(x)}$ ឬ $\frac{G(x)}{A-\sqrt{B}}$ យើងត្រូវ គុណនឹងកន្សោមឆ្លាស់របស់វាគឺ៖ $A+\sqrt{B}$ យើងបាន៖

$(A-\sqrt{B})(A+\sqrt{B}) = A^2 - B$ ឬ $(A-\sqrt{B}) = \frac{A^2-B}{A+\sqrt{B}}$ ។

លំហាត់គំរូ ១២

គណនាលីមីតខាងក្រោម៖

ក. $\lim\limits_{x\to 0}\frac{2-\sqrt{x+4}}{x}$

ខ. $\lim\limits_{x\to 2}\frac{-3+\sqrt{x+7}}{x-2}$

គ. $\lim\limits_{x\to 3}\frac{x-\sqrt{2x+3}}{x-3}$

ឃ. $\lim\limits_{x\to 1}\frac{(x-1)^2}{x-\sqrt{2x-1}}$

ដំណោះស្រាយ

គណនាលីមីតខាងក្រោម៖

៣.៣ របៀបគណនាលីមីតនៃអនុគមន៍អសនិទាន រាង $\sqrt{A}-\sqrt{B}$

សម្គាល់

ប្រសិនបើកន្សោមរាង $\frac{\sqrt{A}-\sqrt{B}}{F(x)}$ ឬ $\frac{G(x)}{\sqrt{A}-\sqrt{B}}$ យើងត្រូវ គុណនឹងកន្សោមឆ្លាស់របស់វាគឺ៖ $\sqrt{A}+\sqrt{B}$ យើងបាន៖
$(\sqrt{A}-\sqrt{B})(\sqrt{A}+\sqrt{B}) = A-B$ ឬ $\sqrt{A}-\sqrt{B} = \frac{A-B}{\sqrt{A}+\sqrt{B}}$ ។

លំហាត់គំរូ ១៣

គណនាលីមីតខាងក្រោម៖

ក. $\lim\limits_{x\to 4}\frac{\sqrt{2x+1}-3}{\sqrt{x-2}-\sqrt{2}}$

ខ. $\lim\limits_{x\to 0}\frac{\sqrt{1+x}+\sqrt{x+4}-3}{x}$

គ. $\lim\limits_{x\to 1}\frac{\sqrt{x+3}-\sqrt{3x+1}}{\sqrt{x}-1}$

ឃ. $\lim\limits_{x\to 1}\frac{\sqrt{x^2+3x}-\sqrt{3x+1}}{\sqrt{x}-1}$

ដំណោះស្រាយ

គណនាលីមីតខាងក្រោម៖

ក. $\lim\limits_{x\to 4}\frac{\sqrt{2x+1}-3}{\sqrt{x-2}-\sqrt{2}}$; (មានរាងមិនកំណត់ $\frac{0}{0}$)

$$\begin{aligned}
\lim_{x\to 4}\frac{\sqrt{2x+1}-3}{\sqrt{x-2}-\sqrt{2}} &= \lim_{x\to 4}\left[\frac{\sqrt{2x+1}-3}{\sqrt{x-2}-\sqrt{2}}\times\frac{\sqrt{2x+1}+3}{\sqrt{2x+1}+3}\times\frac{\sqrt{x-2}+\sqrt{2}}{\sqrt{x-2}+\sqrt{2}}\right]\\
&= \lim_{x\to 4}\frac{(2x+1-3^2)(\sqrt{x-2}+\sqrt{2})}{(x-2-2)(\sqrt{2x+1}+3)}\\
&= \lim_{x\to 4}\frac{2\cancel{(x-4)}(\sqrt{x-2}+\sqrt{2})}{\cancel{(x-4)}(\sqrt{2x+1}+3)}\\
&= \lim_{x\to 4}\frac{2(\sqrt{x-2}+\sqrt{2})}{(\sqrt{2x+1}+3)}\\
&= \frac{2\times 2\sqrt{2}}{3+3} = \frac{2\sqrt{2}}{3}
\end{aligned}$$

ដូចនេះ $\boxed{\lim\limits_{x\to4}\frac{\sqrt{2x+1}-3}{\sqrt{x-2}-\sqrt{2}}=\frac{2\sqrt{2}}{3}}$

ខ. $\lim\limits_{x\to0}\frac{\sqrt{1+x}+\sqrt{x+4}-3}{x}$; (មានរាងមិនកំណត់ $\frac{0}{0}$)

ដោយ $\lim\limits_{x\to0}\frac{\sqrt{1+x}+\sqrt{x+4}-3}{x}=\lim\limits_{x\to0}\frac{\sqrt{1+x}-1+\sqrt{x+4}-2}{x}$

អាចសរសេរទៅជា:

$\lim\limits_{x\to0}\frac{\sqrt{1+x}-1+\sqrt{x+4}-2}{x}=\lim\limits_{x\to0}\frac{\sqrt{1+x}-1}{x}+\lim\limits_{x\to0}\frac{\sqrt{x+4}-2}{x}$

• ករណី $\lim\limits_{x\to0}\frac{\sqrt{1+x}-1}{x}$

$$=\lim_{x\to0}\frac{(\sqrt{1+x}-1)(\sqrt{1+x}+1)}{x(\sqrt{1+x}+1)}$$
$$=\lim_{x\to0}\frac{1+x-1}{x(\sqrt{1+x}+1)}$$
$$=\lim_{x\to0}\frac{\cancel{x}}{\cancel{x}(\sqrt{1+x}+1)}$$
$$=\lim_{x\to0}\frac{1}{(\sqrt{1+x}+1)}$$
$$=\frac{1}{(\sqrt{1+0}+1)}=\frac{1}{2}$$

• ករណី $\lim\limits_{x\to0}\frac{\sqrt{x+4}-2}{x}$

$$=\lim_{x\to0}\frac{(\sqrt{x+4}-2)(\sqrt{x+4}+2)}{x(\sqrt{x+4}+2)}$$
$$=\lim_{x\to0}\frac{x+4-4}{x(\sqrt{x+4}+2)}$$
$$=\lim_{x\to0}\frac{1}{(\sqrt{x+4}+2)}$$
$$=\frac{1}{(\sqrt{0+4}+2)}=\frac{1}{2+2}=\frac{1}{4}$$

នាំឱ្យ យើងបាន៖ $\lim\limits_{x\to0}\frac{\sqrt{1+x}-1}{x}+\lim\limits_{x\to0}\frac{\sqrt{x+4}-2}{x}=\frac{1}{2}+\frac{1}{4}=\frac{3}{4}$

ដូចនេះ $\boxed{\lim\limits_{x\to0}\frac{\sqrt{1+x}+\sqrt{x+4}-3}{x}=\frac{3}{4}}$

គ. $\lim\limits_{x\to1}\frac{\sqrt{x+3}-\sqrt{3x+1}}{\sqrt{x}-1}$; (មានរាងមិនកំណត់ $\frac{0}{0}$)

$$=\lim_{x\to1}\frac{(\sqrt{x+3}-\sqrt{3x+1})(\sqrt{x+3}+\sqrt{3x+1})(\sqrt{x}-1)}{(\sqrt{x}-1)(\sqrt{x+3}+\sqrt{3x+1})(\sqrt{x}-1)}$$
$$=\lim_{x\to1}\frac{[(x+3)-(3x+1)](\sqrt{x}-1)}{(x-1)(\sqrt{x+3}+\sqrt{3x+1})}$$
$$=\lim_{x\to1}\frac{(x+3-3x-1)(\sqrt{x}-1)}{(x-1)(\sqrt{x+3}+\sqrt{3x+1})}$$
$$=\lim_{x\to1}\frac{(2-2x)(\sqrt{x}-1)}{(x-1)(\sqrt{x+3}+\sqrt{3x+1})}$$
$$=\lim_{x\to1}\frac{-2\cancel{(x-1)}(\sqrt{x}-1)}{\cancel{(x-1)}(\sqrt{x+3}+\sqrt{3x+1})}$$
$$=\lim_{x\to1}\frac{-2(\sqrt{x}-1)}{(\sqrt{x+3}+\sqrt{3x+1})}$$
$$=\frac{-2(\sqrt{1}-1)}{(\sqrt{1+3}+\sqrt{3\times1+1})}$$
$$=\frac{-2\times\sqrt{0}}{2+2}=0$$

ដូចនេះ $\boxed{\lim\limits_{x\to1}\frac{\sqrt{x+3}-\sqrt{3x+1}}{\sqrt{x}-1}=0}$

ឃ. $\lim\limits_{x\to1}\frac{\sqrt{x^2+3x}-\sqrt{3x+1}}{\sqrt{x}-1}$; (មានរាងមិនកំណត់ $\frac{0}{0}$)

$$=\lim_{x\to1}\frac{(\sqrt{x^2+3x}-\sqrt{3x+1})(\sqrt{x^2+3x}+\sqrt{3x+1})(\sqrt{x}+1)}{(\sqrt{x}-1)(\sqrt{x^2+3x}+\sqrt{3x+1})(\sqrt{x}+1)}$$
$$=\lim_{x\to1}\frac{[(x^2+3x)-(3x+1)](\sqrt{x}+1)}{(x-1)(\sqrt{x^2+3x}+\sqrt{3x+1})}$$
$$=\lim_{x\to1}\frac{(x^2+3x-3x-1)(\sqrt{x}+1)}{(x-1)(\sqrt{x^2+3x}+\sqrt{3x+1})}$$
$$=\lim_{x\to1}\frac{\cancel{(x-1)}(x+1)(\sqrt{x}+1)}{\cancel{(x-1)}(\sqrt{x^2+3x}+\sqrt{3x+1})}$$
$$=\lim_{x\to1}\frac{(x+1)(\sqrt{x}+1)}{\sqrt{x^2+3x}+\sqrt{3x+1}}$$
$$=\frac{(1+1)(\sqrt{1}+1)}{\sqrt{1^2+3\times1}+\sqrt{3\times1+1}}=1$$

ដូចនេះ $\boxed{\lim\limits_{x\to1}\frac{\sqrt{x^2+3x}-\sqrt{3x+1}}{\sqrt{x}-1}=1}$

៣.៤ របៀបគណនាលីមីតនៃអនុគមន៍អសនិទាន រាង $\sqrt[3]{A}-B$

សម្គាល់

ប្រសិនបើកន្សោមមានរាង $\frac{\sqrt[3]{A}-B}{F(x)}$ ឬ $\frac{G(x)}{\sqrt[3]{A}-B}$ យើងត្រូវគុណនឹងកន្សោមឆ្លាស់របស់វាគឺ $(\sqrt[3]{A^2}+B\sqrt[3]{A}+B^2)$ ។ យើងបាន $(\sqrt[3]{A}-B)(\sqrt[3]{A^2}+B\sqrt[3]{A}+B^2)=A-B^3$ ឬ $\sqrt[3]{A}-B=\frac{A-B^3}{\sqrt[3]{A^2}+B\sqrt[3]{A}+B^2}$

លំហាត់គំរូ ១៤

គណនាលីមីតខាងក្រោម៖

ក. $\lim\limits_{x\to8}\frac{\sqrt[3]{x}-2}{x-8}$

ខ. $\lim\limits_{x\to1}\frac{\sqrt[3]{x}-1}{\sqrt[3]{4x+4}-2}$

គ. $\lim\limits_{x\to-1}\frac{\sqrt[3]{x}+1}{\sqrt{x^2+3}-2}$

ឃ. $\lim\limits_{x\to0}\frac{\sqrt[3]{1+x^2}-1}{x^2}$

ដំណោះស្រាយ

គណនាលីមីតខាងក្រោម៖

ក. $\lim\limits_{x\to 8}\frac{\sqrt[3]{x}-2}{x-8}$; រាងមិនកំណត់ $\frac{0}{0}$

$$\begin{aligned}\lim_{x\to 8}\frac{\sqrt[3]{x}-2}{x-8}&=\lim_{x\to 8}\frac{(\sqrt[3]{x}-2)(\sqrt[3]{x^2}+2\sqrt[3]{x}+4)}{(x-8)(\sqrt[3]{x^2}+2\sqrt[3]{x}+4)}\\&=\lim_{x\to 8}\frac{\cancel{x-8}}{\cancel{(x-8)}(\sqrt[3]{x^2}+2\sqrt[3]{x}+4)}\\&=\lim_{x\to 8}\frac{1}{(\sqrt[3]{x^2}+2\sqrt[3]{x}+4)}\\&=\frac{1}{(\sqrt[3]{8^2}+2\sqrt[3]{8}+4)}\\&=\frac{1}{12}\end{aligned}$$

ដូចនេះ $\boxed{\lim\limits_{x\to 8}\frac{\sqrt[3]{x}-2}{x-8}=\frac{1}{12}}$

ខ. $\lim\limits_{x\to 1}\frac{\sqrt[3]{x}-1}{\sqrt[3]{4x+4}-2}$; រាងមិនកំណត់ $\frac{0}{0}$

$$\begin{aligned}&=\lim_{x\to 1}\frac{(\sqrt[3]{x}-1)(\sqrt[3]{x^2}+\sqrt[3]{x}+1)(\sqrt[3]{(4x+4)^2}+2\sqrt[3]{4x+4}+4)}{(\sqrt[3]{4x+4}-2)(\sqrt[3]{x^2}+\sqrt[3]{x}+1)(\sqrt[3]{(4x+4)^2}+2\sqrt[3]{4x+4}+4)}\\&=\lim_{x\to 1}\frac{(x-1)(\sqrt[3]{(4x+4)^2}+2\sqrt[3]{4x+4}+4)}{(4x+4-8)(\sqrt[3]{x^2}+\sqrt[3]{x}+1)}\\&=\lim_{x\to 1}\frac{(x-1)(\sqrt[3]{(4x+4)^2}+2\sqrt[3]{4x+4}+4)}{(4x-4)(\sqrt[3]{x^2}+\sqrt[3]{x}+1)}\\&=\lim_{x\to 1}\frac{\cancel{(x-1)}(\sqrt[3]{(4x+4)^2}+2\sqrt[3]{4x+4}+4)}{4\cancel{(x-1)}(\sqrt[3]{x^2}+\sqrt[3]{x}+1)}\\&=\lim_{x\to 1}\frac{(\sqrt[3]{(4x+4)^2}+2\sqrt[3]{4x+4}+4)}{4(\sqrt[3]{x^2}+\sqrt[3]{x}+1)}\\&=\frac{(\sqrt[3]{(4+4)^2}+2\sqrt[3]{4\times 1+4}+4)}{4(\sqrt[3]{1^2}+\sqrt[3]{1}+1)}=1\end{aligned}$$

ដូចនេះ $\boxed{\lim\limits_{x\to 1}\frac{\sqrt[3]{x}-1}{\sqrt[3]{4x+4}-2}=1}$

គ. $\lim\limits_{x\to -1}\frac{\sqrt[3]{x}+1}{\sqrt{x^2+3}-2}$; រាងមិនកំណត់ $\frac{0}{0}$

$$\begin{aligned}\lim_{x\to -1}\frac{\sqrt[3]{x}+1}{\sqrt{x^2+3}-2}&=\lim_{x\to -1}\frac{(\sqrt[3]{x}+1)(\sqrt[3]{x^2}-\sqrt[3]{x}+1)(\sqrt{x^2+3}+2)}{(\sqrt{x^2+3}-2)(\sqrt[3]{x^2}-\sqrt[3]{x}+1)(\sqrt{x^2+3}+2)}\\&=\lim_{x\to -1}\frac{(x+1)(\sqrt{x^2+3}+2)}{(x^2+3-4)(\sqrt[3]{x^2}-\sqrt[3]{x}+1)}\\&=\lim_{x\to -1}\frac{(x+1)(\sqrt{x^2+3}+2)}{(x^2-1)(\sqrt[3]{x^2}-\sqrt[3]{x}+1)}\\&=\lim_{x\to -1}\frac{(\sqrt{x^2+3}+2)}{(x-1)(\sqrt[3]{x^2}-\sqrt[3]{x}+1)}\\&=\frac{\sqrt{(-1)^2+3}+2}{(-1-1)(\sqrt[3]{(-1)^2}-\sqrt[3]{-1}+1)}\\&=-\frac{2}{3}\end{aligned}$$

ដូចនេះ $\boxed{\lim\limits_{x\to -1}\frac{\sqrt[3]{x}+1}{\sqrt{x^2+3}-2}=-\frac{2}{3}}$

ឃ. $\lim\limits_{x\to 0}\frac{\sqrt[3]{1+x^2}-1}{x^2}$; រាងមិនកំណត់ $\frac{0}{0}$

$$\begin{aligned}\lim_{x\to 0}\frac{\sqrt[3]{1+x^2}-1}{x^2}&=\lim_{x\to 0}\frac{(\sqrt[3]{1+x^2}-1)(\sqrt[3]{(1+x^2)^2}+\sqrt[3]{1+x^2}+1)}{x^2(\sqrt[3]{(1+x^2)^2}+\sqrt[3]{1+x^2}+1)}\\&=\lim_{x\to 0}\frac{(1+x^2-1)}{x^2(\sqrt[3]{(1+x^2)^2}+\sqrt[3]{1+x^2}+1)}\\&=\lim_{x\to 0}\frac{\cancel{x^2}}{\cancel{x^2}\left(\sqrt[3]{(1+x^2)^2}+\sqrt[3]{1+x^2}+1\right)}\\&=\lim_{x\to 0}\frac{1}{(\sqrt[3]{(1+x^2)^2}+\sqrt[3]{1+x^2}+1)}\\&=\frac{1}{(\sqrt[3]{(1+0^2)^2}+\sqrt[3]{1+0^2}+1)}\\&=\frac{1}{3}\end{aligned}$$

ដូចនេះ $\boxed{\lim\limits_{x\to 0}\frac{\sqrt[3]{1+x^2}-1}{x^2}=\frac{1}{3}}$

៣.៥ របៀបគណនាលីមីតនៃអនុគមន៍អសនិទាន រាង $A-\sqrt[3]{B}$

សម្គាល់

ប្រសិនបើកន្សោមមានរាង $\frac{A-\sqrt[3]{B}}{F(x)}$ ឬ $\frac{G(x)}{A-\sqrt[3]{B}}$ យើងត្រូវគុណនឹងកន្សោមឆ្លាស់របស់វាគឺ $A^2+A\sqrt[3]{B}+\sqrt[3]{B^2}$ ។ យើងបាន $(A-\sqrt[3]{B})(A^2+A\sqrt[3]{B}+\sqrt[3]{B^2})=A^3-B$ ឬ $A-\sqrt[3]{B}=\frac{A^3-B}{A^2+A\sqrt[3]{B}+\sqrt[3]{B^2}}$ ។

លំហាត់គំរូ ១៥

គណនាលីមីតខាងក្រោម៖

ក. $\lim\limits_{x\to 2}\frac{x^3-\sqrt{x^2+60}}{x^2-\sqrt[3]{x^2+60}}$

ខ. $\lim\limits_{x\to 2}\frac{x-\sqrt[3]{x^2+4}}{x-2}$

គ. $\lim\limits_{x\to 0}\frac{1-\sqrt[3]{1-x}}{3x}$

ឃ. $\lim\limits_{x\to 3}\frac{x-3}{x-1-\sqrt[3]{x^2-1}}$

មេរៀនទី ២ វិធីសាស្ត្រក្នុងការគណនា

ដំណោះស្រាយ

គណនាលីមីតខាងក្រោម៖

ក. $\lim\limits_{x\to 2}\frac{x^3-\sqrt{x^2+60}}{x^2-\sqrt[3]{x^2+60}}$; រាងមិនកំណត់ $\frac{0}{0}$

$$
\begin{aligned}
&= \lim_{x\to 2}\frac{(x^3-\sqrt{x^2+60})(x^3+\sqrt{x^2+60})[x^4+x^2\sqrt[3]{x^2+60}+\sqrt[3]{(x^2+60)^2}]}{(x^2-\sqrt[3]{x^2+60})(x^3+\sqrt{x^2+60})[x^4+x^2\sqrt[3]{x^2+60}+\sqrt[3]{(x^2+60)^2}]}\\
&= \lim_{x\to 2}\frac{\cancel{(x^6-x^2-60)}[x^4+x^2\sqrt[3]{x^2+60}+\sqrt[3]{(x^2+60)^2}]}{\cancel{(x^6-x^2-60)}(x^3+\sqrt{x^2+60})}\\
&= \lim_{x\to 2}\frac{x^4+x^2\sqrt[3]{x^2+60}+\sqrt[3]{(x^2+60)^2}}{x^3+\sqrt{x^2+60}}\\
&= \frac{2^4+2^2\sqrt[3]{2^2+60}+\sqrt[3]{(2^2+60)^2}}{(2^3+\sqrt{2^2+60})}\\
&= \frac{16+16+16}{16}\\
&= 3
\end{aligned}
$$

ដូចនេះ $\boxed{\lim\limits_{x\to 2}\frac{x^3-\sqrt{x^2+60}}{x^2-\sqrt[3]{x^2+60}}=3}$

ខ. $\lim\limits_{x\to 2}\frac{x-\sqrt[3]{x^2-4}}{x-2}$; រាងមិនកំណត់ $\frac{0}{0}$

$$
\begin{aligned}
\lim_{x\to 2}\frac{x-\sqrt[3]{x^2-4}}{x-2} &= \lim_{x\to 2}\frac{(x-\sqrt[3]{x^2+4})[x^2+x\sqrt[3]{x^2+4}+\sqrt[3]{(x^2+4)^2}]}{(x-2)[x^2+x\sqrt[3]{x^2+4}+\sqrt[3]{(x^2+4)^2}]}\\
&= \lim_{x\to 2}\frac{x^3-x^2-4}{(x-2)[x^2+x\sqrt[3]{x^2+4}+\sqrt[3]{(x^2+4)^2}]}\\
&= \lim_{x\to 2}\frac{(x^3-2x^2)+(x^2-4)}{(x-2)[x^2+x\sqrt[3]{x^2+4}+\sqrt[3]{(x^2+4)^2}]}\\
&= \lim_{x\to 2}\frac{\cancel{(x-2)}(x^2+x+2)}{\cancel{(x-2)}[x^2+x\sqrt[3]{x^2+4}+\sqrt[3]{(x^2+4)^2}]}\\
&= \lim_{x\to 2}\frac{x^2+x+2}{x^2+x\sqrt[3]{x^2+4}+\sqrt[3]{(x^2+4)^2}}\\
&= \frac{2^2+2+2}{2^2+2\sqrt[3]{2^2+4}+\sqrt[3]{(2^2+4)^2}}\\
&= \frac{2}{3}
\end{aligned}
$$

ដូចនេះ $\boxed{\lim\limits_{x\to 2}\frac{x-\sqrt[3]{x^2-4}}{x-2}=\frac{2}{3}}$

គ. $\lim\limits_{x\to 0}\frac{1-\sqrt[3]{1-x}}{3x}$; រាងមិនកំណត់ $\frac{0}{0}$

$$
\begin{aligned}
\lim_{x\to 0}\frac{1-\sqrt[3]{1-x}}{3x} &= \lim_{x\to 0}\frac{(1-\sqrt[3]{1-x})[1+\sqrt[3]{1-x}+\sqrt[3]{(1-x)^2}]}{3x[1+\sqrt[3]{1-x}+\sqrt[3]{(1-x)^2}]}\\
&= \lim_{x\to 0}\frac{1-1+x}{3x[1+\sqrt[3]{1-x}+\sqrt[3]{(1-x)^2}]}\\
&= \lim_{x\to 0}\frac{\cancel{x}}{3\cancel{x}[1+\sqrt[3]{1-x}+\sqrt[3]{(1-x)^2}]}\\
&= \lim_{x\to 0}\frac{1}{3[1+\sqrt[3]{1-x}+\sqrt[3]{(1-x)^2}]}\\
&= \frac{1}{3[1+\sqrt[3]{1-0}+\sqrt[3]{(1-0)^2}]}=\frac{1}{9}
\end{aligned}
$$

ដូចនេះ $\boxed{\lim\limits_{x\to 0}\frac{1-\sqrt[3]{1-x}}{3x}=\frac{1}{9}}$

ឃ. $\lim\limits_{x\to 3}\frac{x-3}{x-1-\sqrt[3]{x^2-1}}$; រាងមិនកំណត់ $\frac{0}{0}$

យើងមាន $\lim\limits_{x\to 3}\frac{x-3}{x-1-\sqrt[3]{x^2-1}}$ អាចសរសេរទៅជា៖ $\lim\limits_{x\to 3}\frac{x-3}{(x-1)-(\sqrt[3]{x^2-1})}$

$$
\begin{aligned}
&= \lim_{x\to 3}\frac{(x-3)[(x-1)^2+(x-1)(\sqrt[3]{x^2-1})+\sqrt[3]{(x^2-1)^2}]}{[(x-1)-(\sqrt[3]{x^2-1})][(x-1)^2+(x-1)(\sqrt[3]{x^2-1})+\sqrt[3]{(x^2-1)^2}]}\\
&= \lim_{x\to 3}\frac{(x-3)[(x-1)^2+(x-1)(\sqrt[3]{x^2-1})+\sqrt[3]{(x^2-1)^2}]}{(x-1)^3-(x^2-1)}\\
&= \lim_{x\to 3}\frac{(x-3)[(x-1)^2+(x-1)(\sqrt[3]{x^2-1})+\sqrt[3]{(x^2-1)^2}]}{(x-1)(x^2-2x+1)-(x-1)(x+1)}\\
&= \lim_{x\to 3}\frac{(x-3)[(x-1)^2+(x-1)(\sqrt[3]{x^2-1})+\sqrt[3]{(x^2-1)^2}]}{(x-1)[(x^2-2x+1)-(x+1)]}\\
&= \lim_{x\to 3}\frac{(x-3)[(x-1)^2+(x-1)(\sqrt[3]{x^2-1})+\sqrt[3]{(x^2-1)^2}]}{(x-1)(x^2-2x+1-x-1)}\\
&= \lim_{x\to 3}\frac{(x-3)[(x-1)^2+(x-1)(\sqrt[3]{x^2-1})+\sqrt[3]{(x^2-1)^2}]}{(x-1)(x^2-3x)}\\
&= \lim_{x\to 3}\frac{\cancel{(x-3)}[(x-1)^2+(x-1)(\sqrt[3]{x^2-1})+\sqrt[3]{(x^2-1)^2}]}{x(x-1)\cancel{(x-3)}}\\
&= \lim_{x\to 3}\frac{(x-1)^2+(x-1)(\sqrt[3]{x^2-1})+\sqrt[3]{(x^2-1)^2}}{x(x-1)}\\
&= \frac{(3-1)^2+(3-1)(\sqrt[3]{3^2-1})+\sqrt[3]{(3^2-1)^2}}{3(3-1)}\\
&= 2
\end{aligned}
$$

ដូចនេះ $\boxed{\lim\limits_{x\to 3}\frac{x-3}{x-1-\sqrt[3]{x^2-1}}=2}$

៣.៦ របៀបគណនាលីមីតនៃអនុគមន៍អសនិទាន រាង $\sqrt[3]{A}-\sqrt[3]{B}$

⚠ សម្គាល់

ប្រសិនបើកន្សោមមានរាង $\frac{\sqrt[3]{A}-\sqrt[3]{B}}{F(x)}$ ឬ $\frac{G(x)}{\sqrt[3]{A}-\sqrt[3]{B}}$ យើងត្រូវគុណនឹងកន្សោមឆ្លាស់របស់វាគឺ $\sqrt[3]{A^2}+\sqrt[3]{A}\sqrt[3]{B}+\sqrt[3]{B^2}$ ។ យើងបាន $(\sqrt[3]{A}-\sqrt[3]{B})(\sqrt[3]{A^2}+\sqrt[3]{A}\sqrt[3]{B}+\sqrt[3]{B^2})=A-B$ ឬ $\sqrt[3]{A}-\sqrt[3]{B}=\frac{A-B}{\sqrt[3]{A^2}+\sqrt[3]{A}\sqrt[3]{B}+\sqrt[3]{B^2}}$

លំហាត់គំរូ ១៦

គណនាលីមីតខាងក្រោម៖

ក. $\lim\limits_{x\to1}\frac{\sqrt[3]{x-2}+\sqrt[3]{1-x+x^2}}{x^2-1}$

ខ. $\lim\limits_{x\to3}\frac{\sqrt[3]{x^2-1}-\sqrt[3]{x+5}}{x-3}$

គ. $\lim\limits_{x\to1}\frac{\sqrt[3]{x^2}-2\sqrt[3]{x}+1}{(x-1)^2}$

ឃ. $\lim\limits_{x\to2}\frac{\sqrt[3]{x^2+4}-\sqrt[3]{x+6}}{x-2}$

ដំណោះស្រាយ

គណនាលីមីតខាងក្រោម៖

ក. $\lim\limits_{x\to1}\frac{\sqrt[3]{x-2}+\sqrt[3]{1-x+x^2}}{x^2-1}$; រាងមិនកំណត់ $\frac{0}{0}$

យើងមាន $\lim\limits_{x\to1}\frac{\sqrt[3]{x-2}+\sqrt[3]{1-x+x^2}}{x^2-1}$

អាចសរសេរទៅជា: $\lim\limits_{x\to1}\frac{(\sqrt[3]{x-2}+1)+(\sqrt[3]{1-x+x^2}-1)}{x^2-1}$

$=\lim\limits_{x\to1}\frac{\sqrt[3]{x-2}+1}{x^2-1}+\lim\limits_{x\to1}\frac{\sqrt[3]{1-x+x^2}-1}{x^2-1}$

- ករណី $\lim\limits_{x\to1}\frac{\sqrt[3]{x-2}+1}{x^2-1}$

$=\lim\limits_{x\to1}\frac{(\sqrt[3]{x-2}+1)[\sqrt[3]{(x-2)^2}-\sqrt[3]{x-2}+1]}{(x^2-1)[\sqrt[3]{(x-2)^2}-\sqrt[3]{x-2}+1]}$

$=\lim\limits_{x\to1}\frac{x-2+1}{(x^2-1)[\sqrt[3]{(x-2)^2}-\sqrt[3]{x-2}+1]}$

$=\lim\limits_{x\to1}\frac{\cancel{x-1}}{\cancel{(x-1)}(x+1)[\sqrt[3]{(x-2)^2)}-\sqrt[3]{x-2}+1]}$

$=\lim\limits_{x\to1}\frac{1}{(x+1)[\sqrt[3]{(x-2)^2)}-\sqrt[3]{x-2}+1]}$

$=\lim\limits_{x\to1}\frac{1}{(1+1)[\sqrt[3]{(1-2)^2}-\sqrt[3]{1-2}+1]}$

$=\frac{1}{6}$

- ករណី $\lim\limits_{x\to1}\frac{\sqrt[3]{1-x+x^2}-1}{x^2-1}$

$=\lim\limits_{x\to1}\frac{(\sqrt[3]{1-x+x^2}-1)[\sqrt[3]{(1-x+x^2)^2}+\sqrt[3]{1-x+x^2}+1]}{(x^2-1)[\sqrt[3]{(1-x+x^2)^2}+\sqrt[3]{1-x+x^2}+1]}$

$=\lim\limits_{x\to1}\frac{1-x+x^2-1}{(x^2-1)[\sqrt[3]{(1-x+x^2)^2}+\sqrt[3]{1-x+x^2}+1]}$

$=\lim\limits_{x\to1}\frac{x^2-x}{(x^2-1)[\sqrt[3]{(1-x+x^2)^2}+\sqrt[3]{1-x+x^2}+1]}$

$=\lim\limits_{x\to1}\frac{x\cancel{(x-1)}}{\cancel{(x-1)}(x+1)[\sqrt[3]{(1-x+x^2)^2}+\sqrt[3]{1-x+x^2}+1]}$

$=\lim\limits_{x\to1}\frac{x}{(x+1)[\sqrt[3]{(1-x+x^2)^2}+\sqrt[3]{1-x+x^2}+1]}$

$=\frac{x}{(1+1)[\sqrt[3]{(1-1+1^2)^2}+\sqrt[3]{1-1+1^2}+1]}$

$=\frac{1}{6}$

យើងបាន៖

$\lim\limits_{x\to1}\frac{\sqrt[3]{x-2}+1}{x^2-1}+\lim\limits_{x\to1}\frac{\sqrt[3]{1-x+x^2}-1}{x^2-1}=\frac{1}{6}+\frac{1}{6}=\frac{1}{3}$

ដូចនេះ $\boxed{\lim\limits_{x\to1}\frac{\sqrt[3]{x-2}+\sqrt[3]{1-x+x^2}}{x^2-1}=\frac{1}{3}}$

ខ. $\lim\limits_{x\to3}\frac{\sqrt[3]{x^2-1}-\sqrt[3]{x+5}}{x-3}$; រាងមិនកំណត់ $\frac{0}{0}$

$=\lim\limits_{x\to3}\frac{(\sqrt[3]{x^2-1}-\sqrt[3]{x+5})[\sqrt[3]{(x^2-1)^2}+(\sqrt[3]{x^2-1})(\sqrt[3]{x+5})+\sqrt[3]{(x+5)^2}]}{(x-3)[\sqrt[3]{(x^2-1)^2}+(\sqrt[3]{x^2-1})(\sqrt[3]{x+5})+\sqrt[3]{(x+5)^2}]}$

$=\lim\limits_{x\to3}\frac{(x^2-1)-(x+5)}{(x-3)[\sqrt[3]{(x^2-1)^2}+(\sqrt[3]{x^2-1})(\sqrt[3]{x+5})+\sqrt[3]{(x+5)^2}]}$

$=\lim\limits_{x\to3}\frac{x^2-1-x-5}{(x-3)[\sqrt[3]{(x^2-1)^2}+(\sqrt[3]{x^2-1})(\sqrt[3]{x+5})+\sqrt[3]{(x+5)^2}]}$

$=\lim\limits_{x\to3}\frac{x^2-x-6}{(x-3)[\sqrt[3]{(x^2-1)^2}+(\sqrt[3]{x^2-1})(\sqrt[3]{x+5})+\sqrt[3]{(x+5)^2}]}$

$=\lim\limits_{x\to3}\frac{(x+2)\cancel{(x-3)}}{\cancel{(x-3)}[\sqrt[3]{(x^2-1)^2}+(\sqrt[3]{x^2-1})(\sqrt[3]{x+5})+\sqrt[3]{(x+5)^2}]}$

$=\lim\limits_{x\to3}\frac{x+2}{\sqrt[3]{(x^2-1)^2}+(\sqrt[3]{x^2-1})(\sqrt[3]{x+5})+\sqrt[3]{(x+5)^2}}$

$=\frac{3+2}{\sqrt[3]{(3^2-1)^2}+(\sqrt[3]{3^2-1})(\sqrt[3]{3+5})+\sqrt[3]{(3+5)^2}}=\frac{5}{12}$

ដូចនេះ $\boxed{\lim\limits_{x\to3}\frac{\sqrt[3]{x^2-1}-\sqrt[3]{x+5}}{x-3}=\frac{5}{12}}$

គ. $\lim\limits_{x\to1}\frac{\sqrt[3]{x^2}-2\sqrt[3]{x}+1}{(x-1)^2}$; រាងមិនកំណត់ $\frac{0}{0}$

យើងមាន $\lim\limits_{x\to1}\frac{\sqrt[3]{x^2}-2\sqrt[3]{x}+1}{(x-1)^2}$ អាចសរសេរទៅជា:

$$\lim_{x\to1}\frac{(\sqrt[3]{x}-1)^2}{(x-1)^2}=\lim_{x\to1}\frac{(\sqrt[3]{x}-1)^2(\sqrt[3]{x^2}+\sqrt[3]{x}+1)^2}{(x-1)^2(\sqrt[3]{x^2}+\sqrt[3]{x}+1)^2}$$

$$=\lim_{x\to1}\frac{[(\sqrt[3]{x}-1)(\sqrt[3]{x^2}+\sqrt[3]{x}+1)]^2}{(x-1)^2(\sqrt[3]{x^2}+\sqrt[3]{x}+1)^2}$$

$$=\lim_{x\to1}\frac{\cancel{(x-1)^2}}{\cancel{(x-1)^2}(\sqrt[3]{x^2}+\sqrt[3]{x}+1)^2}$$

$$=\lim_{x\to1}\frac{1}{(\sqrt[3]{x^2}+\sqrt[3]{x}+1)^2}$$

$$=\frac{1}{(\sqrt[3]{1^2}+\sqrt[3]{1}+1)^2}$$

$$=\frac{1}{9}$$

ដូចនេះ $\boxed{\lim\limits_{x\to1}\frac{\sqrt[3]{x^2}-2\sqrt[3]{x}+1}{(x-1)^2}=\frac{1}{9}}$

ឃ. $\lim\limits_{x\to2}\frac{\sqrt[3]{x^2+4}-\sqrt[3]{x+6}}{x-2}$; រាងមិនកំណត់ $\frac{0}{0}$

$=\lim\limits_{x\to2}\frac{\left(\sqrt[3]{x^2+4}-\sqrt[3]{x+6}\right)\left[\sqrt[3]{\left(x^2+4\right)^2}+\sqrt[3]{\left(x^2+4\right)(x+6)}+\sqrt[3]{(x+6)^2}\right]}{(x-2)\left[\sqrt[3]{\left(x^2+4\right)^2}+\sqrt[3]{\left(x^2+4\right)(x+6)}+\sqrt[3]{(x+6)^2}\right]}$

$=\lim\limits_{x\to2}\frac{\left(x^2+4\right)-(x+6)}{(x-2)\left[\sqrt[3]{\left(x^2+4\right)^2}+\sqrt[3]{\left(x^2+4\right)(x+6)}+\sqrt[3]{(x+6)^2}\right]}$

$=\lim\limits_{x\to2}\frac{x^2-x-2}{(x-2)\left[\sqrt[3]{\left(x^2+4\right)^2}+\sqrt[3]{\left(x^2+4\right)(x+6)}+\sqrt[3]{(x+6)^2}\right]}$

មេរៀនទី ២ វិធីសាស្ត្រក្នុងការគណនា

$$= \lim_{x\to 2}\frac{(x+1)(x-2)}{(x-2)\left[\sqrt[3]{(x^2+4)^2}+\sqrt[3]{(x^2+4)(x+6)}+\sqrt[3]{(x+6)^2}\right]}$$
$$= \lim_{x\to 2}\frac{x+1}{\sqrt[3]{(x^2+4)^2}+\sqrt[3]{(x^2+4)(x+6)}+\sqrt[3]{(x+6)^2}}$$
$$= \frac{2+1}{\sqrt[3]{(2^2+4)^2}+\sqrt[3]{(2^2+4)(2+6)}+\sqrt[3]{(2+6)^2}}$$
$$= \frac{3}{4+4+4}$$
$$= \frac{1}{4}$$

ដូចនេះ $\boxed{\lim_{x\to 2}\frac{\sqrt[3]{x^2+4}-\sqrt[3]{x+6}}{x-2}=\frac{1}{4}}$

៣.៧ របៀបគណនាលីមីតនៃអនុគមន៍អសនិទាន រាង $\sqrt[3]{A}+\sqrt[3]{B}$

សម្គាល់

ប្រសិនបើកន្សោមមានរាង $\frac{\sqrt[3]{A}+\sqrt[3]{B}}{F(x)}$ ឬ $\frac{G(x)}{\sqrt[3]{A}+\sqrt[3]{B}}$ យើងត្រូវគុណនឹងកន្សោមឆ្លាស់របស់វាគឺ $\sqrt[3]{A^2}-\sqrt[3]{A}\sqrt[3]{B}+\sqrt[3]{B^2}$ ។
យើងបាន $(\sqrt[3]{A}+\sqrt[3]{B})(\sqrt[3]{A^2}-\sqrt[3]{A}\sqrt[3]{B}+\sqrt[3]{B^2})=A+B$។

លំហាត់គំរូ ១៧

គណនាលីមីតខាងក្រោម៖

ក. $\lim_{x\to 0}\frac{\sqrt[3]{x^2+1}+\sqrt[3]{x^2-1}}{x^2}$

ខ. $\lim_{x\to -2}\frac{\sqrt[3]{x^2+4}+\sqrt[3]{4x}}{(x+2)^2}$

គ. $\lim_{x\to 4}\frac{\sqrt[3]{x+4}+\sqrt[3]{8-x^2}}{x-4}$

ឃ. $\lim_{x\to -2}\frac{\sqrt[3]{x^2-2x}+\sqrt[3]{x-6}}{x+2}$

ដំណោះស្រាយ

គណនាលីមីតខាងក្រោម៖

ក. $\lim_{x\to 0}\frac{\sqrt[3]{x^2+1}+\sqrt[3]{x^2-1}}{x^2}$; រាងមិនកំណត់ $\frac{0}{0}$

$$= \lim_{x\to 0}\frac{(\sqrt[3]{x^2+1}+\sqrt[3]{x^2-1})[\sqrt[3]{(x^2+1)^2}-\sqrt[3]{(x^2+1)(x^2-1)}+\sqrt[3]{(x^2-1)^2}]}{x^2[\sqrt[3]{(x^2+1)^2}-\sqrt[3]{(x^2+1)(x^2-1)}+\sqrt[3]{(x^2-1)^2}]}$$
$$= \lim_{x\to 0}\frac{(x^2+1)+(x^2-1)}{x^2[\sqrt[3]{(x^2+1)^2}-\sqrt[3]{(x^2+1)(x^2-1)}+\sqrt[3]{(x^2-1)^2}]}$$
$$= \lim_{x\to 0}\frac{2\cancel{x^2}}{\cancel{x^2}[\sqrt[3]{(x^2+1)^2}-\sqrt[3]{(x^2+1)(x^2-1)}+\sqrt[3]{(x^2-1)^2}]}$$
$$= \lim_{x\to 0}\frac{2}{\sqrt[3]{(x^2+1)^2}-\sqrt[3]{(x^2+1)(x^2-1)}+\sqrt[3]{(x^2-1)^2}}$$
$$= \frac{2}{\sqrt[3]{(0^2+1)^2}-\sqrt[3]{(0^2+1)(0^2-1)}+\sqrt[3]{(0^2-1)^2}}$$
$$= \frac{2}{1+1+1}$$
$$= \frac{2}{3}$$

ដូចនេះ $\boxed{\lim_{x\to 0}\frac{\sqrt[3]{x^2+1}+\sqrt[3]{x^2-1}}{x^2}=\frac{2}{3}}$

ខ. $\lim_{x\to -2}\frac{\sqrt[3]{x^2+4}+\sqrt[3]{4x}}{(x+2)^2}$; រាងមិនកំណត់ $\frac{0}{0}$

$$= \lim_{x\to -2}\frac{(\sqrt[3]{x^2+4}+\sqrt[3]{4x})[\sqrt[3]{(x^2+4)^2}-\sqrt[3]{4x(x^2+4)}+\sqrt[3]{(4x)^2}]}{(x+2)^2[\sqrt[3]{(x^2+4)^2}-\sqrt[3]{4x(x^2+4)}+\sqrt[3]{(4x)^2}]}$$
$$= \lim_{x\to -2}\frac{x^2+4x+4}{(x+2)^2[\sqrt[3]{(x^2+4)^2}-\sqrt[3]{4x(x^2+4)}+\sqrt[3]{(4x)^2}]}$$
$$= \lim_{x\to -2}\frac{\cancel{(x+2)^2}}{\cancel{(x+2)^2}[\sqrt[3]{(x^2+4)^2}-\sqrt[3]{4x(x^2+4)}+\sqrt[3]{(4x)^2}]}$$
$$= \lim_{x\to -2}\frac{1}{\sqrt[3]{(x^2+4)^2}-\sqrt[3]{4x(x^2+4)}+\sqrt[3]{(4x)^2}}$$
$$= \frac{1}{\sqrt[3]{[(-2)^2+4]^2}-\sqrt[3]{4(-2)[(-2)^2+4]}+\sqrt[3]{[4(-2)]^2}}$$
$$= \frac{1}{4+4+4}$$
$$= \frac{1}{12}$$

ដូចនេះ $\boxed{\lim_{x\to -2}\frac{\sqrt[3]{x^2+4}+\sqrt[3]{4x}}{(x+2)^2}=\frac{1}{12}}$

គ. $\lim_{x\to 4}\frac{\sqrt[3]{x+4}+\sqrt[3]{8-x^2}}{x-4}$; រាងមិនកំណត់ $\frac{0}{0}$

$$= \lim_{x\to 4}\frac{(x+4)+(8-x^2)}{(x-4)[\sqrt[3]{(x+4)^2}+\sqrt[3]{(x+4)(8-x^2)}+\sqrt[3]{(x-x^2)^2}]}$$
$$= \lim_{x\to 4}\frac{-x^2+x+12}{(x-4)[\sqrt[3]{(x+4)^2}+\sqrt[3]{(x+4)(8-x^2)}+\sqrt[3]{(x-x^2)^2}]}$$
$$= \lim_{x\to 4}\frac{-(x+3)\cancel{(x-4)}}{\cancel{(x-4)}\sqrt[3]{(x+4)^2}+\sqrt[3]{(x+4)(8-x^2)}+\sqrt[3]{(x-x^2)^2}}$$
$$= \lim_{x\to 4}\frac{-(x+3)}{\sqrt[3]{(x+4)^2}+\sqrt[3]{(x+4)(8-x^2)}+\sqrt[3]{(x-x^2)^2}}$$
$$= \frac{-(4+3)}{\sqrt[3]{(4+4)^2}+\sqrt[3]{(4+4)(8-4^2)}+\sqrt[3]{(4-4^2)^2}}$$
$$= -\frac{7}{4+4+4}$$
$$= -\frac{7}{12}$$

ដូចនេះ $\boxed{\lim\limits_{x\to 4}\frac{\sqrt[3]{x+4}+\sqrt[3]{8-x^2}}{x-4}=-\frac{7}{12}}$

ឃ. $\lim\limits_{x\to -2}\frac{\sqrt[3]{x^2-2x}+\sqrt[3]{x-6}}{x+2}$; រាងមិនកំណត់ $\frac{0}{0}$

$$=\lim_{x\to -2}\frac{\left(\sqrt[3]{x^2-2x}+\sqrt[3]{x-6}\right)\left[\sqrt[3]{\left(x^2-2x\right)^2}-\sqrt[3]{\left(x^2-2x\right)(x-6)}+\sqrt[3]{(x-6)^2}\right]}{(x+2)\left[\sqrt[3]{\left(x^2-2x\right)^2}-\sqrt[3]{\left(x^2-2x\right)(x-6)}+\sqrt[3]{(x-6)^2}\right]}$$

$$=\lim_{x\to -2}\frac{\left(x^2-2x\right)+(x-6)}{(x+2)\left[\sqrt[3]{\left(x^2-2x\right)^2}-\sqrt[3]{\left(x^2-2x\right)(x-6)}+\sqrt[3]{(x-6)^2}\right]}$$

$$=\lim_{x\to -2}\frac{x^2-x-6}{(x+2)\left[\sqrt[3]{\left(x^2-2x\right)^2}-\sqrt[3]{\left(x^2-2x\right)(x-6)}+\sqrt[3]{(x-6)^2}\right]}$$

$$=\lim_{x\to -2}\frac{(x+2)(x-3)}{(x+2)\left[\sqrt[3]{\left(x^2-2x\right)^2}-\sqrt[3]{\left(x^2-2x\right)(x-6)}+\sqrt[3]{(x-6)^2}\right]}$$

$$=\lim_{x\to -2}\frac{x-3}{\sqrt[3]{\left(x^2-2x\right)^2}-\sqrt[3]{\left(x^2-2x\right)(x-6)}+\sqrt[3]{(x-6)^2}}$$

$$=\frac{-2-3}{\sqrt[3]{\left[(-2)^2-2(-2)\right]^2}-\sqrt[3]{\left[(-2)^2-2(-2)\right](-2-6)}+\sqrt[3]{(-2-6)^2}}$$

$$=-\frac{5}{12}$$

ដូចនេះ $\boxed{\lim\limits_{x\to -2}\frac{\sqrt[3]{x^2-2x}+\sqrt[3]{x-6}}{x+2}=-\frac{5}{12}}$

៤ របៀបគណនាលីមីតមិនកំណត់អនន្ត

1. $\lim\limits_{x\to\pm\infty}\frac{a}{x}=0\ ,\ \forall a\in\mathbb{R}$

2. $\lim\limits_{x\to\pm\infty}\frac{x}{a}=\pm\infty\ ,\ a\neq 0$

3. $\lim\limits_{x\to\pm\infty}ax^2=\begin{cases}+\infty\ , & \text{បើ } a>0\\ -\infty\ , & \text{បើ } a<0\end{cases}$

៤.១ របៀបគណនាលីមីតមិនកំណត់អនន្ត រាង $L=\frac{a_mx^m+a_{m-1}x^{m-1}\ldots+a_1x+a_0}{b_mx^m+b_{m-1}x^{m-1}\ldots+b_1x+b_0}$

សម្គាល់

- ដើម្បីគណនាលីមីតរាងប្រភេទនេះគេត្រូវ៖

១. ដាក់ x ដែលមានដឺក្រេខ្ពស់ជាងគេជាកត្តារួម

២. សម្រួលកត្តា x នោះចោល

៣. គណនាលីមីតនៃប្រភាគថ្មីដែលនៅសល់ក្រោយពីសម្រួលរួច។

លំហាត់គំរូ ១៨

គណនាលីមីតខាងក្រោម៖

ក. $\lim\limits_{x\to\infty}\frac{x^3+3x+2}{x^2+1}$

ខ. $\lim\limits_{x\to\infty}\frac{x^3+6x+7}{x^4+2x^2+6}$

គ. $\lim\limits_{x\to\infty}\frac{\sqrt[3]{x^3+2x-1}}{x+2}$

ឃ. $\lim\limits_{x\to\infty}\frac{2x^2-3x+4}{\sqrt{x^4+2x^2}}$

ដំណោះស្រាយ

គណនាលីមីតខាងក្រោម៖

ក. $\lim\limits_{x\to\infty}\frac{x^3+3x+2}{x^2+1}$; រាងមិនកំណត់ $\frac{\infty}{\infty}$

យើងបាន $\displaystyle\lim_{x\to\infty}\frac{x^3+3x+2}{x^2+1}=\lim_{x\to\infty}\frac{x^3(1+\frac{3}{x^2}+\frac{2}{x^3})}{x^2(1+\frac{1}{x^2})}$

$$=\lim_{x\to\infty}\frac{x(1+\frac{3}{x^2}+\frac{2}{x^3})}{(1+\frac{1}{x^2})}$$

$$=\frac{\infty(1+\frac{3}{\infty^2}+\frac{2}{\infty^3})}{1+\frac{1}{\infty^2}}$$

$$=\frac{\infty(1+0+0)}{1+0}$$

$$=\infty$$

ដូចនេះ $\boxed{\lim_{x\to\infty}\frac{x^3+3x+2}{x^2+1}=\infty}$

ខ. $\lim_{x\to\infty}\frac{x^3+6x+7}{x^4+2x^2+6}$; រាងមិនកំណត់ $\frac{\infty}{\infty}$

យើងបាន $\displaystyle\lim_{x\to\infty}\frac{x^3+6x+7}{x^4+2x^2+6}=\lim_{x\to\infty}\frac{(1+\frac{6}{x^2}+\frac{7}{x^3})}{x(1+\frac{2}{x^2}+\frac{6}{x^4})}$

$$=\lim_{x\to\infty}\frac{1+\frac{6}{\infty^2}+\frac{7}{\infty^3}}{\infty(1+\frac{2}{\infty^2}+\frac{6}{\infty^4})}$$

$$=\frac{1+0+0}{\infty(1+0+0)}$$

$$=0$$

ដូចនេះ $\boxed{\lim_{x\to\infty}\frac{x^3+6x+7}{x^4+2x^2+6}=0}$

គ. $\lim_{x\to\infty}\frac{\sqrt[3]{x^3+2x-1}}{x+2}$; រាងមិនកំណត់ $\frac{\infty}{\infty}$

យើងបាន $\displaystyle\lim_{x\to\infty}\frac{\sqrt[3]{x^3+2x-1}}{x+2}=\lim_{x\to\infty}\frac{\sqrt[3]{x^3(1+\frac{2}{x^2}-\frac{1}{x^3})}}{x(1+\frac{1}{x})}$

$$=\lim_{x\to\infty}\frac{\cancel{x}\sqrt[3]{(1+\frac{2}{x^2}-\frac{1}{x^3})}}{\cancel{x}(1+\frac{1}{x})}$$

$$=\lim_{x\to\infty}\frac{\sqrt[3]{1+\frac{2}{x^2}-\frac{1}{x^3}}}{(1+\frac{1}{x})}$$

$$=\frac{\sqrt[3]{1+\frac{2}{\infty^2}-\frac{1}{\infty^3}}}{1+\frac{1}{\infty}}$$

$$=1$$

ដូចនេះ $\boxed{\lim_{x\to\infty}\frac{\sqrt[3]{x^3+2x-1}}{x+2}=1}$

ឃ. $\lim_{x\to\infty}\frac{2x^2-3x+4}{\sqrt{x^4+2x^2}}$; រាងមិនកំណត់ $\frac{\infty}{\infty}$

យើងបាន $\displaystyle\lim_{x\to\infty}\frac{2x^2-3x+4}{\sqrt{x^4+2x^2}}=\lim_{x\to\infty}\frac{x^2(2-\frac{3}{x}+\frac{4}{x^2})}{x^2\sqrt{(1+\frac{2}{x^2})}}$

$$=\lim_{x\to\infty}\frac{2-\frac{3}{x}+\frac{4}{x^2}}{\sqrt{1+\frac{2}{x^2}}}$$

$$=\frac{2-\frac{3}{\infty}+\frac{4}{\infty^2}}{\sqrt{(1+\frac{2}{\infty^2})}}$$

$$=2$$

ដូចនេះ $\boxed{\lim_{x\to\infty}\frac{2x^2-3x+4}{\sqrt{x^4+2}}=2}$

៥ របៀបគណនាលីមីតខិតជិតអនន្តចំពោះអនុគមន៍អសនិទាន

៥.១ របៀបគណនាលីមីតខិតជិតអនន្តអនុគមន៍អសនិទាន រាង $+\infty-\infty$

⚠ សម្គាល់

- ឧបមាថា $\lim_{x\to\infty}[(f(x)+g(x)]$ រាងមិនកំណត់ $+\infty-\infty$ បើ៖

១. បើកន្សោម $f(x)$ និង $g(x)$ ជាពហុធាគេទាញ x ដែលមានដឺក្រេខ្ពស់ជាងគេជាកត្តារួមរួចគណនាកន្សោមដែលសម្រួលរួច។

២. បើកន្សោម $f(x)$ និង $g(x)$ ជាប្រភាគសនិទានគេត្រូវតម្រូវភាគបែងរួម ហើយបំបែកជាផលគុណកត្តា រួចសម្រួលកត្តារួមចោល បន្ទាប់មកគណនាលីមីតថ្មី។

៣. បើកន្សោម $f(x)$ និង $g(x)$ ជាកន្សោមដែលមានរ៉ាឌីកាល់គេត្រូវប្រើរូបមន្ត៖

ក. $\sqrt{A}-B=\frac{A-B^2}{\sqrt{A}+B}$

ខ. $A-\sqrt{B}=\frac{A^2-B}{A+\sqrt{B}}$

គ. $\sqrt[3]{A}-B=\frac{A-B^3}{\sqrt[3]{A^2}+B\sqrt[3]{A}+B^2}$

ឃ. $A-\sqrt[3]{B}=\frac{A^3-B}{A^2+A\sqrt[3]{B}+\sqrt[3]{B^2}}$

ង. $\sqrt{A}-\sqrt{B}=\frac{A-B}{\sqrt{A}+\sqrt{B}}$

ច. $\sqrt[3]{A}-\sqrt[3]{B}=\frac{A^3-B^3}{\sqrt[3]{A^2}+\sqrt[3]{A}\sqrt[3]{B}+\sqrt[3]{B^2}}$

ឆ. $\sqrt[3]{A}+\sqrt[3]{B}=\frac{A^3+B^3}{\sqrt[3]{A^2}-\sqrt[3]{A}\sqrt[3]{B}+\sqrt[3]{B^2}}$

លំហាត់គំរូ ១៩

គណនាលីមីតខាងក្រោម៖

ក. $\lim\limits_{x\to+\infty}\left(\sqrt{x^2+5x-1}-\sqrt{x^2+3x+2}\right)$

ខ. $\lim\limits_{x\to\infty}\left(\frac{x^3}{2x^2-1}-\frac{x^2}{2x+1}\right)$

គ. $\lim\limits_{x\to\infty}\left(x-\sqrt{x^2+1}\right)$ ឃ. $\lim\limits_{x\to3}\left(\frac{1}{x-3}-\frac{6}{x^2-9}\right)$

ដំណោះស្រាយ

គណនាលីមីតខាងក្រោម៖

ក. $\lim\limits_{x\to+\infty}\left(\sqrt{x^2+5x-1}-\sqrt{x^2+3x+2}\right)$, រាងមិនកំណត់$+\infty-\infty$

តាង $A=\lim\limits_{x\to+\infty}\left(\sqrt{x^2+5x-1}-\sqrt{x^2+3x+2}\right)$

យើងប្រើរូបមន្ត $(a-b)(a+b)=a^2-b^2$

នាំឱ្យ យើងបាន៖ $\lim\limits_{x\to+\infty}\left(\sqrt{x^2+5x-1}-\sqrt{x^2+3x+2}\right)$ អាចសរសេរទៅជា

$A=\lim\limits_{x\to+\infty}\frac{\sqrt{\left(x^2+5x-1\right)^2}-\sqrt{\left(x^2+3x+2\right)^2}}{\sqrt{x^2+5x-1}+\sqrt{x^2+3x+2}}$

$A=\lim\limits_{x\to+\infty}\frac{\left(x^2+5x-1\right)-\left(x^2+3x+2\right)}{\sqrt{x^2+5x-1}+\sqrt{x^2+3x+2}}$

$A=\lim\limits_{x\to+\infty}\frac{x^2+5x-1-x^2-3x-2}{\sqrt{x^2+5x-1}+\sqrt{x^2+3x+2}}$

$A=\lim\limits_{x\to+\infty}\frac{2x-3}{\sqrt{x^2\left(1+\frac{5}{x}-\frac{1}{x^2}\right)}+\sqrt{x^2\left(1+\frac{3}{x}+\frac{2}{x^2}\right)}}$; $\left(|x|=x \text{ នៅពេល } x\to+\infty\right)$

$A=\lim\limits_{x\to+\infty}\frac{2x-3}{x\sqrt{1+\frac{5}{x}-\frac{1}{x^2}}+x\sqrt{1+\frac{3}{x}+\frac{2}{x^2}}}$

$A=\lim\limits_{x\to+\infty}\frac{x\left(2-\frac{3}{x}\right)}{x\left(\sqrt{1+\frac{5}{x}-\frac{1}{x^2}}+\sqrt{1+\frac{3}{x}+\frac{2}{x^2}}\right)}$

$A=\lim\limits_{x\to+\infty}\frac{2-\frac{3}{x}}{\sqrt{1+\frac{5}{x}-\frac{1}{x^2}}+\sqrt{1+\frac{3}{x}+\frac{2}{x^2}}}$

$A=\frac{2-\frac{3}{\infty}}{\sqrt{1+\frac{5}{\infty}-\frac{1}{\infty^2}}+\sqrt{1+\frac{3}{\infty}+\frac{2}{\infty^2}}}$

$A=\frac{2-0}{\sqrt{1+0-0}+\sqrt{1+0+0}}$

$A=\frac{2-0}{1+1}$

$A=1$

ដូចនេះ $\boxed{\lim\limits_{x\to+\infty}\left(\sqrt{x^2+5x-1}-\sqrt{x^2+3x+2}\right)=1}$

ខ. $\lim\limits_{x\to\infty}\left(\frac{x^3}{2x^2-1}-\frac{x^2}{2x+1}\right)$, រាងមិនកំណត់$+\infty-\infty$

យើងបាន $\lim\limits_{x\to\infty}\left(\frac{x^3}{2x^2-1}-\frac{x^2}{2x+1}\right)=\lim\limits_{x\to\infty}\frac{x^3(2x+1)-x^2(2x^2-1)}{(2x^2-1)(2x+1)}$

$=\lim\limits_{x\to\infty}\frac{2x^4+x^3-2x^4+x^2}{(2x^2-1)(2x+1)}$

$=\lim\limits_{x\to\infty}\frac{x^3(1+\frac{1}{x})}{x^2(2-\frac{1}{x^2})x(2+\frac{1}{x})}$

$=\lim\limits_{x\to\infty}\frac{\cancel{x^3}(1+\frac{1}{x})}{\cancel{x^3}(2-\frac{1}{x^2})(2+\frac{1}{x})}$

$=\lim\limits_{x\to\infty}\frac{1+\frac{1}{x}}{(2-\frac{1}{x^2})(2+\frac{1}{x})}$

$=\frac{1+\frac{1}{\infty}}{(2-\frac{1}{\infty^2})(2+\frac{1}{\infty})}$

$=\frac{1}{4}$

ដូចនេះ $\boxed{\lim\limits_{x\to\infty}\left(\frac{x^3}{2x^2-1}-\frac{x^2}{2x+1}\right)=\frac{1}{4}}$

គ. $\lim\limits_{x\to\infty}\left(x-\sqrt{x^2+1}\right)$, រាងមិនកំណត់$+\infty-\infty$

៦.១ ការគណនាដោយប្រើរូបមន្តគ្រឹះ ករណី $x \to x_0$

⚠ សម្គាល់

បើរាងមិនកំណត់ $x \to x_0$ ណាមួយ នោះគេត្រូវតាង $k = x - x_0$ ឬ $k = x_0 - x$ កាលណា $x \to x_0$នោះយើងបាន $k \to 0$ បន្ទាប់មក យើងគណនាដោយប្រើរូបមន្តគ្រឹះខាងលើ ។

លំហាត់គំរូ ២០

គណនាលីមីតខាងក្រោម៖

$\lim\limits_{x\to\frac{\pi}{3}} \frac{1-2\cos x}{\sin 3x}$ $\lim\limits_{x\to 2} \frac{2-x}{\sin\frac{2\pi}{x}}$ $\lim\limits_{x\to\pi} \frac{\cos\frac{\pi x}{x+\pi}}{\pi - x}$

ដំណោះស្រាយ

គណនាលីមីតខាងក្រោម៖

ក. $\lim\limits_{x\to\frac{\pi}{2}} \frac{\cos x}{\tan 2x}$, រាងមិនកំណត់ $\frac{0}{0}$

តាង $k = x - \frac{\pi}{2} \Rightarrow x = k + \frac{\pi}{2}$

កាលណា $x \to \frac{\pi}{2}$ នោះ $k \to 0$ យើងបាន៖

$$\begin{aligned}
\lim_{x\to\frac{\pi}{2}} \frac{\cos x}{\tan 2x} &= \lim_{k\to 0} \frac{\cos(k+\frac{\pi}{2})}{\tan 2(k+\frac{\pi}{2})} \\
&= \lim_{k\to 0} \frac{\cos(k+\frac{\pi}{2})}{\tan 2(\pi+2k)} \\
&= \lim_{k\to 0} \frac{-\sin k}{\tan 2k} \\
&= -\lim_{k\to 0}\left[\frac{\sin k}{k} \times \frac{2k}{\tan 2k} \times \frac{1}{2}\right] \\
&= -\frac{1}{2}\lim_{k\to 0}\frac{\sin k}{k} \times \lim_{k\to 0}\frac{2k}{\tan 2k} \\
&= -\frac{1}{2} \times 1 \times 1 \\
&= -\frac{1}{2}
\end{aligned}$$

ដូចនេះ $\boxed{\lim\limits_{x\to\frac{\pi}{2}} \frac{\cos x}{\tan 2x} = -\frac{1}{2}}$

ខ. $\lim\limits_{x\to\frac{\pi}{3}} \frac{1-2\cos x}{\sin 3x}$, រាងមិនកំណត់ $\frac{0}{0}$

តាង $k = x - \frac{\pi}{3} \Rightarrow x = k + \frac{\pi}{3}$

កាលណា $x \to \frac{\pi}{3}$ នោះ $k \to 0$ យើងបាន៖

$$\lim_{x\to\frac{\pi}{3}} \frac{1-2\cos x}{\sin 3x} = \lim_{k\to 0} \frac{1-2\cos(k+\frac{\pi}{3})}{\sin 3(k+\frac{\pi}{3})}$$

យើងមាន

$$\begin{aligned}
\bullet \sin 3(k+\frac{\pi}{3}) &= \sin(\pi+3k) \\
&= -\sin 3k
\end{aligned}$$

$$\begin{aligned}
\bullet \cos(\frac{\pi}{3}+k) &= \cos\frac{\pi}{3}\cos k - \sin\frac{\pi}{3}\sin k \\
&= \frac{1}{2}\cos k - \frac{\sqrt{3}}{2}\sin k \\
&= \frac{1}{2}(\cos k - \sqrt{3}\sin k)
\end{aligned}$$

យើងបាន៖

$$\begin{aligned}
\lim_{k\to 0} \frac{1-2\cos(k+\frac{\pi}{3})}{\sin 3(k+\frac{\pi}{3})} &= \lim_{k\to 0} \frac{1-2\frac{1}{2}(\cos k - \sqrt{3}\sin k)}{-\sin 3k} \\
&= \lim_{k\to 0}\left[\frac{1-\cos k+\sqrt{3}\sin k}{k}\right] \times \frac{k}{-\sin 3k} \\
&= \left[\lim_{k\to 0}\frac{1-\cos k}{k} + \sqrt{3}\lim_{k\to 0}\frac{\sin k}{k}\right] \times \left(-\lim_{k\to 0}\frac{3k}{\sin 3k} \times \frac{1}{3}\right) \\
&= (0+\sqrt{3})(-1\times\frac{1}{3}) \\
&= -\frac{\sqrt{3}}{3}
\end{aligned}$$

ដូចនេះ $\boxed{\lim\limits_{x\to\frac{\pi}{3}} \frac{1-2\cos x}{\sin 3x} = -\frac{\sqrt{3}}{3}}$

គ. $\lim\limits_{x\to 2} \frac{2-x}{\sin\frac{2\pi}{x}}$, រាងមិនកំណត់ $\frac{0}{0}$

តាង $z = \frac{2}{x} \Rightarrow x = \frac{2}{z}$

បើ $x \to 2$ នោះ $z \to 1$

យើងបាន $\lim\limits_{z\to1}\dfrac{2-x}{\sin\frac{2\pi}{x}}=\lim\limits_{z\to1}\dfrac{2-\frac{2}{z}}{\sin\pi z}$

$$=\lim_{z\to1}\frac{2\left(1-\frac{1}{z}\right)}{\sin\pi z}$$
$$=2\lim_{z\to1}\frac{\frac{z-1}{z}}{\sin\pi z}$$
$$=2\lim_{z\to1}\frac{\frac{z-1}{z}}{\sin\pi z}$$
$$=2\lim_{z\to1}\frac{z-1}{z\sin\pi z}$$

តាង $z=1-u\Rightarrow u=1-z$ បើ$z\to1$ នោះ $u\to0$

យើងបាន $2\lim\limits_{z\to1}\dfrac{z-1}{z\sin\pi z}=2\lim\limits_{u\to0}\dfrac{-u}{(1-u)\sin(\pi-\pi u)}$

$$=-2\lim_{u\to0}\left[\frac{1}{1-u}\times\frac{\pi u}{\sin\pi u}\times\frac{1}{\pi}\right]$$
$$=-2\frac{1}{\pi}\lim_{u\to0}\frac{1}{1-u}\lim_{u\to0}\frac{\pi u}{\sin\pi u}$$
$$=-2\times\frac{1}{\pi}\times\frac{1}{1-0}\times1$$
$$=-\frac{2}{\pi}$$

ដូចនេះ $\boxed{\lim\limits_{x\to2}\frac{2-x}{\sin\frac{2\pi}{x}}=-\frac{2}{\pi}}$

ឃ. $\lim\limits_{x\to\pi}\frac{\cos\frac{\pi x}{x+\pi}}{\pi-x}$, រាងមិនកំណត់ $\frac{0}{0}$

តាង $t=\frac{\pi x}{x+\pi}\Rightarrow x=\frac{\pi t}{\pi-t}$ បើ $x\to\pi$ នោះ $t\to\frac{\pi}{2}$ យើងបាន

$\lim\limits_{x\to\pi}\dfrac{\cos\frac{\pi x}{x+\pi}}{\pi-x}=\lim\limits_{t\to\frac{\pi}{2}}\dfrac{\cos t}{\pi-\frac{\pi t}{\pi-t}}$

តាង $u=\frac{\pi}{2}-t\Rightarrow t=\frac{\pi}{2}-u$ បើ $t\to\frac{\pi}{2}$ នោះ $u\to0$ យើងបាន៖

$$\lim_{t\to\frac{\pi}{2}}\frac{\cos t}{\pi-\frac{\pi t}{\pi-t}}=\lim_{u\to0}\frac{\cos\left(\frac{\pi}{2}-u\right)}{\pi-\frac{\pi\left(\frac{\pi}{2}-u\right)}{\pi-\left(\frac{\pi}{2}-u\right)}}=\lim_{u\to0}\frac{\sin u}{\pi-\frac{\frac{\pi^2}{2}-\pi u}{\frac{\pi}{2}+u}}$$
$$=\lim_{u\to0}\frac{\sin u}{\frac{\frac{\pi^2}{2}+\pi u-\frac{\pi^2}{2}+\pi u}{\frac{\pi}{2}+u}}=\lim_{u\to0}\frac{\sin u}{\frac{2\pi u}{\frac{\pi}{2}+u}}$$
$$=\lim_{u\to0}\left[\frac{\sin u}{u}\times\frac{\frac{\pi}{2}+u}{2\pi}\right]$$
$$=\lim_{u\to0}\frac{\sin u}{u}\times\lim_{u\to0}\frac{\frac{\pi}{2}+u}{2\pi}$$
$$=1\times\frac{1}{4}$$
$$=\frac{1}{4}$$

ដូចនេះ $\boxed{\lim\limits_{x\to\pi}\frac{\cos\frac{\pi x}{x+\pi}}{\pi-x}=\frac{1}{4}}$

៦.២ ការគណនាដោយប្រើរូបមន្តគ្រឹះ ករណី $x\to\infty$

សម្គាល់

បើរាងមិនកំណត់ $x\to\infty$ នោះគេត្រូវតាង $k=\frac{1}{x}$ កាលណា $x\to\infty$ នោះយើងបាន $k\to0$ បន្ទាប់មកយើងគណនាដោយប្រើរូបមន្តគ្រឹះខាងលើ ។

លំហាត់គំរូ ២១

គណនាលីមីតខាងក្រោម៖

ក. $\lim\limits_{x\to\infty}(3x+1)\sin\frac{2\pi x}{x-1}$

ខ. $\lim\limits_{x\to\infty}(7x+2)\cos\frac{\pi x}{2(x+1)}$

គ. $\lim\limits_{x\to\infty}(2x+1)\sin\frac{\pi x}{(x+3)}$

ឃ. $\lim\limits_{x\to+\infty}\frac{(1+x)\sin x}{3+x^2}$

ដំណោះស្រាយ

គណនាលីមីតខាងក្រោម៖

ក. $\lim\limits_{x\to\infty}(3x+1)\sin\frac{2\pi x}{x-1}$, រាងមិនកំណត់ $1\times\infty$

$\lim\limits_{x\to\infty}(3x+1)\sin\frac{2\pi x}{x-1}=\lim\limits_{x\to\infty}[3(x-1)+4]\sin\left[2\pi\left(1+\frac{1}{1-x}\right)\right]$

តាង $t=\frac{1}{x-1}\Rightarrow x-1=\frac{1}{t}$, បើ $x\to\infty$ នោះ $t\to0$, យើងបាន៖

$$\lim_{x\to\infty}[3(x-1)+4]\times\sin\left[2\pi\left(1+\frac{1}{1-x}\right)\right]=\lim_{t\to 0}\left(\frac{3}{t}+4\right)\times\sin(2\pi+2\pi t)$$
$$=\lim_{t\to 0}\left(\frac{3}{t}+4\right)\times\sin 2\pi t$$
$$=\lim_{t\to 0}\left[\frac{3\sin 2\pi t}{t}+4\sin 2\pi t\right]$$
$$=\lim_{t\to 0}\left[\frac{\sin 2\pi t}{2\pi t}\times 6\pi+4\sin 2\pi t\right]$$
$$=6\pi\lim_{t\to 0}\frac{\sin 2\pi t}{2\pi t}+4\lim_{t\to 0}\sin 2\pi t$$
$$=6\pi\times 1+4\times 0$$
$$=6\pi$$

ដូចនេះ $\boxed{\lim_{x\to\infty}(3x+1)\sin\frac{2\pi x}{x-1}=6\pi}$

ខ. $\lim_{x\to\infty}(7x+2)\cos\frac{\pi x}{2(x+1)}$, រាងមិនកំណត់ $1\times\infty$

$\lim_{x\to\infty}(7x+2)\cos\frac{\pi x}{2(x+1)}=\lim_{x\to\infty}[7(x+1)-5]\times\cos\left[\frac{\pi}{2}(1-\frac{1}{x+1})\right]$

តាង $k=\frac{1}{x+1}\Rightarrow x+1=\frac{1}{k}$, បើ $x\to\infty$ នោះ $k\to 0$ យើងបាន៖

$$\lim_{x\to\infty}[7(x+1)-5]\times\cos\left[\frac{\pi}{2}(1-\frac{1}{x+1})\right]=\lim_{k\to 0}\left(\frac{7}{k}-5\right)\times\cos\left(\frac{\pi}{2}-\frac{\pi k}{2}\right)$$
$$=\lim_{k\to 0}\left(\frac{7}{k}-5\right)\times\sin\frac{\pi k}{2}$$
$$=\lim_{k\to 0}\left(\frac{7\sin\frac{\pi k}{2}}{k}-5\sin\frac{\pi k}{2}\right)$$
$$=\frac{7\pi}{2}\lim_{k\to 0}\sin\frac{\pi k}{2}-5\lim_{k\to 0}\sin\frac{\pi k}{2}$$
$$=\frac{7\pi}{2}\times 1-5\times 0=\frac{7\pi}{2}$$

ដូចនេះ $\boxed{\lim_{x\to\infty}(7x+2)\cos\frac{\pi x}{2(x-1)}=\frac{7\pi}{2}}$

គ. $\lim_{x\to\infty}(2x+1)\sin\frac{\pi x}{(x+3)}$, រាងមិនកំណត់ $1\times\infty$

$\lim_{x\to\infty}(2x+1)\sin\frac{\pi x}{(x+3)}=\lim_{x\to\infty}[2(x+3)-5]\times\sin\left[\pi\left(1-\frac{3}{x+3}\right)\right]$

តាង $u=\frac{1}{x+3}\Rightarrow x+3=\frac{1}{u}$, បើ $x\to\infty$ នោះ $k\to 0$ យើងបាន៖

$$\lim_{x\to\infty}[2(x+3)-5].\sin\left[\pi\left(1-\frac{3}{x+3}\right)\right]=\lim_{u\to 0}\left(\frac{2}{u}-5\right)\times\sin(\pi-3\pi u)$$
$$=\lim_{u\to 0}\left(\frac{2}{u}-5\right)\times\sin 3\pi u$$
$$=\lim_{u\to 0}\left(\frac{2\sin 3\pi u}{u}-5\sin 3\pi u\right)$$
$$=\lim_{u\to 0}\frac{\sin 3\pi u}{3\pi u}\times 6\pi-\lim_{u\to 0}5\sin 3\pi u$$
$$=1\times 6\pi-5\times 0$$
$$=6\pi$$

ដូចនេះ $\boxed{\lim_{x\to+\infty}(2x+1)\sin\frac{\pi x}{(x+3)}=6\pi}$

ឃ. $\lim_{x\to+\infty}\frac{(1+x)\sin x}{3+x^2}$, រាងមិនកំណត់ $\frac{\infty}{\infty}$

ដោយ $\sin$ មានតម្លៃនៅចន្លោះ -1 និង 1

នាំឱ្យ $\lim_{x\to+\infty}\frac{(1+x)\sin x}{3+x^2}=\lim_{x\to+\infty}\frac{(1+x)}{3+x^2}$ $\quad -1\le\sin x\le 1$

$$=\lim_{x\to+\infty}\frac{x(\frac{1}{x}+1)}{x(\frac{3}{x}+x)}$$
$$=\lim_{x\to+\infty}\frac{\frac{1}{x}+1}{\frac{3}{x}+x}$$
$$=\frac{\frac{1}{+\infty}+1}{\frac{3}{+\infty}+(+\infty)}$$
$$=0$$

ដូចនេះ

$\lim_{x\to\infty}\frac{(1+x)\sin x}{3+x^2}=0$

៧ របៀបគណនាលីមីតនៃអនុគមន៍ អ៊ិចស្ប៉ូណង់ស្យែល

⚠ សម្គាល់

ដើម្បីគណនាលីមីតនៃអនុគមន៍អ៊ិចស្ប៉ូណង់ស្យែលយើងត្រូវ ចងចាំរូបមន្តមួយចំនួនដូចខាងក្រោម៖

; $\lim_{x\to+\infty}\frac{x^n}{e^x}=0$, $n\ge 0$ $\lim_{x\to 0}\frac{a^x-1}{x}=\ln a$, $(a>0)$ $\lim_{x\to 0}\frac{(1+x)^k-1}{x}=k$ $\lim_{x\to 0}(1+x)^{\frac{1}{x}}=e$ $\lim_{x\to\infty}(1+\frac{1}{x})^x=e$ $\lim_{x\to\infty}(1+\frac{a}{x})^x=e^a$

$\lim_{x\to+\infty}a^x=\begin{cases}+\infty & \text{បើ } a>1\\ 0 & \text{បើ } 0<a<1\end{cases}$ $\quad\lim_{x\to-\infty}a^x=\begin{cases}+\infty & \text{បើ } 0<a<1\\ 0 & \text{បើ } a>1\end{cases}$

លំហាត់គំរូ ២២

គណនាលីមីតខាងក្រោម៖

ក. $\lim_{x\to 0}\frac{e^{2x}-1}{x}$ ខ. $\lim_{x\to+\infty}\frac{2e^x+e^{2x}+1}{e^x+1}$ គ. $\lim_{x\to+\infty}\frac{e^x-x^2+x+2}{x^2}$

ឃ. $\lim\limits_{x\to+\infty}\frac{e^x-x}{2e^x+1}$

ដំណោះស្រាយ

គណនាលីមីតខាងក្រោម៖

ក. $\lim\limits_{x\to0}\frac{e^{2x}-1}{x}$, រាងមិនកំណត់ $\frac{0}{0}$

យើងមាន $\lim\limits_{x\to0}\dfrac{e^{2x}-1}{x}=\left[\lim\limits_{x\to0}\dfrac{e^{2x}-1}{2x}\times2\right]$
$=2\lim\limits_{x\to0}\dfrac{e^{2x}-1}{2x}$
$=2$

ដូចនេះ៖ $\boxed{\lim\limits_{x\to0}\frac{e^x-1}{x}=2}$

ខ. $\lim\limits_{x\to+\infty}\frac{2e^x+e^{2x}+1}{e^x+1}$

• របៀបទី១

$\lim\limits_{x\to+\infty}\frac{2e^x+e^{2x}+1}{e^x+1}$, រាងមិនកំណត់ $\frac{\infty}{\infty}$

យើងមាន $\lim\limits_{x\to+\infty}\dfrac{2e^x+e^{2x}+1}{e^x+1}=\lim\limits_{x\to+\infty}\dfrac{e^{2x}+2e^x+1}{e^x+1}$
$=\lim\limits_{x\to+\infty}\dfrac{(e^x+1)^2}{e^x+1}$
$=\lim\limits_{x\to+\infty}e^x+1$
$=+\infty+1$
$=+\infty$

• របៀបទី២

$\lim\limits_{x\to+\infty}\frac{2e^x+e^{2x}+1}{e^x+1}$, រាងមិនកំណត់ $\frac{\infty}{\infty}$

យើងបាន៖ $\lim\limits_{x\to+\infty}\dfrac{2e^x+e^{2x}+1}{e^x+1}=\lim\limits_{x\to+\infty}\dfrac{e^x(2+e^x+\frac{1}{e^x})}{e^x(1+\frac{1}{e^x})}$
$=\lim\limits_{x\to+\infty}\dfrac{2+e^x+\frac{1}{e^x}}{1+\frac{1}{e^x}}$
$=\dfrac{2+\infty+0}{1+0}$
$=+\infty$

ដូចនេះ៖ $\boxed{\lim\limits_{x\to+\infty}\frac{2e^x+e^{2x}+1}{e^x+1}=+\infty}$

គ. $\lim\limits_{x\to+\infty}\frac{e^x-x^2+x+2}{x^2}$, រាងមិនកំណត់ $\frac{\infty}{\infty}$

យើងបាន៖ $\lim\limits_{x\to+\infty}\dfrac{e^x-x^2+x+2}{x^2}=\lim\limits_{x\to+\infty}\dfrac{x^2(\frac{e^x}{x^2}-1+\frac{1}{x^2}+\frac{2}{x^2})}{x^2}$
$=\lim\limits_{x\to+\infty}\left(\dfrac{e^x}{x^2}-1+\dfrac{1}{x^2}+\dfrac{2}{x^2}\right)=+\infty$

ដូចនេះ៖ $\boxed{\lim\limits_{x\to+\infty}\frac{e^x-x^2+x+2}{x^2}=+\infty}$

ឃ. $\lim\limits_{x\to+\infty}\frac{e^x-x}{2e^x+1}$, រាងមិនកំណត់ $\frac{\infty}{\infty}$

យើងបាន៖ $\lim\limits_{x\to+\infty}\dfrac{e^x-x}{2e^x+1}=\lim\limits_{x\to+\infty}\dfrac{e^x(1-\frac{x}{e^x})}{e^x(2+\frac{1}{e^x})}$
$=\lim\limits_{x\to+\infty}\dfrac{1-\frac{x}{e^x}}{2+\frac{1}{e^x}}$
$=\dfrac{1}{2}$

ដូចនេះ៖ $\boxed{\lim\limits_{x\to+\infty}\frac{e^x-x}{2e^x+1}=\frac{1}{2}}$

៨ របៀបគណនាលីមីតនៃអនុគមន៍ លោការីត

⚠ សម្គាល់

ដើម្បីគណនាលីមីតនៃអនុគមន៍លោការីត យើងត្រូវចងចាំរូបមន្តមួយចំនួនដូចខាងក្រោម៖

ក. $\lim\limits_{x\to0}\frac{\ln(1+x)}{x}=1$

ខ. $\lim\limits_{x\to0^+}\ln x=-\infty$

គ. $\lim\limits_{x\to+\infty}\ln x=+\infty$

ឃ. $\lim\limits_{x\to+\infty}\frac{\ln x}{x}=0$

ង. $\lim\limits_{x\to1}\frac{\ln x}{x-1}=1$

ច. $\lim\limits_{x\to0^+}x\ln x=0$

ឆ. $\lim\limits_{x\to0^+}x^n\ln x=0; n>0$

ជ. $\lim\limits_{x\to+\infty}\frac{\ln x}{x^n}=0$, $n>0$

ឈ. $\lim\limits_{x\to+\infty}n\ln x=+\infty$, $n>0$

ញ. $\lim\limits_{x\to0}\frac{\log_a(x+1)}{x}=\log_a e$

ដ. $\lim\limits_{x\to0^+}\log_a x=\begin{cases}-\infty, a>0\\+\infty, 0<a<1\end{cases}$

ឋ. $\lim\limits_{x\to+\infty}\log_a x=\begin{cases}+\infty, a>1\\-\infty, 0<a<1\end{cases}$

ដូចនេះ $\boxed{\lim\limits_{x\to 0}\frac{\ln(x^2-x+1)}{x}=-1}$

៩ របៀបគណនាលីមីតរាងមិនកំណត់ $1^\infty ,\infty^0 ,\infty^\infty$

⚠ សម្គាល់

ដើម្បីគណនាលីមីតរាងមិនកំណត់ $1^\infty,\infty^0 ,\infty^\infty ,0^\infty ,0^0$ យើងត្រូវចាំរូបមន្តសំខាន់ៗមួយចំនួនគឺ៖

ក. $\lim\limits_{x\to 0}(1+x)^{\frac{1}{x}}=e$ ខ. $\lim\limits_{x\to\infty}(1+\frac{1}{x})^x=e$ គ. $\lim\limits_{x\to\infty}(1+\frac{a}{x})^x=e^a$

ឃ. $\lim\limits_{x\to 0}\frac{\ln(1+x)}{x}=1$ ង. $\lim\limits_{x\to 0}\frac{e^x-1}{x}=1$ ច. $\lim\limits_{x\to 0}\frac{a^x-1}{x}=\ln a$

លំហាត់គំរូ ២៤

គណនាលីមីតខាងក្រោម៖

ក. $\lim\limits_{x\to\infty}\left(\frac{x}{x+1}\right)^x$ ខ. $\lim\limits_{x\to 0}(1+\sin x)^{\frac{1}{x}}$ គ. $\lim\limits_{x\to\infty}\left(\frac{2x+3}{2x+1}\right)^{x+1}$

ឃ. $\lim\limits_{x\to 0}(\cos x)^{\frac{1}{x}}$ ង. $\lim\limits_{x\to+\infty}x^{\frac{1}{\ln x}}$ ច. $\lim\limits_{x\to+\infty}x^{\frac{1}{x}}$

ឆ. $\lim\limits_{x\to 0^+}x^x$ ជ. $\lim\limits_{x\to 0^+}x^{\frac{1}{\frac{3\ln x}{2}}}$

ដំណោះស្រាយ

ក. $\lim\limits_{x\to\infty}\left(\frac{x}{x+1}\right)^x$, រាងមិនកំណត់ ∞^∞

យើងបាន
$$\begin{aligned}\lim_{x\to\infty}\left(\frac{x}{x+1}\right)^x &= \lim_{x\to\infty}\left(1-\frac{1}{x+1}\right)^x\\
&=\lim_{x\to\infty}\left(1-\frac{1}{x+1}\right)^{-(x+1)\times\frac{x}{-(x+1)}}\\
&=\lim_{x\to\infty}\left[\left(1-\frac{1}{x+1}\right)^{-(x+1)}\right]^{-\frac{x}{x+1}}\\
&=e^{\lim\limits_{x\to\infty}-\frac{x}{x+1}}\\
&=e^{-\lim\limits_{x\to\infty}\frac{\cancel{x}}{\cancel{x}(1+\frac{1}{x})}}\\
&=e^{-\lim\limits_{x\to\infty}\frac{1}{1+\frac{1}{x}}}\\
&=e^{-\frac{1}{1+\frac{1}{\infty}}}\\
&=e^{-1}\\
&=\frac{1}{e}\end{aligned}$$

ដូចនេះ $\boxed{\lim\limits_{x\to\infty}\left(\frac{x}{x+1}\right)^x=\frac{1}{e}}$

ខ. $\lim\limits_{x\to 0}(1+\sin x)^{\frac{1}{x}}$,រាងមិនកំណត់ 1^∞

ដោយ $\lim\limits_{x\to 0}(1+\sin x)^{\frac{1}{x}}=\lim\limits_{x\to 0}(1+\sin x)^{\frac{1}{\sin x}\times\frac{\sin x}{x}}$

យើងបាន
$$\begin{aligned}\lim_{x\to 0}(1+\sin x)^{\frac{1}{\sin x}\times\frac{1}{x}}&=\lim_{x\to 0}\left[(1+\sin x)^{\frac{1}{\sin x}}\right]^{\frac{\sin x}{x}}\\
&=e^{\lim\limits_{x\to 0}\frac{\sin x}{x}}\\
&=e\end{aligned}$$

ដូចនេះ $\boxed{\lim\limits_{x\to 0}(1+\sin x)^{\frac{1}{x}}=e}$

គ. $\lim\limits_{x\to\infty}\left(\frac{2x+3}{2x+1}\right)^{x+1}$,រាងមិនកំណត់ ∞^∞

យើងបាន $\lim\limits_{x\to\infty}\left(\frac{2x+3}{2x+1}\right)^{x+1}=\lim\limits_{x\to\infty}\left(1+\frac{2x+3}{2x+1}-1\right)^{x+1}$

$$=\lim_{x\to\infty}\left(1+\frac{2}{2x+1}\right)^{x+1}$$

$$=\lim_{x\to\infty}\left(1+\frac{1}{\frac{2x+1}{2}}\right)^{x+1}$$

$$=\lim_{x\to\infty}\left(1+\frac{1}{\frac{2x+1}{2}}\right)^{\frac{2x+1}{2}\times\frac{2}{2x+1}\times(x+1)}$$

$$=\lim_{x\to\infty}\left[\left(1+\frac{1}{\frac{2x+1}{2}}\right)^{\frac{2x+1}{2}}\right]^{\frac{2(x+1)}{2x+1}}$$

$$=e^{\lim\limits_{x\to\infty}\frac{\cancel{x}(2+\frac{2}{x})}{\cancel{x}(2+\frac{1}{x})}}$$

$$=e^{\frac{2+\frac{2}{\infty}}{2+\frac{1}{\infty}}}$$

$$=e^{\frac{1+0}{1+0}}$$

$$=e$$

ដូចនេះ $\boxed{\lim\limits_{x\to\infty}\left(\frac{2x+3}{2x+1}\right)^{x+1}=e}$

ឃ. $\lim\limits_{x\to0}(\cos x)^{\frac{1}{x}}$,រាងមិនកំណត់ 1^{∞}

យើងបាន $\lim\limits_{x\to0}(\cos x)^{\frac{1}{x}}=\lim\limits_{x\to0}(1-2\sin^2\frac{x}{2})^{\frac{1}{x}}$

$$=\left[\left(1-2\sin^2\tfrac{x}{2}\right)^{\frac{1}{-2\sin^2\frac{x}{2}}}\right]^{\frac{-2\sin^2\frac{x}{2}}{x}}$$

$$=e^{\lim\limits_{x\to0}\frac{-2\sin^2\frac{x}{2}}{x}}$$

$$=e^{\lim\limits_{x\to0}\left[\frac{\sin\frac{x}{2}}{\frac{x}{2}}\times(-\sin\frac{x}{2})\right]}$$

$$=e^{1\times0}=1$$

ដូចនេះ $\boxed{\lim\limits_{x\to0}(\cos x)^{\frac{1}{x}}=1}$

ង. $\lim\limits_{x\to+\infty}x^{\frac{1}{\ln x}}$, រាងមិនកំណត់ ∞^0

យើងបាន $\lim\limits_{x\to+\infty}x^{\frac{1}{\ln x}}=\lim\limits_{x\to+\infty}\left(e^{\ln x}\right)^{\frac{1}{\ln x}}$

$$=\lim_{x\to+\infty}e^{\frac{\ln x}{\ln x}}$$

$$=e$$

ដូចនេះ $\boxed{\lim\limits_{x\to+\infty}x^{\frac{1}{\ln x}}=e}$

ច. $\lim\limits_{x\to+\infty}x^{\frac{1}{x}}$, រាងមិនកំណត់ ∞^0

យើងបាន $\lim\limits_{x\to+\infty}x^{\frac{1}{x}}=\lim\limits_{x\to+\infty}\left(e^{\ln x}\right)^{\frac{1}{x}}$

$$=\lim_{x\to+\infty}e^{\frac{\ln x}{x}}$$

$$=e^0$$

$$=1$$

ដូចនេះ $\boxed{\lim\limits_{x\to+\infty}x^{\frac{1}{x}}=1}$

ឆ. $\lim\limits_{x\to0^+}x^x$, រាងមិនកំណត់ 0^0

• របៀបទី១

យើងបាន $\lim\limits_{x\to0^+}x^x=\lim\limits_{x\to0^+}[1+(x-1)]^x$

$$=\lim_{x\to0^+}\left\{[1+(x-1)]^{\frac{1}{x-1}}\right\}^{x(x-1)}$$

$$=e^{\lim\limits_{x\to0^+}x(x-1)}$$

$$=e^0$$

$$=1$$

• របៀបទី២

យើងបាន $\lim\limits_{x\to0^+}x^x=\lim\limits_{x\to0^+}\left(e^{\ln x}\right)^x$

$$=\lim_{x\to0^+}e^{\lim\limits_{x\to0^+}x\ln x}$$

$$=e^0$$

$$=1$$

ដូចនេះ $\boxed{\lim\limits_{x\to0^+}x^x=1}$

ជ. $\lim\limits_{x\to0^+}x^{\frac{1}{\frac{3\ln x}{2}}}$

យើងបាន $\lim\limits_{x\to0^+}x^{\frac{1}{\frac{3\ln x}{2}}}=\lim\limits_{x\to0^+}\left(e^{\ln x}\right)^{\frac{1}{\frac{3\ln x}{2}}}$

$$=e^{\lim\limits_{x\to0^+}\frac{\ln x}{\frac{3\ln x}{2}}}$$

$$=e^{\lim\limits_{x\to0^+}\frac{2\ln x}{3\ln x}}$$

$$=e^{\frac{2}{3}}$$

ដូចនេះ $\boxed{\lim\limits_{x\to0^+}x^{\frac{1}{3\ln x}}=e^{\frac{2}{3}}}$

១០ របៀបគណនាលីមីតរាងមិនកំណត់ $0\times\infty$

⚠ សម្គាល់

ដើម្បីគណនាគណនាលីមីតរាងមិនកំណត់ $0\times\infty$ យើងត្រូវបំលែងរាងយ៉ាងណាឲ្យចូលរាង $\frac{0}{0},\frac{\infty}{\infty}$។

លំហាត់គំរូ ២៥

គណនាលីមីតខាងក្រោម៖

ក. $\lim\limits_{x\to+\infty}(x^3+x)e^{-x}$ ខ. $\lim\limits_{x\to+\infty}(x^3e^{-x}+xe^{-1})$ គ. $\lim\limits_{x\to0}x\left(\frac{1+\cos x}{\sin x}\right)$

ឃ. $\lim\limits_{x\to0}\frac{1}{x}\left(\frac{e^x+3x^2}{x^3}\right)$

ដំណោះស្រាយ

គណនាលីមីតខាងក្រោម៖

ក. $\lim\limits_{x\to+\infty}(x^3+x)e^{-x}$, រាងមិនកំណត់ $0\times\infty$

យើងបាន $$\begin{aligned}\lim_{x\to+\infty}(x^3+x)e^{-x} &= \lim_{x\to+\infty}\left[(x^3+x)\times\frac{1}{e^x}\right]\\ &= \lim_{x\to+\infty}\left[\frac{x^3\left(1+\frac{1}{x^2}\right)}{e^x}\right]\\ &= \lim_{x\to+\infty}\frac{x^3}{e^x}\\ &= 0\end{aligned}$$

ដូចនេះ $\boxed{\lim\limits_{x\to+\infty}(x^3+x)e^{-x}=0}$

ខ. $\lim\limits_{x\to+\infty}(x^3e^{-x}+xe^{-1})$, រាងមិនកំណត់ $0\times\infty$

យើងបាន $$\begin{aligned}\lim_{x\to+\infty}(x^3e^{-x}+xe^{-1}) &= \lim_{x\to+\infty}\left(\frac{x^3}{e^x}+\frac{x}{e^x}\right)\\ &= \lim_{x\to+\infty}\frac{x^3}{e^x}+\lim_{x\to+\infty}\frac{x}{e^x}\\ &= 0+0=0\end{aligned}$$

ដូចនេះ $\boxed{\lim\limits_{x\to+\infty}(x^3e^{-x}+xe^{-1})=0}$

គ. $\lim\limits_{x\to0}x\left(\frac{1+\cos x}{\sin x}\right)$, រាងមិនកំណត់ $0\times\infty$

យើងបាន $$\begin{aligned}\lim_{x\to0}x\left(\frac{1+\cos x}{\sin x}\right) &= \lim_{x\to0}\left(\frac{x+x\cos x}{\sin x}\right)\\ &= \lim_{x\to0}\left[\frac{x}{\sin x}+\frac{x\cos x}{\sin x}\right]\\ &= \lim_{x\to0}\frac{x}{\sin x}+\lim_{x\to0}\frac{x\cos x}{\sin x}\\ &= \lim_{x\to0}\frac{x}{\sin x}+\lim_{x\to0}x\cot x\\ &= 1+0\\ &= 1\end{aligned}$$

ដូចនេះ $\boxed{\lim\limits_{x\to0}x\left(\frac{1+\cos x}{\sin x}\right)=1}$

ឃ. $\lim\limits_{x\to0^+}\frac{1}{x}\left(\frac{e^x+3x^2}{x^3}\right)$, រាងមិនកំណត់ $0\times\infty$

យើងបាន $\lim\limits_{x\to+\infty}\frac{1}{x}\left(\frac{e^x+3x^2}{x^3}\right)=\lim\limits_{x\to+\infty}\left(\frac{e^x+3x^2}{x^4}\right)$

$$
\begin{aligned}
&=\lim_{x\to+\infty}\left(\frac{e^x}{x^4}+\frac{3x^2}{x^4}\right)\\
&=\lim_{x\to+\infty}\frac{e^x}{x^4}+\lim_{x\to+\infty}\frac{3x^2}{x^4}\\
&=\lim_{x\to+\infty}\frac{e^x}{x^4}+\lim_{x\to+\infty}\frac{3}{x^2}\\
&=+\infty+0\\
&=+\infty
\end{aligned}
$$

ដូចនេះ $\boxed{\lim\limits_{x\to0^+}\frac{1}{x}\left(\frac{e^x+3x^2}{x^3}\right)=+\infty}$

១១ របៀបគណនាលីមីតខិតជិតអនន្តចំពោះស្វ៊ីតចំនួនពិត

សម្គាល់

រូបមន្តផលបូកស្វ៊ីតសំខាន់មួយចំនួនសម្រាប់ដោះស្រាយលំហាត់

១. $1+2+3+\ldots+n=\frac{n(n+1)}{2}$

២. $2+4+6+\ldots+2n=n(n+1)$

៣. $1+3+5+\ldots+(2n-1)=n^2$

៤. $k+(k+1)+(k+2)+\ldots+(k+n-1)=\frac{n(2k+n-1)}{2}$

៥. $1^2+2^2+3^2+\ldots+n^2=\frac{n(n+1)(2n+1)}{6}$

៦. $1^2+3^2+5^2+\ldots+(2n-1)^2=\frac{n(4n^2-1)}{3}$

៧. $1^3+2^3+3^3+\ldots+n^3=\left[\frac{n(n+1)}{2}\right]^2=\frac{n^2(n+1)^2}{4}$

៨. $1^3+3^3+5^3+\ldots+(2n-1)^3=n^2(2n^2-1)$

៩. $1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\ldots+\frac{1}{2^n}+\ldots=2$

១០. $\frac{1}{1\times2}+\frac{1}{2\times3}+\frac{1}{3\times4}+\ldots+\frac{1}{n\times(n+1)}+\ldots=1$

១១. $1+\frac{1}{1!}+\frac{1}{2!}+\frac{1}{3!}+\ldots+\frac{1}{(n-1)!}+\ldots=e$

១២. ផលបូកស្វ៊ីតនព្វន្ត

- $S_n=a_1+a_2+a_3+\ldots+a_n=\frac{n(a_1+a_n)}{2}$

១៣. ផលបូកស្វ៊ីតធរណីមាត្រ

- $S_n=a_1+a_2+a_3+\ldots+a_n=a_1\times\frac{1-q^n}{1-q}$

លំហាត់គំរូ ២៦

គណនាលីមីតខាងក្រោម៖

ក. $\lim\limits_{n\to\infty}\frac{1+2+3+\ldots+n}{n^2}$ ខ. $\lim\limits_{n\to\infty}\frac{1^2+2^2+3^2+\ldots+n^2}{n^3}$

គ. $\lim\limits_{n\to\infty}\left(\frac{1^3+2^3+3^3+\ldots+n^3}{n^3}-\frac{n}{4}\right)$

ឃ. $\lim\limits_{n\to\infty}\left[\frac{1}{1\times2}+\frac{1}{2\times3}+\frac{1}{3\times4}+\ldots+\frac{1}{n\times(n+1)}\right]$

ដំណោះស្រាយ

គណនាលីមីតខាងក្រោម៖

ក. $\lim\limits_{n\to\infty}\frac{1+2+3+\ldots+n}{n^2}$, រាងមិនកំណត់ $\frac{\infty}{\infty}$

យើងមាន $\lim\limits_{n\to\infty}\dfrac{1+2+3+\ldots+n}{n^2}=\lim\limits_{n\to\infty}\dfrac{n(n+1)}{2n^2}$

$$=\lim_{n\to\infty}\frac{n^2+n}{2n^2}$$

$$=\lim_{n\to\infty}\frac{\cancel{n^2}(1+\frac{1}{n})}{2\cancel{n^2}}$$

$$=\lim_{n\to\infty}\frac{(1+\frac{1}{n})}{2}$$

$$=\frac{(1+\frac{1}{\infty})}{2}$$

$$=\frac{1}{2}$$

ដូចនេះ $\boxed{\lim\limits_{n\to\infty}\frac{1+2+3+\ldots+n}{n^2}=\frac{1}{2}}$

ខ. $\lim\limits_{n\to\infty}\frac{1^2+2^2+3^2+\ldots+n^2}{n^3}$, រាងមិនកំណត់ $\frac{\infty}{\infty}$

យើងបាន $\lim\limits_{n\to\infty}\dfrac{1^2+2^2+3^2+\ldots+n^2}{n^3}=\lim\limits_{n\to\infty}\dfrac{n(n+1)(2n+1)}{6n^3}$

$$=\lim_{n\to\infty}\frac{\cancel{n^3}(1+\frac{1}{n})(2+\frac{1}{n})}{6\cancel{n^3}}$$

$$=\lim_{n\to\infty}\frac{(1+\frac{1}{n})(2+\frac{1}{n})}{6}$$

$$=\frac{(1+\frac{1}{\infty})(2+\frac{1}{\infty})}{6}$$

$$=\frac{(1+0)(2+0)}{6}$$

$$=\frac{1}{3}$$

ដូចនេះ $\boxed{\lim\limits_{n\to\infty}\frac{1^2+2^2+3^2+\ldots+n^2}{n^3}=\frac{1}{3}}$

គ. $\lim\limits_{n\to\infty}\left(\frac{1^3+2^3+3^3+\ldots+n^3}{n^3}-\frac{n}{4}\right)$, រាងមិនកំណត់ $\infty-\infty$

យើងបាន $\lim\limits_{n\to\infty}\left(\dfrac{1^3+2^3+3^3+\ldots+n^3}{n^3}-\dfrac{n}{4}\right)=\lim\limits_{n\to\infty}\left[\dfrac{n^2(n+1)^2}{4n^3}-\dfrac{n}{4}\right]$

$$=\lim_{n\to\infty}\left[\frac{(n+1)^2}{4n}-\frac{n}{4}\right]$$

$$=\lim_{n\to\infty}\left(\frac{n^2+2n+1}{4n}-\frac{n}{4}\right)$$

$$=\lim_{n\to\infty}\frac{n^2+2n+1-n^2}{4n}$$

$$=\lim_{n\to\infty}\frac{2n+1}{4n}$$

$$=\lim_{n\to\infty}\frac{\cancel{n}(2+\frac{1}{n})}{4\cancel{n}}$$

$$=\lim_{n\to\infty}\frac{2+\frac{1}{n}}{4}$$

$$=\frac{2+\frac{1}{\infty}}{4}$$

$$=\frac{1}{2}$$

ដូចនេះ $\boxed{\lim\limits_{n\to\infty}\left(\frac{1^3+2^3+3^3+\ldots+n^3}{n^3}-\frac{n}{4}\right)=\frac{1}{2}}$

ឃ. $\lim\limits_{n\to\infty}\left[\frac{1}{1\times2}+\frac{1}{2\times3}+\frac{1}{3\times4}+\ldots+\frac{1}{n\times(n+1)}\right]$, រាងមិនកំណត់ $\frac{\infty}{\infty}$

យើង មាន $\lim\limits_{n\to\infty}\left[\frac{1}{1\times2}+\frac{1}{2\times3}+\frac{1}{3\times4}+\ldots+\frac{1}{n\times(n+1)}\right]$ អាច សរសេរទៅជា

$$=\lim_{n\to\infty}\left[\left(1-\tfrac{1}{2}\right)+\left(\tfrac{1}{2}-\tfrac{1}{3}\right)+\left(\tfrac{1}{3}-\tfrac{1}{4}+\ldots+\tfrac{1}{n}\right)+\tfrac{1}{n+1}\right]$$

$$=\lim_{n\to\infty}\left(1-\tfrac{1}{n+1}\right)$$

$$=1-\tfrac{1}{\infty}$$

$$=1$$

ដូចនេះ $\boxed{\lim\limits_{n\to\infty}\left[\frac{1}{1\times2}+\frac{1}{2\times3}+\frac{1}{3\times4}+\ldots+\frac{1}{n\times(n+1)}\right]=1}$

១២ របៀបគណនាលីមីតរាងមិនកំណត់ ដោយប្រើទ្រឹស្តីបទឡូពីតាល់

រូបមន្តដេរីវេមួយចំនួនដែលចាំបាច់សម្រាប់ដោះស្រាយលំហាត់

អនុគមន៍	ដេរីវេ
$y=a$	$y'=0$
$y=x^n$	$y'=nx^{n-1}$
$y=\frac{1}{x}$	$y'=-\frac{1}{x^2}$
$y=\sqrt{x}$	$y'=\frac{1}{2\sqrt{x}}$
$y=e^x$	$y'=e^x$
$y=a^x$	$y'=a^x\ln a$
$y=\ln x$	$y'=\frac{1}{x}$
$y=\sin x$	$y'=\cos x$
$y=\cos x$	$y'=-\sin x$
$y=\tan x$	$y'=\frac{1}{\cos^2 x}=1+\tan^2 x$
$y=\cot x$	$y'=-\frac{1}{\sin^2 x}=1+\cot^2 x$
$y=u^n$	$y'=nu'u^{n-1}$
$y=\sqrt{u}$	$y'=\frac{u'}{2\sqrt{u}}$
$y=u.v$	$y'=u'v+v'u$
$y=\frac{u}{v}$	$y'=\frac{u'v-v'u}{v^2}$
$y=\ln u$	$y'=\frac{u'}{u}$
$y=\sin u$	$y'=u'\cos u$
$y=\cos u$	$y'=-u'\sin u$
$y=\tan u$	$y'=u'(1+tan^2 u)$
$y=\cot u$	$y'=-u'(1+\cot^2 u)$
$y=e^u$	$y'=u'e^u$

⚠ សម្គាល់

• ជាទូទៅ៖ ដើម្បីគណនាលីមីតដោយប្រើទ្រឹស្តីបទឡូពីតាល់ អាចគណនាបានគឺ ទម្រង់$\frac{0}{0}$ និង $\frac{\infty}{\infty}$ ។ ប៉ុន្តែបើក្រៅពីរាងទាំងពីរ យើងត្រូវបំលែងយ៉ាងណាឲ្យចូលទម្រង់ $\frac{0}{0}$ និង $\frac{\infty}{\infty}$ ។

• ដើម្បីគណនាលីមីតដោយប្រើទ្រឹស្តីបទឡូពីតាល់យើងត្រូវធ្វើដេរីវេ ភាគយក និង ភាគបែង រហូតក្លាយជារាងកំណត់ថ្មីមួយរួច គណនាលីមីតថ្មីនោះ។

$\frac{f(x)}{g(x)}=\frac{f(x')}{g(x')}=\frac{f(x")}{g(x")}=\ldots=\frac{f(x^n)}{g(x^n)}=A,\forall\in\mathbb{R}$

លំហាត់គំរូ ២៧

គណនាលីមីខាងក្រោមដោយប្រើទ្រឹស្តីបទឡូពីតាល់

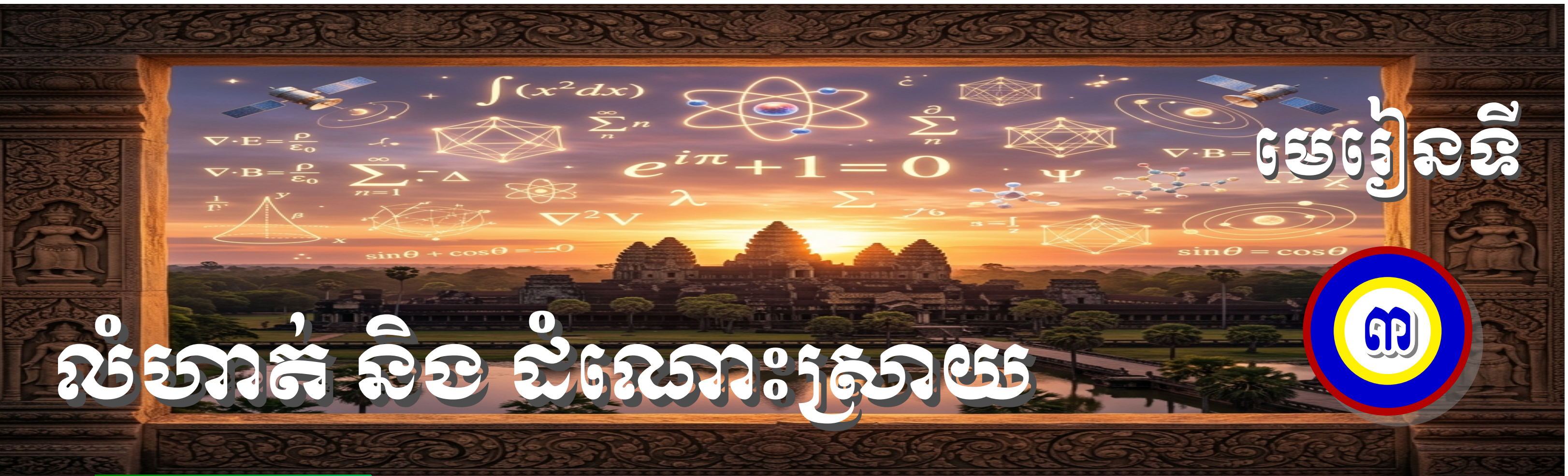
ក. $\lim_{x\to\infty}\frac{x^2-1}{2x^2+x+2}$ ខ. $\lim_{x\to 2}\frac{\sqrt{x+7}-3}{x-2}$ គ. $\lim_{x\to 1}\frac{1+\cos\pi x}{(x-1)^2}$

ឃ. $\lim_{x\to 0}\frac{e^{ax}-1}{x}$

ដំណោះស្រាយ

គណនាលីមីខាងក្រោមដោយប្រើទ្រឹស្តីបទឡូពីតាល់

ដូចនេះ: $\boxed{\lim_{x\to 0}\frac{e^{ax}-1}{x}=a}$



លំហាត់គំរូទី១

ដោយប្រើនិយមន័យចូរបង្ហាញថា

ក. $\lim\limits_{x\to -2}\frac{x^2-1}{x+1}=-3$ ខ. $\lim\limits_{x\to a}(mx+b)=ma+b$ គ. $\lim\limits_{x\to 0}\sqrt[3]{x}=0$

ឃ. $\lim\limits_{x\to 2}\sqrt{x}=\sqrt{2}$ ង. $\lim\limits_{x\to 2^-}\sqrt{(x-2)(x-3)}=0$ ច. $\lim\limits_{x\to 2}\frac{x^2-1}{x}=\frac{3}{2}$

ដំណោះស្រាយ

ដោយប្រើនិយមន័យចូរបង្ហាញថា

ក. $\lim\limits_{x\to -2}\frac{x^2-1}{x+1}=-3$

តាមនិយមន័យ: $\forall\,\varepsilon>0\,,\,|f(x)-L|<\varepsilon$

យើងមាន$|f(x)-L|<\varepsilon=\left|\dfrac{x^2-1}{x+1}-(-3)\right|<\varepsilon$

$$\Leftrightarrow\left|\frac{(x+2)(x+1)}{x+1}\right|<\varepsilon$$

$$\Leftrightarrow|x+2|<\varepsilon=\alpha$$

ដោយ$\forall\,\varepsilon>0\,,\exists\,\alpha=\varepsilon>0$ ដែល $|x+2|<\varepsilon$ នាំឱ្យ $|f(x)-L|<\varepsilon$

ដូចនេះ: $\boxed{\lim\limits_{x\to -2}\frac{x^2-1}{x+1}=-3}$

ខ. $\lim\limits_{x\to a}(mx+b)=ma+b$

តាមនិយមន័យ: $\forall\,\varepsilon>0\,,\,|f(x)-L|<\varepsilon$

យើងមាន$|f(x)-L|<\varepsilon\Leftrightarrow|(mx+b)-(ma+b)|<\varepsilon$

$$\Leftrightarrow|mx+b-ma-b|<\varepsilon$$

$$\Leftrightarrow|m(x-a)|<\varepsilon$$

$$\Leftrightarrow|m||x-a|<\varepsilon$$

$$\Leftrightarrow|x-a|<\frac{\varepsilon}{|m|}$$

ដោយ$\forall\ \varepsilon>0\ ,\ \exists\ \alpha=\frac{\varepsilon}{|m|}>0$ ដែល $|x-a|<\varepsilon$ នាំឱ្យ $|f(x)-L|<\varepsilon$

ដូចនេះ: $\boxed{\lim\limits_{x\to a}(mx+b)=ma+b}$

គ. $\lim\limits_{x\to 0}\sqrt[3]{x}=0$

តាមនិយមន័យ: $\forall\,\varepsilon>0\,,\,|f(x)-L|<\varepsilon$

យើងមាន$|f(x)-L|<\varepsilon\Leftrightarrow|\sqrt[3]{x}-0|<\varepsilon$

$$\Leftrightarrow|\sqrt[3]{x}|<\varepsilon$$

$$\Leftrightarrow(|\sqrt[3]{x}|)^3<\varepsilon^3$$

$$\Leftrightarrow|x|<\varepsilon^3$$

ដោយ$\forall\,\varepsilon>0\,,\exists\,\alpha=\varepsilon^3>0$ ដែល $|x-0|<\varepsilon$ ឬ $|x|<\varepsilon$ នាំឱ្យ $|f(x)-L|<\varepsilon$

ដូចនេះ: $\boxed{\lim\limits_{x\to 0}\sqrt[3]{x}=0}$

ឃ. $\lim\limits_{x\to 2}\sqrt{x}=\sqrt{2}$

តាមនិយមន័យ: $\forall\,\varepsilon>0\,,\,|f(x)-L|<\varepsilon$

យើងមាន$|f(x)-L|<\varepsilon\Leftrightarrow|\sqrt{x}-\sqrt{2}|<\varepsilon$

$$\Leftrightarrow|(\sqrt{x}-\sqrt{2})(\sqrt{x}+\sqrt{2})|<\varepsilon(\sqrt{x}+\sqrt{2})$$

$$\Leftrightarrow|x-2|<\varepsilon(\sqrt{x}+\sqrt{2})$$

ដោយ$\forall\,\varepsilon>0\,,\exists\,\alpha=\varepsilon\left(\sqrt{x}+\sqrt{2}\right)>0$ ដែល $|x-2|<\varepsilon$ នាំឱ្យ $|f(x)-L|<\varepsilon$

ដូចនេះ: $\boxed{\lim\limits_{x\to 2}\sqrt{x}=\sqrt{2}}$

ង. $\lim\limits_{x\to 2^-}\sqrt{(x-2)(x-3)}=0$

តាមនិយមន័យ: $\forall\ \varepsilon>0\ ,\ |f(x)-L|<\varepsilon$

យើងមាន$|f(x)-L|<\varepsilon\Leftrightarrow|\sqrt{(x-2)(x-3)}-0|<\varepsilon$

$$\Leftrightarrow|\sqrt{(x-2)(x-3)}|<\varepsilon$$
$$\Leftrightarrow(|\sqrt{(x-2)(x-3)}|)^2<\varepsilon^2$$
$$\Leftrightarrow|(x-2)(x-3)|<\varepsilon^2$$
$$\Leftrightarrow|x-2||x-3|<\varepsilon^2$$
$$\Leftrightarrow|x-2|<\frac{\varepsilon^2}{|x-3|}(*)$$

ដោយ $x\to 2^-$ យើងបាន៖

$$1<x<2\Leftrightarrow 1-3<x-3<2-3$$
$$\Leftrightarrow -2<x-3<-1$$
$$\Leftrightarrow\frac{-2}{-1}<\frac{x-3}{-1}<\frac{-1}{-1}$$
$$\Leftrightarrow 1<-(x-3)<2$$
$$\Leftrightarrow|-(x-3)|<2$$
$$\Leftrightarrow|x-3|<2(**)$$

តាម $(*)$ និង $(**)$ យើងបាន $|x-2^-|<\frac{\varepsilon^2}{2}$

ដោយ$\forall\ \varepsilon>0\ ,\ \exists\ \alpha=\frac{\varepsilon^2}{2}>0$ ដែល $|x-2|<\varepsilon$

នាំឱ្យ $|f(x)-L|<\varepsilon$

ដូចនេះ: $\boxed{\lim\limits_{x\to 2^-}\sqrt{(x-2)(x-3)}=0}$

ច. $\lim\limits_{x\to 2}\frac{x^2-1}{x}=\frac{3}{2}$

តាមនិយមន័យ: $\forall\ \varepsilon>0\ ,\ |f(x)-L|<\varepsilon$

យើងមាន$|f(x)-L|<\varepsilon\Leftrightarrow\left|\frac{x^2-1}{x}-\frac{3}{2}\right|<\varepsilon$

$$\Leftrightarrow\left|\frac{2(x^2-1)-3x}{2x}\right|<\varepsilon$$
$$\Leftrightarrow\left|\frac{2x^2-3x-2}{2x}\right|<\varepsilon$$
$$\Leftrightarrow\left|\frac{(x-2)(2x+1)}{2x}\right|<\varepsilon$$
$$\Leftrightarrow\frac{|x-2||2x+1|}{|2x|}<\varepsilon$$
$$\Leftrightarrow\frac{|x-2||2x+1|}{|2x|}\times\frac{|2x|}{|2x+1|}<\varepsilon\frac{|2x|}{|2x+1|}$$
$$\Leftrightarrow|x-2|<\varepsilon\frac{|2x|}{|2x+1|}$$

ដោយ $x\to 2$ យើងបាន៖

ចំពោះ $|2x|$

$$1<x<3\Leftrightarrow 2<2x<6$$
$$\Leftrightarrow|2x|<6(*)$$

ហើយម្យ៉ាងទៀតចំពោះ $|2x+1|$

$$1<x<3\Leftrightarrow 2<2x<6$$
$$\Leftrightarrow 2+1<2x+1<6+1$$
$$\Leftrightarrow 3<2x+1<7$$
$$\Leftrightarrow|2x+1|<7(**)$$

តាម$(*)$ និង $(**)$យើងបាន$|x-2|<\frac{6}{7}\varepsilon$

ដោយ$\forall\ \varepsilon>0\ ,\ \exists\ \alpha=|x-2|<\frac{6}{7}\varepsilon$ ដែល $|x-2|<\frac{6}{7}\varepsilon$ នាំឱ្យ $|f(x)-L|<\varepsilon$

ដូចនេះ: $\boxed{\lim\limits_{x\to 2}\frac{x^2-1}{x}=\frac{3}{2}}$

លំហាត់គំរូទី២

ដោយប្រើនិយមន័យចូរបង្ហាញថា

ក. $\lim\limits_{x\to 2^-}\frac{3x-1}{x-2}=-\infty$ ខ. $\lim\limits_{x\to 2^+}\frac{4x+1}{x-2}=+\infty$ គ. $\lim\limits_{x\to-\infty}\frac{x^2-2}{x^2+2}=1$

ឃ. $\lim\limits_{x\to+\infty}\frac{3x+1}{x}=3$ ង. $\lim\limits_{x\to-\infty}(x^2-x-1)=+\infty$ ច. $\lim\limits_{x\to+\infty}\frac{x^2+3}{2-x}=-\infty$

ដំណោះស្រាយ

ដោយប្រើនិយមន័យចូរបង្ហាញថា

ក. $\lim\limits_{x\to 2^-}\frac{3x-1}{x-2}=-\infty$

តាមនិយមន័យ $\forall M>0\ ,\ f(x)<-M$

យើងមាន$f(x)<-M\Leftrightarrow\frac{3x-1}{x-2}<-M$

$$\Leftrightarrow 3+\frac{5}{x-2}<-M$$
$$\Leftrightarrow\frac{5}{x-2}<-M-3$$
$$\Leftrightarrow\frac{5}{x-2}<-(M+3)$$
$$\Leftrightarrow\left|\frac{5}{x-2}\right|<|-(M+3)|$$
$$\Leftrightarrow\frac{5}{|x-2|}<M+3$$
$$\Leftrightarrow|x-2|>\frac{5}{M+3}$$

យក $\alpha = \frac{5}{M+3} > 0$
យើងបាន $\forall M > 0$, $\exists \alpha = \frac{5}{M+3}$ ដែល $|x-2| < \alpha$
នាំឱ្យ $f(x) < -M$

ដូចនេះ $\boxed{\lim\limits_{2^-} \frac{3x-1}{x-2} = -\infty}$

ខ. $\lim\limits_{x\to 2^+} \frac{4x+1}{x-2} = +\infty$
តាមនិយមន័យ $\forall M > 0$, $f(x) > M$
យើងមាន $f(x) > M \Leftrightarrow \dfrac{4x+1}{x-2} > M$
$$\Leftrightarrow 4 + \frac{9}{x-2} > M$$
ដើម្បីឲ្យ $4 + \frac{9}{x-2} > M$ យើងគ្រាន់តែយក $\frac{9}{x-2} > M$
យើងបាន $\frac{9}{x-2} > M \Leftrightarrow |x-2| < \frac{9}{M}$ យក $\alpha = \frac{9}{M} > 0$
យើងបាន $\forall M > 0$, $\exists \alpha = \frac{9}{M}$ ដែល $|x-2| < \alpha$ នាំឱ្យ $f(x) > M$

ដូចនេះ $\boxed{\lim\limits_{2^+} \frac{4x+1}{x-2} = +\infty}$

គ. $\lim\limits_{x\to -\infty} \frac{x^2-2}{x^2+2} = 1$
តាមនិយមន័យ $\forall \varepsilon > 0$, $|f(x) - L| < \varepsilon$
យើងមាន $|f(x)-L| < \varepsilon \Leftrightarrow \left|\dfrac{x^2-2}{x^2+2} - 1\right| < \varepsilon$
$$\Leftrightarrow \left|\frac{-4}{x^2+2}\right| < \varepsilon$$
$$\Leftrightarrow \frac{4}{x^2+2} < \varepsilon$$
$$\Leftrightarrow x^2 + 2 > \frac{4}{\varepsilon}$$
$$\Leftrightarrow x^2 < \frac{4}{\varepsilon}$$
$$\Leftrightarrow |x| < \sqrt{\frac{4}{\varepsilon}}$$
ដោយ $x \to -\infty$ នោះ $|x| = -x$
យើងបាន $-x < \sqrt{\frac{4}{\varepsilon}} \Leftrightarrow x > -\frac{2}{\sqrt{\varepsilon}}$ យក $N = \frac{2}{\sqrt{\varepsilon}} > 0$
យើងបាន $\forall \varepsilon > 0 < \exists N = \frac{2}{\varepsilon}$ ដែល $x < -N$ នាំឱ្យ $|f(x) - L| < \varepsilon$

ដូចនេះ $\boxed{\lim\limits_{x\to -\infty} \frac{x^2-2}{x^2+2} = 1}$

ឃ. $\lim\limits_{x\to +\infty} \frac{3x+1}{x} = 3$
តាមនិយមន័យ $\forall \varepsilon > 0$, $|f(x) - L| < \varepsilon$
យើងមាន $|f(x)-L| < \varepsilon \Leftrightarrow \left|\dfrac{3x+1}{x} - 3\right| < \varepsilon$
$$\Leftrightarrow \left|\frac{(3x+1)-3x}{x}\right| < \varepsilon$$
$$\Leftrightarrow \left|\frac{3x+1-3x}{x}\right| < \varepsilon$$
$$\Leftrightarrow \left|\frac{1}{x}\right| < \varepsilon$$
$$\Leftrightarrow \frac{1}{|x|} < \varepsilon$$
$$\Leftrightarrow |x| > \frac{1}{\varepsilon}$$
ដោយ $x \to +\infty$ នោះ $|x| = x \Rightarrow |x| > \frac{1}{\varepsilon} = x > \frac{1}{\varepsilon}$ យក $N = \frac{1}{\varepsilon} > 0$
យើងបាន $\forall \varepsilon > 0$, $\exists N = \frac{1}{\varepsilon}$ ដែល $x > N$ នាំឱ្យ $|f(x) - L| < \varepsilon$

ដូចនេះ $\boxed{\lim\limits_{x\to +\infty} \frac{3x+1}{x} = 3}$

ង. $\lim\limits_{x\to -\infty} (x^2 - x - 1) = +\infty$
តាមនិយមន័យ $\forall M > 0$, $f(x) > M$
យើងមាន $|f(x) > M| \Leftrightarrow x^2 - x - 1 > M$
$$\Leftrightarrow \left(x - \frac{1}{2}\right)^2 - \frac{5}{4} > M$$
$$\Leftrightarrow \left(x - \frac{1}{2}\right)^2 > M + \frac{5}{4}$$
$$\Leftrightarrow \sqrt{\left(x-\frac{1}{2}\right)^2} > \sqrt{M + \frac{5}{4}}$$
$$\Leftrightarrow \left|x - \frac{1}{2}\right| > \sqrt{M + \frac{5}{4}}$$
ដោយ $x \to -\infty$ នោះ $|x - \frac{1}{2} \to| = -(x - \frac{1}{2})$ យើងបាន៖ $-(x-\frac{1}{2}) > \sqrt{M + \frac{5}{4}} \Leftrightarrow x - \frac{1}{2} < -\sqrt{M+\frac{5}{4}}$, យក $N = \sqrt{M + \frac{5}{4}} > 0$
យើង បាន $\forall M > 0$, $\exists N = \sqrt{M+\frac{5}{4}}$ ដែល $x < -N$ នាំឱ្យ $f(x) > M$

ដូចនេះ $\boxed{\lim\limits_{x\to -\infty} (x^2 - x - 1) = +\infty}$

ច. $\lim\limits_{x\to +\infty} \frac{x^2+3}{2-x} = -\infty$
តាមនិយមន័យ $\forall M > 0$, $f(x) > M$
យើងមាន $|f(x), < -M \Leftrightarrow \dfrac{x^2+3}{2-x} < -M$
$$\Leftrightarrow -x - 2 + \frac{7}{2-x} < -M$$
ដោយ $x \to +\infty$ នោះ $\frac{7}{2-x} \to 0$ យើងបាន $-x - 2 + \frac{7}{2-x} < -M$
អាចសរសេរ ៖ $-x-2 < -M \Leftrightarrow x > M+2$ យក $N = M+2 > 0$
យើង បាន $\forall M + 2 > 0$, $\exists N = M+2$ ដែល $x > N$ នាំឱ្យ $f(x) < -M$

ដូចនេះ $\boxed{\lim\limits_{x\to +\infty} \frac{x^2+3}{2-x} = -\infty}$

លំហាត់គំរូទី៣

គណនាតម្លៃ $\alpha > 0$ ទៅតាមតម្លៃ ε នីមួយៗដែលគេឲ្យដូចខាងក្រោម៖

ក. $\lim\limits_{x\to 2}(x^2+x+6)=0 \quad ,\varepsilon=10^{-7}$

ខ. $\lim\limits_{x\to -3}(x^2+x-6)=0 \quad ,\varepsilon=10^{-9}$

គ. $\lim\limits_{x\to -1}(3-4x)=7 \quad ,\varepsilon=2\times 10^{-2}$

ឃ. $\lim\limits_{x\to 3}x^2=9 \quad ,\varepsilon=5\times 10^{-3}$

ង. $\lim\limits_{x\to -2}\frac{x^2-4}{x+2}=-4 \quad ,\varepsilon=10^{-2}$

ច. $\lim\limits_{x\to -2}(2+5x)=-8 \quad ,\varepsilon=2\times 10^{-5}$

ដំណោះស្រាយ

គណនាតម្លៃ $\alpha > 0$ ទៅតាមតម្លៃ ε នីមួយៗដែលគេឲ្យ

ក. $\lim\limits_{x\to 2}(x^2+x+6)=0 \quad ,\varepsilon=10^{-7}$

តាមនិយមន័យ

យើងបាន$|f(x)-L|<\varepsilon \Leftrightarrow |(x^2+x-6)-0|<\varepsilon$

$\Leftrightarrow |(x+3)(x-2)|<\varepsilon$

$\Leftrightarrow |x+3||x-2|<\varepsilon(*)$

ដោយ$x\to 2$

យើងបាន $1<x<3 \Leftrightarrow 1+3<x+3<3+3$

$\Leftrightarrow 4<|x+3|<6$

$\Leftrightarrow |x+3|<6(**)$

តាម $(*)$ និង $(**)$ យើងបាន

$|x+3||x-2|<\varepsilon \Leftrightarrow 6|x-2|<\varepsilon$

$\Leftrightarrow |x-2|<\frac{\varepsilon}{6}=\alpha$,តែ$\varepsilon=10^{-7}$

$\Leftrightarrow \alpha=\frac{10^{-7}}{6}$

ដូចនេះ $\boxed{\text{បើ } \varepsilon=10^{-7} \text{ នោះ } \alpha=\frac{10^{-7}}{6}}$

ខ. $\lim\limits_{x\to -3}(x^2+x-6)=0 \quad ,\varepsilon=10^{-9}$

តាមនិយមន័យ

យើងបាន$|f(x)-L|<\varepsilon \Leftrightarrow |(x^2+x-6)-0|<\varepsilon$

$\Leftrightarrow |(x+3)(x-2)|<\varepsilon$

$\Leftrightarrow |x+3||x-2|<\varepsilon(*)$

ដោយ$x\to -3$

យើងបាន $-4<x<-2 \Leftrightarrow -6<x-2<-4$

$\Leftrightarrow 4<|x-2|<6$

$\Leftrightarrow |x-2|<6(**)$

តាម $(*)$ និង $(**)$

យើងបាន$|x+3||x-2|<\varepsilon \Leftrightarrow 6|x+3|<\varepsilon$

$\Leftrightarrow |x+3|<\frac{\varepsilon}{6}=\alpha$,តែ$\varepsilon=10^{-9}$

$\Leftrightarrow \alpha=\frac{10^{-9}}{6}$

ដូចនេះ $\boxed{\text{បើ } \varepsilon=10^{-9} \text{ នោះ } \alpha=\frac{10^{-9}}{6}}$

គ. $\lim\limits_{x\to -1}(3-4x)=7 \quad ,\varepsilon=2\times 10^{-2}$

តាមនិយមន័យ

យើងបាន$|f(x)-L|<\varepsilon \Leftrightarrow |(3-4x)-7|<\varepsilon$

$\Leftrightarrow |-4(x+1)|<\varepsilon$

$\Leftrightarrow |x+1|<\frac{\varepsilon}{4}=\alpha$,តែ$\varepsilon=10^{-2}$

$\Leftrightarrow \alpha=\frac{10^{-2}}{4}$

ដូចនេះ $\boxed{\text{បើ } \varepsilon=10^{-2} \text{ នោះ } \alpha=\frac{10^{-2}}{4}}$

ឃ. $\lim\limits_{x\to 3}x^2=9 \quad ,\varepsilon=5\times 10^{-3}$ តាមនិយមន័យ

យើងបាន$|f(x)-L|<\varepsilon \Leftrightarrow |x^2-9|<\varepsilon$

$\Leftrightarrow |(x+3)(x-3)|<\varepsilon(*)$

ដោយ $x\to 3$

យើងបាន $3<x<4 \Leftrightarrow 6<x+3<7$

$\Leftrightarrow |x+3|<7(**)$

តាម $(*)$ និង $(**)$ យើងបាន

$|(x+3)(x-3)|<\varepsilon \Leftrightarrow 7|x-3|<\varepsilon$

$\Leftrightarrow |x-3|=\frac{\varepsilon}{7}=\alpha$,តែ$\varepsilon=5\times 10^{-3}$

$\Leftrightarrow \alpha=\frac{5\times 10^{-3}}{7}$

ដូចនេះ $\boxed{\text{បើ } \varepsilon=5\times 10^{-3} \text{ នោះ } \alpha=\frac{5\times 10^{-3}}{7}}$

ង. $\lim_{x\to -2}\frac{x^2-4}{x+2}=-4\ \ ,\varepsilon=10^{-2}$

តាមនិយមន័យ

យើងបាន$|f(x)-L|<\varepsilon \Leftrightarrow \left|\dfrac{x^2-4}{x+2}-(-4)\right|<\varepsilon$

$$\Leftrightarrow \left|\dfrac{(x^2-4)+4(x+2)}{x+2}\right|<\varepsilon$$

$$\Leftrightarrow \left|\dfrac{x^2-4+4x+8}{x+2}\right|<\varepsilon$$

$$\Leftrightarrow \left|\dfrac{x^2+4x+4}{x+2}\right|<\varepsilon$$

$$\Leftrightarrow \left|\dfrac{(x+2)^2}{x+2}\right|<\varepsilon$$

$$\Leftrightarrow |x+2|<\varepsilon=\alpha$$

ដូចនេះ $\boxed{\text{បើ } \varepsilon=10^{-2} \text{ នោះ } \alpha=10^{-2}}$

ច. $\lim_{x\to -2}(2+5x)=-8\ \ ,\varepsilon=2\times 10^{-5}$

តាមនិយមន័យ

យើងបាន$|f(x)-L|<\varepsilon \Leftrightarrow |(2+5x)-(-8)<\varepsilon$

$$\Leftrightarrow |2+5x+8|<\varepsilon$$

$$\Leftrightarrow |5x+10|<\varepsilon$$

$$\Leftrightarrow |5(x+2)|<\varepsilon$$

$\Leftrightarrow |x+2|<\dfrac{\varepsilon}{5}=\alpha,$តែ $\varepsilon=2\times 10^{-2}$

$$\Leftrightarrow \alpha=\dfrac{2\times 10^{-2}}{5}$$

ដូចនេះ $\boxed{\text{បើ } \varepsilon=2\times 10^{-2} \text{ នោះ } \alpha=\frac{2\times 10^{-2}}{5}}$

លំហាត់គំរូទី៤

ចូររកតម្លៃធំបំផុតនៃ $\alpha>0$ ដែលផ្ទៀងផ្ទាត់លក្ខខណ្ឌខាងក្រោម៖

ក. $0<|x+2|<\alpha$ ដែល $0<|3x+6|<0.000009$

ខ. $0<|x+3|<\alpha$ ដែល $0<\left|\frac{4x+12}{x+3}\right|<0.000000044$

ដំណោះស្រាយ

ចូររកតម្លៃធំបំផុតនៃ $\alpha>0$ ដែលផ្ទៀងផ្ទាត់លក្ខខណ្ឌខាងក្រោម៖

ក. $0<|x+2|<\alpha$ ដែល $0<|3x+6|<0.000009$

យើមាន$0<|3x+6|<0.000009 \Leftrightarrow 0<|3(x+2)|<0.000009$

$$\Leftrightarrow 0<|x+2|<\tfrac{0.000009}{3}$$

$$\Leftrightarrow 0<|x+2|<0.000003$$

យើងទាញបាន $\alpha=0.000003$

ដូចនេះ $\boxed{\text{តម្លៃធំបំផុតនៃ } \alpha=0.000003}$

ខ. $0<|x+3|<\alpha$ ដែល $0<\left|\frac{4x^2+24x+36}{x+3}\right|<0.000000044$

យើងមាន $0<\left|\dfrac{4x^2+24x+36}{x+3}\right|<0.000000044$

$$\Leftrightarrow 0<\left|\dfrac{4(x^2+6x+9)}{x+3}\right|<0.000000044$$

$$\Leftrightarrow 0<\left|\dfrac{4(x+3)^2}{x+3}\right|<0.000000044$$

$$\Leftrightarrow 0<|4(x+3)|<0.000000044$$

$$\Leftrightarrow 0<|(x+3)|<\tfrac{0.000000044}{4}$$

$$\Leftrightarrow 0<|(x+3)|<0.000000011$$

យើងទាញបាន $\alpha=0.000000011$

ដូចនេះ $\boxed{\text{តម្លៃធំបំផុតនៃ } \alpha=0.000000011}$

លំហាត់គំរូទី៥

គណនាលីមីតខាងក្រោម៖

ក. $\lim_{x\to 0}2020$

ខ. $\lim_{x\to 0}\frac{2020}{2021}$

គ. $\lim_{x\to -2}(3x+4)$

ឃ. $\lim_{x\to 2}(x^2-x)$

ង. $\lim_{x\to 2}(5x^4-3x^3-x+2)$

ច. $\lim_{x\to 1}(x^2+x)(3x-4)$

ឆ. $\lim_{x\to 2}\frac{2x^2-1}{x-4}$

ជ. $\lim_{x\to 0}\frac{\sin x}{x+1}$

ឈ. $\lim_{x\to \frac{\pi}{3}}(\sin x+\cos x)$

ញ. $\lim\limits_{x\to\frac{\pi}{3}}\frac{\tan x+2}{\sin x+1}$ ដ. $\lim\limits_{x\to\frac{\pi}{6}}\frac{\tan(2x)+\cot 3x}{\sin x}$ ឋ. $\lim\limits_{x\to 1}(\ln x+\ln 4x)$

ឌ. $\lim\limits_{x\to 3}e^{x+1}.x$ ឍ. $\lim\limits_{x\to 3}\frac{xe^{\frac{x}{3}}}{x+1}$

ដំណោះស្រាយ

គណនាលីមីតខាងក្រោម៖

ក. $\lim\limits_{x\to 0}2020$

តាមរូបមន្ត $\lim\limits_{x\to a}K=K$

យើងបាន $\lim\limits_{x\to 0}2020=2020$

ដូចនេះ $\boxed{\lim\limits_{x\to 0}2020=2020}$

ខ. $\lim\limits_{x\to 0}\frac{2020}{2021}$

យើងបាន៖ $\lim\limits_{x\to 0}\frac{2020}{2021}=\frac{2020}{2021}$

ដូចនេះ $\boxed{\lim\limits_{x\to 0}\frac{2020}{2021}=\frac{2020}{2021}}$

គ. $\lim\limits_{x\to -2}(3x+4)$

យើងបាន $\lim\limits_{x\to -2}(3x+4)=3(-2)+4$

ដូចនេះ $\boxed{\lim\limits_{x\to -2}(3x+4)=-2}$

ឃ. $\lim\limits_{x\to 2}(x^2-x)$

យើងបាន $\lim\limits_{x\to 2}(x^2-x)=2^2-2$

$=2$

ដូចនេះ $\boxed{\lim\limits_{x\to 2}(x^2-x)=2}$

ង. $\lim\limits_{x\to 2}(5x^4-3x^3-x+2)$

យើងបាន $\lim\limits_{x\to 2}(5x^4-3x^3-x+2)=2\times 2^4-3\times 2^3-2+2$

$=8$

ដូចនេះ $\boxed{\lim\limits_{x\to 2}(5x^4-3x^3-x+2)=8}$

ច. $\lim\limits_{x\to 1}(x^2+x)(3x-4)$

យើងបាន $\lim\limits_{x\to 1}(x^2+x)(3x-4)=(1^2+1)(3\times 1-4)$

$=-2$

ដូចនេះ $\boxed{\lim\limits_{x\to 1}(x^2+x)(3x-4)=-2}$

ឆ. $\lim\limits_{x\to 2}\frac{2x^2-1}{x-4}$

យើងបាន $\lim\limits_{x\to 2}\frac{2x^2-1}{x-4}=\frac{2\times 2^2-1}{2-4}$

$=\frac{7}{-2}$

ដូចនេះ $\boxed{\lim\limits_{x\to 2}\frac{2x^2-1}{x-4}=-\frac{7}{2}}$

ជ. $\lim\limits_{x\to 0}\frac{\sin x}{x+1}$

យើងបាន $\lim\limits_{x\to 0}\frac{\sin x}{x+1}=\frac{\sin 0}{0+1}$

$=\frac{0}{0+1}=0$

ដូចនេះ $\boxed{\lim\limits_{x\to 0}\frac{\sin x}{x+1}=0}$

ឈ. $\lim\limits_{x\to\frac{\pi}{3}}(\sin x+\cos x)$

យើងបាន $\lim\limits_{x\to\frac{\pi}{3}}(\sin x+\cos x)=\sin\frac{\pi}{3}+\cos\frac{\pi}{3}$

$=\frac{\sqrt{3}}{2}+\frac{1}{2}$

$=\frac{\sqrt{3}+1}{2}$

ដូចនេះ $\boxed{\lim\limits_{x\to\frac{\pi}{3}}(\sin x+\cos x)=\frac{\sqrt{3}+1}{2}}$

ញ. $\lim\limits_{x\to\frac{\pi}{3}}\frac{\tan x+2}{\sin x+1}$

យើងបាន $\lim\limits_{x\to\frac{\pi}{3}}\frac{\tan x+2}{\sin x+1}=\frac{\tan\frac{\pi}{3}+2}{\sin\frac{\pi}{3}+1}$

$=\frac{\sqrt{3}+2}{\frac{\sqrt{3}}{2}+1}$

$=2$

ដូចនេះ $\boxed{\lim\limits_{x\to\frac{\pi}{3}}\frac{\tan x+2}{\sin x+1}=2}$

ដ. $\lim\limits_{x\to\frac{\pi}{6}}\frac{\tan(2x)+\cot 3x}{\sin x}$

យើងបាន $\lim\limits_{x\to\frac{\pi}{6}}\frac{\tan(2x)+\cot 3x}{\sin x}=\frac{\tan(2\times\frac{\pi}{6})+\cot(3\times\frac{\pi}{6})}{\sin\frac{\pi}{6}}$

$=\frac{\tan(\frac{\pi}{3})+\cot(\frac{\pi}{2})}{\sin\frac{\pi}{6}}$

$=\frac{\sqrt{3}+0}{\frac{1}{2}}$

$=2\sqrt{3}$

ដូចនេះ $\boxed{\lim\limits_{x\to\frac{\pi}{6}}\frac{\tan(2x)+\cot 3x}{\sin x}=2\sqrt{3}}$

មេរៀនទី 3 លំហាត់ និង ដំណោះស្រាយ

ឋ. $\lim\limits_{x\to1}(\ln x+\ln 4x)$

យើងបាន $\lim\limits_{x\to1}(\ln x+\ln 4x)=(\ln 1+\ln 4)$

$=(0+\ln 4)$

$=\ln 4$

ដូចនេះ $\boxed{\lim\limits_{x\to1}(\ln x+\ln 4x)=\ln 4}$

ឌ. $\lim\limits_{x\to3}e^{x+1}.x$

យើងបាន $\lim\limits_{x\to3}e^{x+1}.x=e^{3+1}\times 3$

$=3e^4$

ដូចនេះ $\boxed{\lim\limits_{x\to3}e^{x+1}.x=3e^4}$

ឍ. $\lim\limits_{x\to3}\frac{xe^{\frac{x}{3}}}{x+1}$

យើងបាន $\lim\limits_{x\to3}\dfrac{xe^{\frac{x}{3}}}{x+1}=\dfrac{3e^{\frac{3}{3}}}{3+1}$

$=\dfrac{3e}{4}$

ដូចនេះ $\boxed{\lim\limits_{x\to3}\frac{xe^{\frac{x}{3}}}{x+1}=\frac{3e}{4}}$

លំហាត់គំរូទី៦

គណនាលីមីតខាងក្រោម៖

ក. $\lim\limits_{x\to4}\frac{x^2-6x+8}{x^2-16}$

ខ. $\lim\limits_{x\to-3}\frac{x^2+x-6}{9-x^2}$

គ. $\lim\limits_{x\to5}\frac{x^2-7x+10}{x^2-25}$

ឃ. $\lim\limits_{x\to2}\frac{x^2-8x+12}{x^2-4}$

ង. $\lim\limits_{x\to-3}\frac{x^2-2x-15}{x^2+10x+21}$

ច. $\lim\limits_{x\to4}\frac{x^2+x-20}{2x^2-3x-20}$

ឆ. $\lim\limits_{x\to-\frac{3}{2}}\frac{2x^2+9x+9}{6x^2+7x-3}$

ជ. $\lim\limits_{x\to4}\frac{2x^2-3x-20}{x^2+x-20}$

ឈ. $\lim\limits_{x\to-\frac{5}{2}}\frac{25-4x^2}{2x^2+9x+10}$

ញ. $\lim\limits_{x\to3}\frac{9-x^2}{x^2-7x+12}$

ដ. $\lim\limits_{x\to4}\frac{16-x^2}{3x^2-10x-8}$

ឋ. $\lim\limits_{x\to8}\frac{(x+1)(x^2-64)}{x^2-3x-40}$

ឌ. $\lim\limits_{x\to5}\frac{x^2-2x-15}{-x^2+7x-10}$

ឍ. $\lim\limits_{x\to6}\frac{2x^2-16x+24}{36-x^2}$

ណ. $\lim\limits_{x\to\frac{4}{3}}\frac{-6x^2+17x-12}{-3x^2-8x+16}$

ត. $\lim\limits_{x\to1}\frac{x^2-1}{x^2+2x-3}$

ថ. $\lim\limits_{x\to-1}\frac{x^2+3x+2}{x^2-2x-3}$

ទ. $\lim\limits_{x\to3}\frac{x^2-9}{x^2-x-6}$

ធ. $\lim\limits_{x\to1}\frac{x^2+3x-4}{2x^2-5x+3}$

ន. $\lim\limits_{x\to2}\frac{\frac{1}{x^2}-\frac{1}{4}}{x-2}$

ដំណោះស្រាយ

គណនាលីមីតខាងក្រោម៖

ក. $\lim\limits_{x\to4}\frac{x^2-6x+8}{x^2-16}$, រាងមិនកំណត់ $\frac{0}{0}$

យើងបាន $\lim\limits_{x\to4}\dfrac{x^2-6x+8}{x^2-16}=\lim\limits_{x\to4}\dfrac{(x-4)(x-2)}{(x-4)(x+4)}$

$=\lim\limits_{x\to4}\dfrac{x-2}{x+4}$

$=\dfrac{1}{4}$

ដូចនេះ $\boxed{\lim\limits_{x\to4}\frac{x^2-6x+8}{x^2-16}=\frac{1}{4}}$

ខ. $\lim\limits_{x\to-3}\frac{x^2+x-6}{9-x^2}$, រាងមិនកំណត់ $\frac{0}{0}$

យើងបាន $\lim\limits_{x\to-3}\dfrac{x^2+x-6}{9-x^2}=\lim\limits_{x\to-3}\dfrac{(x+3)(x-2)}{(3+x)(3-x)}$

$=\lim\limits_{x\to-3}\dfrac{x-2}{3-x}$

$=\dfrac{-3-2}{3-(-3)}$

$=-\dfrac{5}{6}$

ដូចនេះ $\boxed{\lim\limits_{x\to-3}\frac{x^2+x-6}{9-x^2}=-\frac{5}{6}}$

គ. $\lim\limits_{x\to5}\frac{x^2-7x+10}{x^2-25}$, រាងមិនកំណត់ $\frac{0}{0}$

យើងបាន $\lim\limits_{x\to5}\dfrac{x^2-7x+10}{x^2-25}=\lim\limits_{x\to5}\dfrac{(x-5)(x-2)}{(x-5)(x+5)}$
$=\lim\limits_{x\to5}\dfrac{x-2}{x+5}$
$=\dfrac{5-2}{5+5}$
$=\dfrac{3}{10}$

ដូចនេះ $\boxed{\lim\limits_{x\to5}\frac{x^2-7x+10}{x^2-25}=\frac{3}{10}}$

ឃ. $\lim\limits_{x\to2}\frac{x^2-8x+12}{x^2-4}$, រាងមិនកំណត់ $\frac{0}{0}$

យើងបាន $\lim\limits_{x\to2}\dfrac{x^2-8x+12}{x^2-4}=\lim\limits_{x\to2}\dfrac{(x-6)(x-2)}{(x-2)(x+2)}$
$=\lim\limits_{x\to2}\dfrac{x-6}{x+2}$
$=-1$

ដូចនេះ $\boxed{\lim\limits_{x\to2}\frac{x^2-8x+12}{x^2-4}=-1}$

ង. $\lim\limits_{x\to-3}\frac{x^2-2x-15}{x^2+10x+21}$, រាងមិនកំណត់ $\frac{0}{0}$

យើងបាន $\lim\limits_{x\to-3}\dfrac{x^2-2x-15}{x^2+10x+21}=\lim\limits_{x\to-3}\dfrac{(x+3)(x-5)}{(x+3)(x+7)}$
$=\lim\limits_{x\to-3}\dfrac{x-5}{x+7}$
$=\dfrac{-3-5}{-3+7}$
$=-2$

ដូចនេះ $\boxed{\lim\limits_{x\to-3}\frac{x^2-2x-15}{x^2+10x+21}=-2}$

ច. $\lim\limits_{x\to4}\frac{x^2+x-20}{2x^2-3x-20}$, រាងមិនកំណត់ $\frac{0}{0}$

យើងបាន $\lim\limits_{x\to4}\dfrac{x^2+x-20}{2x^2-3x-20}=\lim\limits_{x\to4}\dfrac{(x-4)(x+5)}{(x-4)(2x+5)}$
$=\lim\limits_{x\to4}\dfrac{x+5}{2x+5}$
$=\dfrac{4+5}{2\times4+5}$
$=\dfrac{9}{13}$

ដូចនេះ $\boxed{\lim\limits_{x\to4}\frac{x^2+x-20}{2x^2-3x-20}=\frac{9}{13}}$

ឆ. $\lim\limits_{x\to-\frac{3}{2}}\frac{2x^2+9x+9}{6x^2+7x-3}$, រាងមិនកំណត់ $\frac{0}{0}$

យើងបាន $\lim\limits_{x\to-\frac{3}{2}}\dfrac{2x^2+9x+9}{6x^2+7x-3}=\lim\limits_{x\to-\frac{3}{2}}\dfrac{(x+3)(2x+3)}{(3x-1)(2x+3)}$
$=\lim\limits_{x\to-\frac{3}{2}}\dfrac{x+3}{3x-1}$
$=-\dfrac{3}{11}$

ដូចនេះ $\boxed{\lim\limits_{x\to-\frac{3}{2}}\frac{2x^2+9x+9}{6x^2+7x-3}=-\frac{3}{11}}$

ជ. $\lim\limits_{x\to4}\frac{2x^2-3x-20}{x^2+x-20}$, រាងមិនកំណត់ $\frac{0}{0}$

យើងបាន $\lim\limits_{x\to4}\dfrac{2x^2-3x-20}{x^2+x-20}=\lim\limits_{x\to4}\dfrac{(x-4)(2x+5)}{(x-4)(x+5)}$
$=\lim\limits_{x\to4}\dfrac{2x+5}{x+5}$
$=\lim\limits_{x\to4}\dfrac{2\times4+5}{4+5}$
$=\dfrac{13}{9}$

ដូចនេះ $\boxed{\lim\limits_{x\to4}\frac{2x^2-3x-20}{x^2+x-20}=\frac{13}{9}}$

ឈ. $\lim\limits_{x\to-\frac{5}{2}}\frac{25-4x^2}{2x^2+9x+10}$, រាងមិនកំណត់ $\frac{0}{0}$

យើងបាន $\lim\limits_{x\to-\frac{5}{2}}\dfrac{25-4x^2}{2x^2+9x+10}=\lim\limits_{x\to-\frac{5}{2}}\dfrac{(5-2x)(5+2x)}{(x+2)(2x+5)}$
$=\lim\limits_{x\to-\frac{5}{2}}\dfrac{5-2x}{x+2}$
$=\dfrac{5-2(-\frac{5}{2})}{-\frac{5}{2}+2}$
$=-20$

ដូចនេះ $\boxed{\lim\limits_{x\to-\frac{5}{2}}\frac{25-4x^2}{2x^2+9x+10}=-20}$

ញ. $\lim\limits_{x\to3}\frac{9-x^2}{x^2-7x+12}$, រាងមិនកំណត់ $\frac{0}{0}$

យើងបាន $\lim\limits_{x\to3}\dfrac{9-x^2}{x^2-7x+12}=\lim\limits_{x\to3}\dfrac{(3-x)(3+x)}{(x-3)(x-4)}$
$=\lim\limits_{x\to3}\dfrac{-(3+x)}{x-4}$
$=6$

ដូចនេះ $\boxed{\lim\limits_{x\to3}\frac{9-x^2}{x^2-7x+12}=6}$

ដ. $\lim\limits_{x\to4}\frac{16-x^2}{3x^2-10x-8}$, រាងមិនកំណត់ $\frac{0}{0}$

យើងបាន $\lim\limits_{x\to4}\dfrac{16-x^2}{3x^2-10x-8}=\lim\limits_{x\to4}\dfrac{(4-x)(4+x)}{(x-4)(3x+2)}$
$=\lim\limits_{x\to4}\dfrac{-(x-4)(4+x)}{(x-4)(3x+2)}$
$=\lim\limits_{x\to4}\dfrac{-(4+x)}{3x+2}$
$=\dfrac{-(4+4)}{3\times4+2}$
$=\dfrac{-8}{14}$
$=-\dfrac{4}{7}$

ដូចនេះ $\boxed{\lim\limits_{x\to4}\frac{16-x^2}{3x^2-10x-8}=-\frac{4}{7}}$

ឋ. $\lim\limits_{x\to8}\frac{(x+1)(x^2-64)}{x^2-3x-40}$, រាងមិនកំណត់ $\frac{0}{0}$

យើងបាន $\lim_{x\to8}\frac{(x+1)(x^2-64)}{x^2-3x-40}=\lim_{x\to8}\frac{(x+1)(x-8)(x+8)}{(x-8)(x+5)}$

$=\lim_{x\to8}\frac{(x+1)(x+8)}{x+5}$

$=\frac{(8+1)(8+8)}{8+5}$

$=\frac{9\times16}{13}$

$=\frac{144}{13}$

ដូចនេះ $\boxed{\lim_{x\to8}\frac{(x+1)(x^2-64)}{x^2-3x-40}=\frac{144}{13}}$

ខ. $\lim_{x\to5}\frac{x^2-2x-15}{-x^2+7x-10}$, រាងមិនកំណត់ $\frac{0}{0}$

យើងបាន $\lim_{x\to5}\frac{x^2-2x-15}{-x^2+7x-10}=\lim_{x\to5}\frac{(x-5)(x+3)}{(x-5)(-x+2)}$

$=\lim_{x\to5}\frac{x+3}{-x+2}$

$=-\frac{8}{3}$

ដូចនេះ $\boxed{\lim_{x\to5}\frac{x^2-2x-15}{-x^2+7x-10}=-\frac{8}{3}}$

ឃ. $\lim_{x\to6}\frac{2x^2-16x+24}{36-x^2}$, រាងមិនកំណត់ $\frac{0}{0}$

យើងបាន $\lim_{x\to6}\frac{2x^2-16x+24}{36-x^2}=\lim_{x\to6}\frac{2(x^2-8x+12)}{36-x^2}$

$=\lim_{x\to6}\frac{2(x-6)(x-2)}{(6-x)(6+x)}$

$=\lim_{x\to6}\frac{2(x-6)(x-2)}{(-x-6)(x-6)}$

$=\lim_{x\to6}\frac{2(x-2)}{-x-6}$

$=-\frac{2}{3}$

ដូចនេះ $\boxed{\lim_{x\to6}\frac{2x^2-16x+24}{36-x^2}=-\frac{2}{3}}$

ណ. $\lim_{x\to\frac{4}{3}}\frac{-6x^2+17x-12}{-3x^2-8x+16}$, រាងមិនកំណត់ $\frac{0}{0}$

យើងបាន $\lim_{x\to\frac{4}{3}}\frac{-6x^2+17x-12}{-3x^2-8x+16}=\lim_{x\to\frac{4}{3}}\frac{(3x-4)(3-2x)}{(3x-4)(-x-4)}$

$=\lim_{x\to\frac{4}{3}}\frac{3-2x}{-x-4}$

$=-\frac{1}{16}$

ដូចនេះ $\boxed{\lim_{x\to\frac{4}{3}}\frac{-6x^2+17x-12}{-3x^2-8x+16}=-\frac{1}{16}}$

ត. $\lim_{x\to1}\frac{x^2-1}{x^2+2x-3}$, រាងមិនកំណត់ $\frac{0}{0}$

យើងបាន $\lim_{x\to1}\frac{x^2-1}{x^2+2x-3}=\lim_{x\to1}\frac{(x-1)(x+1)}{(x-1)(x+3)}$

$=\lim_{x\to1}\frac{x+1}{x+3}$

$=\frac{1}{2}$

ដូចនេះ $\boxed{\lim_{x\to1}\frac{x^2-1}{x^2+2x-3}=\frac{1}{2}}$

ថ. $\lim_{x\to-1}\frac{x^2+3x+2}{x^2-2x-3}$, រាងមិនកំណត់ $\frac{0}{0}$

យើងបាន $\lim_{x\to-1}\frac{x^2+3x+2}{x^2-2x-3}=\lim_{x\to-1}\frac{(x+1)(x+2)}{(x-3)(x+1)}$

$=\lim_{x\to-1}\frac{x+2}{x-3}$

$=-\frac{1}{4}$

ដូចនេះ $\boxed{\lim_{x\to-1}\frac{x^2+3x+2}{x^2-2x-3}=-\frac{1}{4}}$

ទ. $\lim_{x\to3}\frac{x^2-9}{x^2-x-6}$, រាងមិនកំណត់ $\frac{0}{0}$

យើងបាន $\lim_{x\to3}\frac{x^2-9}{x^2-x-6}=\lim_{x\to3}\frac{(x-3)(x+3)}{(x-3)(x+2)}$

$=\lim_{x\to3}\frac{x+3}{x+2}$

$=\frac{6}{5}$

ដូចនេះ $\boxed{\lim_{x\to3}\frac{x^2-9}{x^2-x-6}=\frac{6}{5}}$

ធ. $\lim_{x\to1}\frac{x^2+3x-4}{2x^2-5x+3}$, រាងមិនកំណត់ $\frac{0}{0}$

យើងបាន $\lim_{x\to1}\frac{x^2+3x-4}{2x^2-5x+3}=\lim_{x\to1}\frac{(x-1)(x+4)}{(x-1)(2x-3)}$

$=\lim_{x\to1}\frac{x+4}{2x-3}$

$=\frac{1+4}{2-3}$

$=-5$

ដូចនេះ $\boxed{\lim_{x\to1}\frac{x^2+3x-4}{2x^2-5x+3}=-5}$

ន. $\lim_{x\to2}\frac{\frac{1}{x^2}-\frac{1}{4}}{x-2}$, រាងមិនកំណត់ $\frac{0}{0}$

យើងបាន $\lim_{x\to2}\frac{\frac{1}{x^2}-\frac{1}{4}}{x-2}=\lim_{x\to2}\frac{(-x-2)\left(\frac{1}{4x}-\frac{1}{2x^2}\right)}{4x^2\left(\frac{1}{4x}-\frac{1}{2x^2}\right)}$

$=\lim_{x\to2}\frac{-x-2}{4x^2}$

$=\frac{-2-2}{4\times2^2}$

$=-\frac{1}{4}$

ដូចនេះ $\boxed{\lim_{x\to2}\frac{\frac{1}{x^2}-\frac{1}{4}}{x-2}=-\frac{1}{4}}$

លំហាត់គំរូទី៧

ដោះស្រាយលីមីតខាងក្រោម៖

ក. $\lim\limits_{x\to 2}\frac{x^3-8}{x-2}$ ខ. $\lim\limits_{x\to 3}\frac{x^3-27}{x-3}$ គ. $\lim\limits_{x\to -4}\frac{x^3+64}{x+4}$

ឃ. $\lim\limits_{x\to 1}\frac{x^3-3x^2+3x-1}{x^3-x}$ ង. $\lim\limits_{x\to -5}\frac{x^2+7x+10}{x^3+5x^2-x-5}$ ច. $\lim\limits_{x\to a}\frac{x^n-a^n}{x-a}, n\geq 1$

ដំណោះស្រាយ

ដោះស្រាយលីមីតខាងក្រោម៖

ក. $\lim\limits_{x\to 2}\frac{x^3-8}{x-2}$, រាងមិនកំណត់ $\frac{0}{0}$

យើងបាន
$$\begin{aligned}\lim_{x\to 2}\frac{x^3-8}{x-2}&=\lim_{x\to 2}\frac{(x-2)(x^2+2x+4)}{x-2}\\&=\lim_{x\to 2}(x^2+2x+4)\\&=12\end{aligned}$$

ដូចនេះ $\boxed{\lim\limits_{x\to 2}\frac{x^3-8}{x-2}=12}$

ខ. $\lim\limits_{x\to 3}\frac{x^3-27}{x-3}$, រាងមិនកំណត់ $\frac{0}{0}$

យើងបាន
$$\begin{aligned}\lim_{x\to 3}\frac{x^3-27}{x-3}&=\lim_{x\to 3}\frac{x^3-3^3}{x-3}\\&=\lim_{x\to 3}\frac{(x-3)(x^2+3x+9)}{x-3}\\&=\lim_{x\to 3}(x^2+3x+9)\\&=27\end{aligned}$$

ដូចនេះ $\boxed{\lim\limits_{x\to 3}\frac{x^3-27}{x-3}=27}$

គ. $\lim\limits_{x\to -4}\frac{x^3+64}{x+4}$, រាងមិនកំណត់ $\frac{0}{0}$

យើងបាន
$$\begin{aligned}\lim_{x\to -4}\frac{x^3+64}{x+4}&=\lim_{x\to -4}\frac{x^3+4^3}{x+4}\\&=\lim_{x\to -4}\frac{(x+4)(x^2-4x+16)}{x+4}\\&=\lim_{x\to -4}(x^2-4x+16)\\&=48\end{aligned}$$

ដូចនេះ $\boxed{\lim\limits_{x\to -4}\frac{x^3+64}{x+4}=48}$

ឃ. $\lim\limits_{x\to 1}\frac{x^3-3x^2+3x-1}{x^3-x}$, រាងមិនកំណត់ $\frac{0}{0}$

យើងបាន
$$\begin{aligned}\lim_{x\to 1}\frac{x^3-3x^2+3x-1}{x^3-x}&=\lim_{x\to 1}\frac{(x-1)(x^2-2x+1)}{(x-1)(x^2+x)}\\&=\lim_{x\to 1}\frac{x^2-2x+1}{x^2+x}\\&=0\end{aligned}$$

ដូចនេះ $\boxed{\lim\limits_{x\to 1}\frac{x^3-3x^2+3x-1}{x^3-x}=0}$

ង. $\lim\limits_{x\to -5}\frac{x^2+7x+10}{x^3+5x^2-x-5}$, រាងមិនកំណត់ $\frac{0}{0}$

យើងបាន
$$\begin{aligned}\lim_{x\to -5}\frac{x^2+7x+10}{x^3+5x^2-x-5}&=\lim_{x\to -5}\frac{(x+5)(x+2)}{(x+5)(x^2-1)}\\&=\lim_{x\to -5}\frac{x+2}{x^2-1}\\&=\frac{-5+2}{(-5)^2-1}\\&=-\frac{1}{8}\end{aligned}$$

ដូចនេះ $\boxed{\lim\limits_{x\to -5}\frac{x^2+7x+10}{x^3+5x^2-x-5}=-\frac{1}{8}}$

ច. $\lim\limits_{x\to a}\frac{x^n-a^n}{x-a}, n\geq 1$, រាងមិនកំណត់ $\frac{0}{0}$

យើងបាន
$$\begin{aligned}\lim_{x\to a}\frac{x^n-a^n}{x-a}&=\lim_{x\to a}\frac{(x-a)(x^{n-1}+x^{n-2}a+\ldots+xa^{n-2}+xa^{n-1})}{x-a}\\&=\lim_{x\to a}(x^{n-1}+x^{n-2}a+\ldots+xa^{n-2}+a^{n-1})\\&=a^{n-1}+a^{n-2}a+\ldots+aa^{n-2}+a^{n-1}\\&=\underbrace{a^{n-1}+a^{n-1}+\ldots+a^{n-1}}_{n}\\&=na^{n-1}\end{aligned}$$

ដូចនេះ $\boxed{\lim\limits_{x\to a}\frac{x^n-a^n}{x-a}=na^{n-1}}$

លំហាត់គំរូទី៨

ដោះស្រាយលីមីតខាងក្រោម៖

ក. $\lim\limits_{x\to -2}\frac{x^3+8}{x^2+2x}$ ខ. $\lim\limits_{x\to -2}\frac{x^3+3x^2+2x}{x^2-x-6}$ គ. $\lim\limits_{x\to \frac{1}{2}}\frac{8x^3-1}{6x^2-5x+1}$

ឃ. $\lim\limits_{x\to1}\frac{(x-1)(x^3+x-2)}{x^3-x^2-x+1}$ ង. $\lim\limits_{x\to2}\frac{x^3-4x^2+x+6}{x^3-4x^2-11x+30}$ ច. $\lim\limits_{x\to-5}\frac{x^3+6x^2+3x-10}{2x^3+15x^2+22x-15}$

ឆ. $\lim\limits_{x\to-3}\frac{x^3+3x^2-4x-12}{3x^3+11x^2+5x-3}$ ជ. $\lim\limits_{x\to\frac{1}{2}}\frac{2x^3+3x^2-1}{2x^3-x^2-18x+9}$ ឈ. $\lim\limits_{x\to1}\frac{x^3+3x^2-9x+5}{x^3-5x^2+7x-3}$

ញ. $\lim\limits_{x\to-2}\frac{x^3+7x^2+16x+12}{2x^3+9x^2+12x+4}$ ដ. $\lim\limits_{x\to-2}\frac{x+2}{x^3+7x^2+16x+12}$ ឋ. $\lim\limits_{x\to3}\frac{x^3-2x^2-15x+36}{-x^3+11x^2-39x+45}$

ដំណោះស្រាយ

ដោះស្រាយលីមីតខាងក្រោម៖

ក. $\lim\limits_{x\to-2}\frac{x^3+8}{x^2+2x}$, រាងមិនកំណត់ $\frac{0}{0}$

យើងបាន
$$\begin{aligned}\lim_{x\to-2}\frac{x^3+8}{x^2+2x}&=\lim_{x\to-2}\frac{x^3+2^3}{x^2+2x}\\&=\lim_{x\to-2}\frac{(x+2)(x^2-2x+4)}{x(x+2)}\\&=\lim_{x\to-2}\frac{x^2-2x+4}{x}\\&=\frac{(-2)^2-2(-2)+4}{-2}\\&=\frac{4+4+4}{-2}\\&=-6\end{aligned}$$

ដូចនេះ $\boxed{\lim\limits_{x\to-2}\frac{x^3+8}{x^2+2x}=-6}$

ខ. $\lim\limits_{x\to-2}\frac{x^3+3x^2+2x}{x^2-x-6}$, រាងមិនកំណត់ $\frac{0}{0}$

យើងបាន
$$\begin{aligned}\lim_{x\to-2}\frac{x^3+3x^2+2x}{x^2-x-6}&=\lim_{x\to-2}\frac{(x+2)(x^2+x)}{(x+2)(x-3)}\\&=\lim_{x\to-2}\frac{x^2+x}{x-3}\\&=\frac{(-2)^2-2}{-2-3}=-\frac{2}{5}\end{aligned}$$

ដូចនេះ $\boxed{\lim\limits_{x\to-2}\frac{x^3+3x^2+2x}{x^2-x-6}=-\frac{2}{5}}$

គ. $\lim\limits_{x\to\frac{1}{2}}\frac{8x^3-1}{6x^2-5x+1}$, រាងមិនកំណត់ $\frac{0}{0}$

យើងបាន
$$\begin{aligned}\lim_{x\to\frac{1}{2}}\frac{8x^3-1}{6x^2-5x+1}&=\lim_{x\to\frac{1}{2}}\frac{(2x-1)(4x^2+2x+1)}{(2x-1)(3x-1)}\\&=\lim_{x\to\frac{1}{2}}\frac{4x^2+2x+1}{3x-1}\\&=\frac{4\left(\frac{1}{2}\right)^2+2\left(\frac{1}{2}\right)+1}{3\left(\frac{1}{2}\right)-1}=6\end{aligned}$$

ដូចនេះ $\boxed{\lim\limits_{x\to\frac{1}{2}}\frac{8x^3-1}{6x^2-5x+1}=6}$

ឃ. $\lim\limits_{x\to1}\frac{(x-1)(x^3+x-2)}{x^3-x^2-x+1}$, រាងមិនកំណត់ $\frac{0}{0}$

យើងបាន
$$\begin{aligned}\lim_{x\to1}\frac{(x-1)(x^3+x-2)}{x^3-x^2-x+1}&=\lim_{x\to1}\frac{(x-1)(x-1)(x^2+x+2)}{(x-1)(x^2-1)}\\&=\lim_{x\to1}\frac{(x-1)(x-1)(x^2+x+2)}{(x-1)(x-1)(x+1)}\\&=\lim_{x\to1}\frac{x^2+x+2}{x+1}\\&=\frac{1^2+1+2}{1+1}=2\end{aligned}$$

ដូចនេះ $\boxed{\lim\limits_{x\to1}\frac{(x-1)(x^3+x-2)}{x^3-x^2-x+1}=2}$

ង. $\lim\limits_{x\to2}\frac{x^3-4x^2+x+6}{x^3-4x^2-11x+30}$, រាងមិនកំណត់ $\frac{0}{0}$

យើងបាន
$$\begin{aligned}\lim_{x\to2}\frac{x^3-4x^2+x+6}{x^3-4x^2-11x+30}&=\lim_{x\to2}\frac{(x-2)(x^2-2x-3)}{(x-2)(x^2-2x-15)}\\&=\lim_{x\to2}\frac{x^2-2x-3}{x^2-2x-15}\\&=\frac{1}{5}\end{aligned}$$

ដូចនេះ $\boxed{\lim\limits_{x\to2}\frac{x^3-4x^2+x+6}{x^3-4x^2-11x+30}=\frac{1}{5}}$

ច. $\lim\limits_{x\to-5}\frac{x^3+6x^2+3x-10}{2x^3+15x^2+22x-15}$, រាងមិនកំណត់ $\frac{0}{0}$

យើងបាន
$$\begin{aligned}\lim_{x\to-5}\frac{x^3+6x^2+3x-10}{2x^3+15x^2+22x-15}&=\lim_{x\to-5}\frac{x^3+6x^2+3x-10}{2x^3+15x^2+22x-15}\\&=\lim_{x\to-5}\frac{(x+5)(x^2+x-2)}{(x+5)(2x^2+5x-3)}\\&=\lim_{x\to-5}\frac{x^2+x-2}{2x^2+5x-3}\\&=\frac{9}{11}\end{aligned}$$

ដូចនេះ $\boxed{\lim\limits_{x\to-5}\frac{x^3+6x^2+3x-10}{2x^3+15x^2+22x-15}=\frac{9}{11}}$

ឆ. $\lim\limits_{x\to-3}\frac{x^3+3x^2-4x-12}{3x^3+11x^2+5x-3}$, រាងមិនកំណត់ $\frac{0}{0}$

យើងបាន $\lim\limits_{x\to-3}\dfrac{x^3+3x^2-4x-12}{3x^3+11x^2+5x-3}=\lim\limits_{x\to-3}\dfrac{(x+3)(x^2-4)}{(x+3)(3x^2+2x-1)}$

$=\lim\limits_{x\to-3}\dfrac{x^2-4}{3x^2+2x-1}$

$=\dfrac{(-3)^2-4}{3(-3)^2+2(-3)-1}$

$=\dfrac{1}{4}$

ដូចនេះ $\boxed{\lim\limits_{x\to-3}\frac{x^3+3x^2-4x-12}{3x^3+11x^2+5x-3}=\frac{1}{4}}$

ជ. $\lim\limits_{x\to\frac{1}{2}}\frac{2x^3+3x^2-1}{2x^3-x^2-18x+9}$, រាងមិនកំណត់ $\frac{0}{0}$

យើងបាន $\lim\limits_{x\to\frac{1}{2}}\dfrac{2x^3+3x^2-1}{2x^3-x^2-18x+9}=\lim\limits_{x\to\frac{1}{2}}\dfrac{(2x-1)(x^2+2x+1)}{(2x-1)(x^2-9)}$

$=\lim\limits_{x\to\frac{1}{2}}\dfrac{x^2+2x+1}{x^2-9}$

$=-\dfrac{9}{35}$

ដូចនេះ $\boxed{\lim\limits_{x\to\frac{1}{2}}\frac{2x^3+3x^2-1}{2x^3-x^2-18x+9}=-\frac{9}{35}}$

ឈ. $\lim\limits_{x\to1}\frac{x^3+3x^2-9x+5}{x^3-5x^2+7x-3}$, រាងមិនកំណត់ $\frac{0}{0}$

យើងបាន $\lim\limits_{x\to1}\dfrac{x^3+3x^2-9x+5}{x^3-5x^2+7x-3}=\lim\limits_{x\to1}\dfrac{(x+5)(x^2-2x+1)}{(x-3)(x^2-2x+1)}$

$=\lim\limits_{x\to1}\dfrac{x+5}{x-3}$

$=-3$

ដូចនេះ $\boxed{\lim\limits_{x\to1}\frac{x^3+3x^2-9x+5}{x^3-5x^2+7x-3}=-3}$

ញ. $\lim\limits_{x\to-2}\frac{x^3+7x^2+16x+12}{2x^3+9x^2+12x+4}$, រាងមិនកំណត់ $\frac{0}{0}$

យើងបាន $\lim\limits_{x\to-2}\dfrac{x^3+7x^2+16x+12}{2x^3+9x^2+12x+4}=\lim\limits_{x\to-2}\dfrac{(x+3)(x^2+4x+4)}{(2x+1)(x^2+4x+4)}$

$=\lim\limits_{x\to-2}\dfrac{x+3}{2x+1}$

$=\dfrac{-2+3}{2(-2)+1}$

$=-\dfrac{1}{3}$

ដូចនេះ $\boxed{\lim\limits_{x\to-2}\frac{x^3+7x^2+16x+12}{2x^3+9x^2+12x+4}=-\frac{1}{3}}$

ដ. $\lim\limits_{x\to-2}\frac{x+2}{x^3+7x^2+16x+12}$, រាងមិនកំណត់ $\frac{0}{0}$

យើងបាន $\lim\limits_{x\to-2}\dfrac{x+2}{x^3+7x^2+16x+12}=\lim\limits_{x\to-2}\dfrac{x+2}{(x+2)(x^2+5x+6)}$

$=\lim\limits_{x\to-2}\dfrac{1}{x^2+5x+6}$

• ករណី $x\to-2^-$

យើងបាន $\lim\limits_{x\to-2^-}\dfrac{1}{x^2+5x+6}=\lim\limits_{x\to-2^-}\dfrac{1}{(x+2)(x+3)}$

$=\dfrac{1}{0^-}$

$=-\infty$

ដូចនេះ $\boxed{\lim\limits_{x\to-2^-}\frac{x+2}{x^3+7x^2+16x+12}=-\infty}$

• ករណី $x\to-2^+$

យើងបាន $\lim\limits_{x\to-2^+}\dfrac{1}{x^2+5x+6}=\lim\limits_{x\to-2^+}\dfrac{1}{(x+2)(x+3)}$

$=\dfrac{1}{0^+}$

$=+\infty$

ដូចនេះ $\boxed{\lim\limits_{x\to-2^+}\frac{x+2}{x^3+7x^2+16x+12}=+\infty}$

ឋ. $\lim\limits_{x\to3}\frac{x^3-2x^2-15x+36}{-x^3+11x^2-39x+45}$, រាងមិនកំណត់ $\frac{0}{0}$

យើងបាន $\lim\limits_{x\to3}\dfrac{x^3-2x^2-15x+36}{-x^3+11x^2-39x+45}=\lim\limits_{x\to3}\dfrac{(x+4)(x^2-6x+9)}{(-x+5)(x^2-6x+9)}$

$=\lim\limits_{x\to3}\dfrac{x+4}{-x+5}$

$=\dfrac{3+4}{-3+5}$

$=\dfrac{7}{2}$

ដូចនេះ $\boxed{\lim\limits_{x\to3}\frac{x^3-2x^2-15x+36}{-x^3+11x^2-39x+45}=\frac{7}{2}}$

លំហាត់គំរូទី៩

ដោះស្រាយលីមីតខាងក្រោម៖

ក. $\lim\limits_{x\to4}\frac{\sqrt{x}-2}{x-4}$

ខ. $\lim\limits_{x\to2}\frac{\sqrt{2x}-2}{x-2}$

គ. $\lim\limits_{x\to-1}\frac{1+x}{\sqrt{x^2+3}-2}$

ឃ. $\lim\limits_{x\to0}\frac{\sqrt{x+2}-\sqrt{2}}{x}$

ង. $\lim\limits_{x\to0}\frac{2-\sqrt{x+4}}{x}$

ច. $\lim\limits_{x\to2}\frac{\sqrt{7+x}-3}{x-2}$

ឆ. $\lim\limits_{x\to1}\frac{\sqrt{x+3}-\sqrt{3x+1}}{\sqrt{x}-1}$

ជ. $\lim\limits_{x\to2}\frac{\sqrt{2x}-2}{3\sqrt{x+2}-6}$

ឈ. $\lim\limits_{x\to-1}\frac{\sqrt{x+2}-1}{1-\sqrt{2x+3}}$

ញ. $\lim\limits_{x\to\frac{1}{2}}\frac{\sqrt{2x+1}-\sqrt{2}}{2-\sqrt{8x}}$

ដំណោះស្រាយ

ដោះស្រាយលីមីតខាងក្រោម៖

ក. $\lim\limits_{x\to4}\frac{\sqrt{x}-2}{x-4}$, រាងមិនកំណត់ $\frac{0}{0}$

យើងបាន $\lim\limits_{x\to4}\dfrac{\sqrt{x}-2}{x-4}=\lim\limits_{x\to4}\left[\dfrac{\sqrt{x}-2}{x-4}\times\dfrac{\sqrt{x}+2}{\sqrt{x}+2}\right]$

$$=\lim_{x\to4}\frac{x-4}{(x-4)(\sqrt{x}+2)}$$
$$=\lim_{x\to4}\frac{1}{\sqrt{x}+2}$$
$$=\frac{1}{\sqrt{4}+2}$$
$$=\frac{1}{4}$$

ដូចនេះ $\boxed{\lim\limits_{x\to4}\frac{\sqrt{x}-2}{x-4}=\frac{1}{4}}$

ខ. $\lim\limits_{x\to2}\frac{\sqrt{2x}-2}{x-2}$, រាងមិនកំណត់ $\frac{0}{0}$

យើងបាន $\lim\limits_{x\to2}\dfrac{\sqrt{2x}-2}{x-2}=\lim\limits_{x\to2}\left[\dfrac{\sqrt{2x}-2}{x-2}\times\dfrac{\sqrt{2x}+2}{\sqrt{2x}+2}\right]$

$$=\lim_{x\to2}\frac{2x-4}{(x-2)(\sqrt{2x}+2)}$$
$$=\lim_{x\to2}\frac{2(x-2)}{(x-2)(\sqrt{2x}+2)}$$
$$=\lim_{x\to2}\frac{2}{\sqrt{2x}+2}$$
$$=\frac{2}{\sqrt{2\times2}+2}$$
$$=\frac{1}{2}$$

ដូចនេះ $\boxed{\lim\limits_{x\to2}\frac{\sqrt{2x}-2}{x-2}=\frac{1}{2}}$

គ. $\lim\limits_{x\to-1}\frac{1+x}{\sqrt{x^2+3}-2}$, រាងមិនកំណត់ $\frac{0}{0}$

យើងបាន $\lim\limits_{x\to-1}\dfrac{1+x}{\sqrt{x^2+3}-2}=\lim\limits_{x\to-1}\left[\dfrac{1+x}{\sqrt{x^2+3}-2}\times\dfrac{\sqrt{x^2+3}+2}{\sqrt{x^2+3}+2}\right]$

$$=\lim_{x\to-1}\frac{(1+x)(\sqrt{x^2+3}+2)}{x^2+3-4}$$
$$=\lim_{x\to-1}\frac{(1+x)(\sqrt{x^2+3}+2)}{x^2-1}$$
$$=\lim_{x\to-1}\frac{(1+x)(\sqrt{x^2+3}+2)}{(x-1)(x+1)}$$
$$=\lim_{x\to-1}\frac{\sqrt{x^2+3}+2}{x-1}$$
$$=\frac{\sqrt{(-1)^2+3}+2}{-1-1}$$
$$=-2$$

ដូចនេះ $\boxed{\lim\limits_{x\to-1}\frac{1+x}{\sqrt{x^2+3}-2}=-2}$

ឃ. $\lim\limits_{x\to0}\frac{\sqrt{x+2}-\sqrt{2}}{x}$, រាងមិនកំណត់ $\frac{0}{0}$

យើងបាន $\lim\limits_{x\to0}\dfrac{\sqrt{x+2}-\sqrt{2}}{x}=\lim\limits_{x\to0}\left[\dfrac{\sqrt{x+2}-\sqrt{2}}{x}\times\dfrac{\sqrt{x+2}+\sqrt{2}}{\sqrt{x+2}+\sqrt{2}}\right]$

$$=\lim_{x\to0}\frac{x+2-2}{x(\sqrt{x+2}+\sqrt{2})}$$
$$=\lim_{x\to0}\frac{x}{x(\sqrt{x+2}+\sqrt{2})}$$
$$=\lim_{x\to0}\frac{1}{\sqrt{x+2}+\sqrt{2}}$$
$$=\frac{1}{\sqrt{0+2}+\sqrt{2}}$$
$$=\frac{1}{2\sqrt{2}}$$
$$=\frac{\sqrt{2}}{4}$$

ដូចនេះ $\boxed{\lim\limits_{x\to0}\frac{\sqrt{x+2}-\sqrt{2}}{x}=\frac{\sqrt{2}}{4}}$

ង. $\lim\limits_{x\to0}\frac{2-\sqrt{x+4}}{x}$, រាងមិនកំណត់ $\frac{0}{0}$

យើងបាន $\lim\limits_{x\to0}\dfrac{2-\sqrt{x+4}}{x}=\lim\limits_{x\to0}\left[\dfrac{2-\sqrt{x+4}}{x}\times\dfrac{2+\sqrt{x+4}}{2+\sqrt{x+4}}\right]$

$$=\lim_{x\to0}\frac{4-x-4}{x(2+\sqrt{x+4})}$$
$$=\lim_{x\to0}\frac{-x}{x(2+\sqrt{x+4})}$$
$$=\lim_{x\to0}\frac{-1}{2+\sqrt{x+4}}$$
$$=\frac{-1}{2+\sqrt{0+4}}$$
$$=\frac{-1}{4}$$

ដូចនេះ $\boxed{\lim\limits_{x\to0}\frac{2-\sqrt{x+4}}{x}=-\frac{1}{4}}$

ច. $\lim\limits_{x\to2}\frac{\sqrt{7+x}-3}{x-2}$, រាងមិនកំណត់ $\frac{0}{0}$

យើងបាន $\lim\limits_{x\to2}\dfrac{\sqrt{7+x}-3}{x-2}=\lim\limits_{x\to2}\left[\dfrac{\sqrt{7+x}-3}{x-2}\times\dfrac{\sqrt{7+x}+3}{\sqrt{7+x}+3}\right]$

$$=\lim_{x\to2}\frac{7+x-9}{(x-2)(\sqrt{7+x}+3)}$$
$$=\lim_{x\to2}\frac{x-2}{(x-2)(\sqrt{7+x}+3)}$$
$$=\lim_{x\to2}\frac{1}{\sqrt{7+x}+3}$$
$$=\frac{1}{\sqrt{7+2}+3}$$
$$=\frac{1}{6}$$

ដូចនេះ $\boxed{\lim\limits_{x\to2}\frac{\sqrt{7+x}-3}{x-2}=\frac{1}{6}}$

ឆ. $\lim\limits_{x\to1}\frac{\sqrt{x+3}-\sqrt{3x+1}}{\sqrt{x}-1}$, រាងមិនកំណត់ $\frac{0}{0}$

យើងបាន $\lim\limits_{x\to1}\frac{\sqrt{x+3}-\sqrt{3x+1}}{\sqrt{x}-1}=\lim\limits_{x\to1}\frac{(x+3-3x-1)(\sqrt{x}+1)}{(x-1)(\sqrt{x+3}+\sqrt{3x+1})}$

$$=\lim_{x\to1}\frac{(-2x+2)(\sqrt{x}+1)}{(x-1)(\sqrt{x+3}+\sqrt{3x+1})}$$
$$=\lim_{x\to1}\frac{-2(x-1)(\sqrt{x}+1)}{(x-1)(\sqrt{x+3}+\sqrt{3x+1})}$$
$$=\lim_{x\to1}\frac{-2(\sqrt{x}+1)}{\sqrt{x+3}+\sqrt{3x+1}}$$
$$=\frac{-2(\sqrt{1}+1)}{\sqrt{1+3}+\sqrt{3+1}}$$
$$=-1$$

ដូចនេះ $\boxed{\lim\limits_{x\to1}\frac{\sqrt{x+3}-\sqrt{3x+1}}{\sqrt{x}-1}=-1}$

ជ. $\lim\limits_{x\to2}\frac{\sqrt{2x}-2}{3\sqrt{x+2}-6}$, រាងមិនកំណត់ $\frac{0}{0}$

យើងបាន $\lim\limits_{x\to2}\frac{\sqrt{2x}-2}{3\sqrt{x+2}-6}=\lim\limits_{x\to2}\left[\frac{\sqrt{2x}-2}{3(\sqrt{x+2}-2)}\times\frac{\sqrt{2x}+2}{\sqrt{2x}+2}\times\frac{\sqrt{x+2}+2}{\sqrt{x+2}+2}\right]$

$$=\lim_{x\to2}\frac{(2x-4)(\sqrt{x+2}+2)}{3(x+2-4)(\sqrt{2x}+2)}$$
$$=\lim_{x\to2}\frac{2(x-2)(\sqrt{x+2}+2)}{3(x-2)(\sqrt{2x}+2)}$$
$$=\frac{2(\sqrt{2+2}+2)}{3(\sqrt{2\times2}+2)}$$
$$=\frac{2}{3}$$

ដូចនេះ $\boxed{\lim\limits_{x\to2}\frac{\sqrt{2x}-2}{3\sqrt{x+2}-6}=\frac{2}{3}}$

ឈ. $\lim\limits_{x\to-1}\frac{\sqrt{x+2}-1}{1-\sqrt{2x+3}}$, រាងមិនកំណត់ $\frac{0}{0}$

យើងបាន $\lim\limits_{x\to-1}\frac{\sqrt{x+2}-1}{1-\sqrt{2x+3}}=\lim\limits_{x\to-1}\left[\frac{\sqrt{x+2}-1}{1-\sqrt{2x+3}}\times\frac{\sqrt{x+2}+1}{\sqrt{x+2}+1}\times\frac{1+\sqrt{2x+3}}{1+\sqrt{2x+3}}\right]$

$$=\lim_{x\to-1}\frac{(x+2-1)(1+\sqrt{2x+3})}{(1-2x-3)(\sqrt{x+2}+1)}$$
$$=\lim_{x\to-1}\frac{(x+1)(1+\sqrt{2x+3})}{(-2x-2)(\sqrt{x+2}+1)}$$
$$=\lim_{x\to-1}\frac{(x+1)(1+\sqrt{2x+3})}{-2(x+1)(\sqrt{x+2}+1)}$$
$$=\lim_{x\to-1}\frac{1+\sqrt{2x+3}}{-2(\sqrt{x+2}+1)}$$
$$=\frac{1+\sqrt{2(-1)+3}}{-2(\sqrt{-1+2}+1)}$$
$$=-\frac{1}{2}$$

ដូចនេះ $\boxed{\lim\limits_{x\to-1}\frac{\sqrt{x+2}-1}{1-\sqrt{2x+3}}=-\frac{1}{2}}$

ញ. $\lim\limits_{x\to\frac{1}{2}}\frac{\sqrt{2x+1}-\sqrt{2}}{2-\sqrt{8x}}$, រាងមិនកំណត់ $\frac{0}{0}$

យើងបាន

$$\lim_{x\to\frac{1}{2}}\frac{\sqrt{2x+1}-\sqrt{2}}{2-\sqrt{8x}}=\lim_{x\to\frac{1}{2}}\left[\frac{\sqrt{2x+1}-\sqrt{2}}{2-\sqrt{8x}}\times\frac{\sqrt{2x+1}+\sqrt{2}}{\sqrt{2x+1}+\sqrt{2}}\times\frac{2+\sqrt{8x}}{2+\sqrt{8x}}\right]$$
$$=\lim_{x\to\frac{1}{2}}\frac{(2x-1)(2+\sqrt{8x})}{-4(2x-1)(\sqrt{2x+1}+\sqrt{2})}$$
$$=\lim_{x\to\frac{1}{2}}\frac{2+\sqrt{8x}}{-4(\sqrt{2x+1}+\sqrt{2})}$$
$$=\frac{2+\sqrt{8\left(\frac{1}{2}\right)}}{-4\left[\sqrt{2\left(\frac{1}{2}\right)+1}+\sqrt{2}\right]}$$
$$=-\frac{\sqrt{2}}{4}$$

ដូចនេះ $\boxed{\lim\limits_{x\to\frac{1}{2}}\frac{\sqrt{2x+1}-\sqrt{2}}{2-\sqrt{8x}}=-\frac{\sqrt{2}}{4}}$

លំហាត់គំរូទី១០

ដោះស្រាយលីមីតខាងក្រោម៖

ក. $\lim\limits_{x\to1}\frac{\sqrt[3]{x-2}+\sqrt[3]{1-x+x^2}}{x^2-1}$

ខ. $\lim\limits_{x\to64}\frac{\sqrt{x}-8}{\sqrt[3]{x}-4}$

គ. $\lim\limits_{x\to1}\frac{\sqrt[3]{7+x^3}-\sqrt{3+x^2}}{x-1}$

ឃ. $\lim\limits_{x\to1}\frac{\sqrt[3]{x^3+7x}-\sqrt[3]{10x-2}}{(x-1)^2}$

ង. $\lim\limits_{x\to0}\frac{\sqrt[3]{x^2+1}+\sqrt[3]{x^2-1}}{x^2}$

ច. $\lim\limits_{x\to4}\frac{\sqrt[3]{x+4}+\sqrt[3]{8-x^2}}{x-4}$

ឆ. $\lim\limits_{x\to-2}\frac{\sqrt[3]{x^2-2x}+\sqrt[3]{x-6}}{x+2}$

ជ. $\lim\limits_{x\to0}\frac{\sqrt[3]{x+1}-1}{x(x+2)}$

ឈ. $\lim\limits_{x\to1}\frac{\sqrt[5]{x^2+x-1}-1}{1+\sqrt[3]{1-2x}}$

ញ. $\lim\limits_{x\to0}\frac{\sqrt[5]{(x+1)^3}-1}{x}$

ដំណោះស្រាយ

ដោះស្រាយលីមីតខាងក្រោម៖

ក. $\lim\limits_{x\to1}\frac{\sqrt[3]{x-2}+\sqrt[3]{1-x+x^2}}{x^2-1}$, រាងមិនកំណត់ $\frac{0}{0}$

យើងមាន $\lim\limits_{x\to1}\frac{\sqrt[3]{x-2}+1+\sqrt[3]{1-x+x^2}-1}{x^2-1}$

អាចសរសេរទៅជា $=\lim\limits_{x\to1}\frac{\sqrt[3]{x-2}+1}{x^2-1}+\lim\limits_{x\to1}\frac{\sqrt[3]{1-x+x^2}-1}{x^2-1}$

តាង $A=\lim\limits_{x\to1}\frac{\sqrt[3]{x-2}+1}{x^2-1}$ និង តាង $B=\lim\limits_{x\to1}\frac{\sqrt[3]{1-x+x^2}-1}{x^2-1}$

គណនា $A=\lim\limits_{x\to1}\frac{\sqrt[3]{x-2}+1}{x^2-1}$

$$\lim_{x\to1}\frac{\sqrt[3]{x-2}+1}{x^2-1}=\lim_{x\to1}\left[\frac{\sqrt[3]{x-2}+1}{x^2-1}\times\frac{\sqrt[3]{(x-2)^2}-\sqrt[3]{x-2}+1}{\sqrt[3]{(x-2)^2}-\sqrt[3]{x-1}+1}\right]$$

$$=\lim_{x\to1}\frac{x-1}{(x^2-1)(\sqrt[3]{(x-2)^2}-\sqrt[3]{x-2}+1)}$$

$$=\lim_{x\to1}\frac{x-1}{(x-1)(x+1)(\sqrt[3]{(x-2)^2}-\sqrt[3]{x-2}+1)}$$

$$=\lim_{x\to1}\frac{1}{(x+1)(\sqrt[3]{(x-2)^2}-\sqrt[3]{x-2}+1)}$$

$$=\frac{1}{(1+1)(\sqrt[3]{(1-2)^2}-\sqrt[3]{1-2}+1)}$$

$$=\frac{1}{6}$$

គណនា $B=\lim\limits_{x\to1}\frac{\sqrt[3]{1-x+x^2}-1}{x^2-1}$

$$B=\lim_{x\to1}\left[\frac{\sqrt[3]{1-x+x^2}-1}{x^2-1}\times\frac{\sqrt[3]{(1-x+x^2)^2}+\sqrt[3]{1-x+x^2}+1}{\sqrt[3]{(1-x+x^2)^2}+\sqrt[3]{1-x+x^2}+1}\right]$$

$$B=\lim_{x\to1}\frac{1-x+x^2-1}{(x^2-1)(\sqrt[3]{(1-x+x^2)^2}+\sqrt[3]{1-x+x^2}+1)}$$

$$B=\lim_{x\to1}\frac{x^2-x}{(x^2-1)(\sqrt[3]{(1-x+x^2)^2}+\sqrt[3]{1-x+x^2}+1)}$$

$$B=\lim_{x\to1}\frac{x(x-1)}{(x-1)(x+1)(\sqrt[3]{(1-x+x^2)^2}+\sqrt[3]{1-x+x^2}+1)}$$

$$B=\lim_{x\to1}\frac{x}{(x+1)(\sqrt[3]{(1-x+x^2)^2}+\sqrt[3]{1-x+x^2}+1)}$$

$$B=\lim_{x\to1}\frac{1}{(1+1)(\sqrt[3]{(1-1+1^2)^2}+\sqrt[3]{1-1+1^2}+1)}$$

$$B=\frac{1}{6}$$

យើងបាន $A+B=\frac{1}{6}+\frac{1}{6}=\frac{1}{3}$

ដូចនេះ $\boxed{\lim\limits_{x\to1}\frac{\sqrt[3]{x-2}+\sqrt[3]{1-x+x^2}}{x^2-1}=\frac{1}{3}}$

ខ. $\lim\limits_{x\to64}\frac{\sqrt{x}-8}{\sqrt[3]{x}-4}$, រាងមិនកំណត់ $\frac{0}{0}$

យើងបាន $\lim\limits_{x\to64}\frac{\sqrt{x}-8}{\sqrt[3]{x}-4}=\lim\limits_{x\to1}\left[\frac{\sqrt{x}-8}{\sqrt[3]{x}-4}\times\frac{\sqrt{x}+8}{\sqrt{x}+8}\times\frac{\sqrt[3]{x^2}+4\sqrt[3]{x}+16}{\sqrt[3]{x^2}+4\sqrt[3]{x}+16}\right]$

$$=\lim_{x\to64}\frac{(x-64)(\sqrt[3]{x^2}+4\sqrt[3]{x}+16)}{(x-64)(\sqrt{x}+8)}$$

$$=\lim_{x\to64}\frac{\sqrt[3]{x^2}+4\sqrt[3]{x}+16}{\sqrt{x}+8}$$

$$=\frac{\sqrt[3]{64^2}+4\sqrt[3]{64}+16}{\sqrt{64}+8}$$

$$=3$$

ដូចនេះ $\boxed{\lim\limits_{x\to64}\frac{\sqrt{x}-8}{\sqrt[3]{x}-4}=3}$

គ. $\lim\limits_{x\to1}\frac{\sqrt[3]{7+x^3}-\sqrt{3+x^2}}{x-1}$, រាងមិនកំណត់ $\frac{0}{0}$

យើងមាន $\lim\limits_{x\to1}\frac{\sqrt[3]{7+x^3}-\sqrt{3+x^2}}{x-1}$

អាចសរសេរទៅជា $\lim\limits_{x\to1}\frac{\sqrt[3]{7+x^3}-2}{x-1}-\lim\limits_{x\to1}\frac{\sqrt{3+x^2}-2}{x-1}$

តាង $A=\lim\limits_{x\to1}\frac{\sqrt[3]{7+x^3}-2}{x-1}$ និង តាង $B=\lim\limits_{x\to1}\frac{\sqrt{3+x^2}-2}{x-1}$

គណនា $A=\lim\limits_{x\to1}\frac{\sqrt[3]{7+x^3}-2}{x-1}$

$$\lim_{x\to1}\frac{\sqrt[3]{7+x^3}-2}{x-1}=\lim_{x\to1}\left[\frac{\sqrt[3]{7+x^3}-2}{x-1}\times\frac{\sqrt[3]{(7+x^3)^2}+2\sqrt[3]{7+x^3}+4}{\sqrt[3]{(7+x^3)^2}+2\sqrt[3]{7+x^3}+4}\right]$$

$$=\lim_{x\to1}\frac{x^3-1}{(x-1)(\sqrt[3]{(7+x^3)^2}+2\sqrt[3]{7+x^3}+4)}$$

$$=\lim_{x\to1}\frac{(x-1)(x^2+x+1)}{(x-1)(\sqrt[3]{(7+x^3)^2}+2\sqrt[3]{7+x^3}+4)}$$

$$=\lim_{x\to1}\frac{x^2+x+1}{\sqrt[3]{(7+x^3)^2}+2\sqrt[3]{7+x^3}+4}$$

$$=\frac{1^2+1+1}{\sqrt[3]{(7+1^3)^2}+2\sqrt[3]{7+1^3}+4}$$

$$=\frac{1}{4}$$

គណនា $B=\lim\limits_{x\to1}\frac{\sqrt{3+x^2}-2}{x-1}$

$$\lim_{x\to1}\frac{\sqrt{3+x^2}-2}{x-1}=\lim_{x\to1}\left[\frac{\sqrt{3+x^2}-2}{x-1}\times\frac{\sqrt{3+x^2}+2}{\sqrt{3+x^2}+2}\right]$$

$$=\lim_{x\to1}\frac{x^2-1}{(x-1)(\sqrt{3+x^2}+2)}$$

$$=\lim_{x\to1}\frac{(x-1)(x+1)}{(x-1)(\sqrt{3+x^2}+2)}$$

$$=\lim_{x\to1}\frac{x+1}{\sqrt{3+x^2}+2}$$

$$=\frac{1+1}{\sqrt{3+1^2}+2}$$

$$=\frac{1}{2}$$

នោះយើងបាន $A-B=\frac{1}{4}-\frac{1}{2}$

$$=-\frac{1}{4}$$

ដូចនេះ $\boxed{\lim\limits_{x\to1}\frac{\sqrt[3]{7+x^3}-\sqrt{3+x^2}}{x-1}=-\frac{1}{4}}$

ឃ. $\lim\limits_{x\to1}\frac{\sqrt[3]{x^3+7x}-\sqrt[3]{10x-2}}{(x-1)^2}$, រាងមិនកំណត់ $\frac{0}{0}$

តាង $A=\lim\limits_{x\to1}\frac{\sqrt[3]{x^3+7x}-\sqrt[3]{10x-2}}{(x-1)^2}$ យើងបាន

$$A=\lim_{x\to1}\frac{(x^3+7x)-(10x-2)}{(x-1)^2\left[\sqrt[3]{(x^3+7x)^2}+\sqrt[3]{(x^3+7x)(10x-2)}+\sqrt[3]{(10x-2)^2}\right]}$$
$$=\lim_{x\to1}\frac{x^3-3x+2}{(x-1)^2\left[\sqrt[3]{(x^3+7x)^2}+\sqrt[3]{(x^3+7x)(10x-2)}+\sqrt[3]{(10x-2)^2}\right]}$$
$$=\lim_{x\to1}\frac{x^3-x-2x+2}{(x-1)^2\left[\sqrt[3]{(x^3+7x)^2}+\sqrt[3]{(x^3+7x)(10x-2)}+\sqrt[3]{(10x-2)^2}\right]}$$
$$=\lim_{x\to1}\frac{(x-1)^2(x+2)}{(x-1)^2\left[\sqrt[3]{(x^3+7x)^2}+\sqrt[3]{(x^3+7x)(10x-2)}+\sqrt[3]{(10x-2)^2}\right]}$$
$$=\lim_{x\to1}\frac{x+2}{\sqrt[3]{(x^3+7x)^2}+\sqrt[3]{(x^3+7x)(10x-2)}+\sqrt[3]{(10x-2)^2}}$$
$$=\frac{1+2}{\sqrt[3]{(1^3+7)^2}+\sqrt[3]{(1^3+7)(10-2)}+\sqrt[3]{(10-2)^2}}$$
$$=\frac{3}{\sqrt[3]{64}+\sqrt[3]{64}+\sqrt[3]{64}}$$
$$=\frac{3}{4+4+4}$$
$$=\frac{1}{4}$$

ដូចនេះ $\boxed{\lim_{x\to1}\frac{\sqrt[3]{x^3+7x}-\sqrt[3]{10x-2}}{(x-1)^2}=\frac{1}{4}}$

ង. $\lim_{x\to0}\frac{\sqrt[3]{x^2+1}+\sqrt[3]{x^2-1}}{x^2}$, រាងមិនកំណត់ $\frac{0}{0}$

តាង $A=\lim_{x\to0}\frac{\sqrt[3]{x^2+1}+\sqrt[3]{x^2-1}}{x^2}$

យើងបាន $A=\lim_{x\to0}\frac{x^2+1+x^2-1}{x^2\left(\sqrt[3]{(x^2+1)^2}-\sqrt[3]{(x^2+1)(x^2-1)}+\sqrt[3]{(x^2-1)^2}\right)}$

$$=\lim_{x\to0}\frac{2x^2}{x^2\left(\sqrt[3]{(x^2+1)^2}-\sqrt[3]{(x^2+1)(x^2-1)}+\sqrt[3]{(x^2-1)^2}\right)}$$
$$=\frac{2}{\sqrt[3]{(0^2+1)^2}-\sqrt[3]{(0^2+1)(0^2-1)}+\sqrt[3]{(0^2-1)^2}}$$
$$=\frac{2}{3}$$

ដូចនេះ $\boxed{\lim_{x\to0}\frac{\sqrt[3]{x^2+1}+\sqrt[3]{x^2-1}}{x^2}=\frac{2}{3}}$

ច. $\lim_{x\to4}\frac{\sqrt[3]{x+4}+\sqrt[3]{8-x^2}}{x-4}$, រាងមិនកំណត់ $\frac{0}{0}$

តាង$A=\lim_{x\to4}\frac{\sqrt[3]{x+4}+\sqrt[3]{8-x^2}}{x-4}$ យើងបាន

$$A=\lim_{x\to4}\left[\frac{\sqrt[3]{x+4}+\sqrt[3]{8-x^2}}{x-4}\times\frac{\sqrt[3]{(x+4)^2}-\sqrt[3]{(x+4)(8-x^2)}+\sqrt[3]{(8-x^2)^2}}{\sqrt[3]{(x+4)^2}-\sqrt[3]{(x+4)(8-x^2)}+\sqrt[3]{(8-x^2)^2}}\right]$$
$$=\lim_{x\to4}\frac{(x+4)+(8-x^2)}{(x-4)\left[\sqrt[3]{(x+4)^2}-\sqrt[3]{(x+4)(8-x^2)}+\sqrt[3]{(8-x^2)^2}\right]}$$
$$=\lim_{x\to4}\frac{-(x-4)(x+3)}{(x-4)\left[\sqrt[3]{(x+4)^2}-\sqrt[3]{(x+4)(8-x^2)}+\sqrt[3]{(8-x^2)^2}\right]}$$
$$=\lim_{x\to4}\frac{-(x+3)}{\sqrt[3]{(x+4)^2}-\sqrt[3]{(x+4)(8-x^2)}+\sqrt[3]{(8-x^2)^2}}$$
$$=\frac{-(4+3)}{\sqrt[3]{(4+4)^2}-\sqrt[3]{(4+4)(8-4^2)}+\sqrt[3]{(8-4^2)^2}}$$
$$=-\frac{7}{12}$$

ដូចនេះ $\boxed{\lim_{x\to4}\frac{\sqrt[3]{x+4}+\sqrt[3]{8-x^2}}{x-4}=-\frac{7}{12}}$

ឆ. $\lim_{x\to-2}\frac{\sqrt[3]{x^2-2x}+\sqrt[3]{x-6}}{x+2}$, រាងមិនកំណត់ $\frac{0}{0}$

តាង $A=\lim_{x\to-2}\frac{\sqrt[3]{x^2-2x}+\sqrt[3]{x-6}}{x+2}$

យើងបាន $A=\lim_{x\to-2}\frac{x^2-x-6}{(x+2)\left[\sqrt[3]{(x^2-2x)^2}-\sqrt[3]{(x^2-2x)(x-6)}+\sqrt[3]{(x-6)^2}\right]}$

$$=\lim_{x\to-2}\frac{(x-3)(x+2)}{(x+2)\left[\sqrt[3]{(x^2-2x)^2}-\sqrt[3]{(x^2-2x)(x-6)}+\sqrt[3]{(x-6)^2}\right]}$$
$$=\lim_{x\to-2}\frac{x-3}{\sqrt[3]{(x^2-2x)^2}-\sqrt[3]{(x^2-2x)(x-6)}+\sqrt[3]{(x-6)^2}}$$
$$=\frac{-2-3}{\sqrt[3]{(4+4)^2}-\sqrt[3]{(4+4)(-2-6)}+\sqrt[3]{(-2-6)^2}}$$
$$=\frac{-5}{4+4+4}$$
$$=-\frac{5}{12}$$

ដូចនេះ $\boxed{\lim_{x\to-2}\frac{\sqrt[3]{x^2-2x}+\sqrt[3]{x-6}}{x+2}=-\frac{5}{12}}$

ជ. $\lim_{x\to0}\frac{\sqrt[3]{x+1}-1}{x(x+2)}$, រាងមិនកំណត់ $\frac{0}{0}$

យើងបាន $\lim_{x\to0}\frac{\sqrt[3]{x+1}-1}{x(x+2)}=\lim_{x\to0}\left[\frac{\sqrt[3]{x+1}-1}{x(x+2)}\times\frac{\sqrt[3]{(x+1)^2}+\sqrt[3]{x+1}+1}{\sqrt[3]{(x+1)^2}+\sqrt[3]{x+1}+1}\right]$

$$=\lim_{x\to0}\frac{x}{x(x+2)\left[\sqrt[3]{(x+1)^2}+\sqrt[3]{x+1}+1\right]}$$
$$=\lim_{x\to0}\frac{1}{(x+2)\left[\sqrt[3]{(x+1)^2}+\sqrt[3]{x+1}+1\right]}$$
$$=\frac{1}{(0+2)\left[\sqrt[3]{(0+1)^2}+\sqrt[3]{0+1}+1\right]}=\frac{1}{6}$$

ដូចនេះ $\boxed{\lim_{x\to0}\frac{\sqrt[3]{x+1}-1}{x(x+2)}=\frac{1}{6}}$

ឈ. $\lim_{x\to1}\frac{\sqrt[5]{x^2+x-1}-1}{1+\sqrt[3]{1-2x}}$, រាងមិនកំណត់ $\frac{0}{0}$

តាង $A=\lim_{x\to1}\frac{\sqrt[5]{x^2+x-1}-1}{1+\sqrt[3]{1-2x}}$ យើងបាន

$$A=\lim_{x\to1}\frac{(x^2+x-1-1)\left[1-\sqrt[3]{1-2x}+\sqrt[3]{(1-2x)^2}\right]}{(2-2x)\left[\sqrt[5]{(x^2+x-1)^4}+\sqrt[5]{(x^2+x-1)^3}+\ldots+1\right]}$$
$$A=\lim_{x\to1}\frac{(x^2+x-2)\left[1-\sqrt[3]{1-2x}+\sqrt[3]{(1-2x)^2}\right]}{(2-2x)\left[\sqrt[5]{(x^2+x-1)^4}+\sqrt[5]{(x^2+x-1)^3}+\ldots+1\right]}$$
$$A=\lim_{x\to1}\frac{(x+2)(x-1)\left[1-\sqrt[3]{1-2x}+\sqrt[3]{(1-2x)^2}\right]}{-2(x-1)\left[\sqrt[5]{(x^2+x-1)^4}+\sqrt[5]{(x^2+x-1)^3}+\ldots+1\right]}$$
$$A=\lim_{x\to1}\frac{(x+2)\left[1-\sqrt[3]{1-2x}+\sqrt[3]{(1-2x)^2}\right]}{-2\left[\sqrt[5]{(x^2+x-1)^4}+\sqrt[5]{(x^2+x-1)^3}+\ldots+1\right]}$$
$$A=\frac{(1+2)\left[1-\sqrt[3]{1-2}+\sqrt[3]{(1-2)^2}\right]}{-2\left[\sqrt[5]{(1^2+1-1)^4}+\sqrt[5]{(1^2+1-1)^3}+\ldots+1\right]}$$
$$A=\frac{3(1+1+1)}{-2(\underbrace{1+1+\cdots+1}_{5\text{តួ}})}=-\frac{9}{10}$$

ដូចនេះ $\boxed{\lim_{x\to1}\frac{\sqrt[5]{x^2+x-1}-1}{1+\sqrt[3]{1-2x}}=-\frac{9}{10}}$

ញ. $\lim_{x\to0}\frac{\sqrt[5]{(x+1)^3}-1}{x}$, រាងមិនកំណត់ $\frac{0}{0}$

តាង $A\lim_{x\to0}\frac{\sqrt[5]{(x+1)^3}-1}{x}$ យើងបាន

$$A=\lim_{x\to0}\left[\frac{\sqrt[5]{(x+1)^3}-1}{x}\times\frac{\sqrt[5]{(x+1)^{12}}+\sqrt[5]{(x+1)^9}+\cdots+1}{\sqrt[5]{(x+1)^{12}}+\sqrt[5]{(x+1)^9}+\cdots+1}\right]$$

មេរៀនទី 3 លំហាត់ និង ដំណោះស្រាយ

$$A=\lim_{x\to 0}\frac{(x+1)^3-1}{x\left[\sqrt[5]{(x+1)^{12}}+\sqrt[5]{(x+1)^{9}}+\cdots+1\right]}$$
$$A=\lim_{x\to 0}\frac{x^3+3x^2+3x+1-1}{x\left[\sqrt[5]{(x+1)^{12}}+\sqrt[5]{(x+1)^{9}}+\cdots+1\right]}$$
$$A=\lim_{x\to 0}\frac{x(x^2+3x+3)}{x\left[\sqrt[5]{(x+1)^{12}}+\sqrt[5]{(x+1)^{9}}+\cdots+1\right]}$$
$$A=\lim_{x\to 0}\frac{x^2+3x+3}{\sqrt[5]{(x+1)^{12}}+\sqrt[5]{(x+1)^{9}}+\cdots+1}$$
$$A=\frac{0^2+0+3}{\sqrt[5]{(0+1)^{12}}+\sqrt[5]{(0+1)^{9}}+\cdots+1}$$
$$A=\frac{3}{\underbrace{1+1+\cdots+1}_{5\text{តួ}}}=\frac{3}{5}$$

ដូចនេះ $\boxed{\lim_{x\to 0}\frac{\sqrt[5]{(x+1)^3}-1}{x}=\frac{3}{5}}$

លំហាត់គំរូទី១១

ដោះស្រាយលីមីតខាងក្រោម៖

ក. $\lim_{x\to+\infty}\frac{(x-1)(2x+3)(2-x)}{(x^2+1)(2x+1)}$

ខ. $\lim_{x\to-\infty}\frac{(2x^2-5x)(3x^5-2)}{-8x^7-x^5}$

គ. $\lim_{x\to+\infty}\frac{x^2-7x^3}{4x^4+7x^2-5}$

ឃ. $\lim_{x\to-\infty}\frac{(2x^3+4x)(-3x^2-1)}{7x^6-4x}$

ង. $\lim_{x\to+\infty}\frac{(x^2+4)(-2-3x^3)}{(x^2-2)(4x^2+2020)}$

ច. $\lim_{x\to-\infty}\frac{(2x-1)(3x-4)(1-x)}{(x-4)(2020-3x)}$

ដំណោះស្រាយ

ដោះស្រាយលីមីតខាងក្រោម៖

ក. $\lim_{x\to+\infty}\frac{(x-1)(2x+3)(2-x)}{(x^2+1)(2x+1)}$, រាងមិនកំណត់ $\frac{\infty}{\infty}$

យើងបាន $\lim_{x\to+\infty}\frac{(x-1)(2x+3)(2-x)}{(x^2+1)(2x+1)}=\lim_{x\to+\infty}\frac{x(1-\frac{1}{x})x(2+\frac{1}{x})x(\frac{2}{x}-1)}{x^2(1+\frac{1}{x^2})x(2+\frac{1}{x})}$

$$=\lim_{x\to+\infty}\frac{x^3(1-\frac{1}{x})(2+\frac{1}{x})(\frac{2}{x}-1)}{x^3(1+\frac{1}{x^2})(2+\frac{1}{x})}$$
$$=\frac{(1-0)(2+0)(0-1)}{(1+0)(2+0)}$$
$$=-1$$

ដូចនេះ $\boxed{\lim_{x\to+\infty}\frac{(x-1)(2x+3)(2-x)}{(x^2+1)(2x+1)}=-1}$

ខ. $\lim_{x\to-\infty}\frac{(2x^2-5x)(3x^5-2)}{-8x^7-x^5}$, រាងមិនកំណត់ $\frac{\infty}{\infty}$

យើងបាន $\lim_{x\to-\infty}\frac{(2x^2-5x)(3x^5-2)}{-8x^7-x^5}=\lim_{x\to-\infty}\frac{x^2(2-\frac{5}{x})x^5(3-\frac{2}{x^5})}{-x^7(8+\frac{1}{x^2})}$

$$=\lim_{x\to-\infty}\frac{x^7(2-\frac{5}{x})(3-\frac{2}{x^5})}{-x^7(8+\frac{1}{x^2})}$$
$$=\frac{(2-0)(3-0)}{-(8+0)}=-\frac{3}{4}$$

ដូចនេះ $\boxed{\lim_{x\to-\infty}\frac{(2x^2-5x)(3x^5-2)}{-8x^7-x^5}=-\frac{3}{4}}$

គ. $\lim_{x\to+\infty}\frac{x^2-7x^3}{4x^4+7x^2-5}$, រាងមិនកំណត់ $\frac{\infty}{\infty}$

យើងបាន $\lim_{x\to+\infty}\frac{x^2-7x^3}{4x^4+7x^2-5}=\lim_{x\to+\infty}\frac{x^3\left(\frac{1}{x}-7\right)}{x^4\left(4+\frac{7}{x^2}-\frac{5}{x^4}\right)}$

$$=\lim_{x\to+\infty}\frac{\frac{1}{x}-7}{x\left(4+\frac{7}{x^2}-\frac{5}{x^4}\right)}$$
$$=\frac{0-7}{+\infty(4+0-0)}=0$$

ដូចនេះ $\boxed{\lim_{x\to+\infty}\frac{x^2-7x^3}{4x^4+7x^2-5}=0}$

ឃ. $\lim_{x\to-\infty}\frac{(2x^3+4x)(-3x^2-1)}{7x^6-4x}$, រាងមិនកំណត់ $\frac{\infty}{\infty}$

យើងបាន $\lim_{x\to-\infty}\frac{(2x^3+4x)(-3x^2-1)}{7x^6-4x}=\lim_{x\to-\infty}\frac{x^3(2+\frac{4}{x^2})x^2(-3-\frac{1}{x^2})}{x^6(7-\frac{4}{x^5})}$

$$=\lim_{x\to-\infty}\frac{x^5(2+\frac{4}{x^2})(-3-\frac{1}{x^2})}{x^6(7-\frac{4}{x^5})}$$
$$=\lim_{x\to-\infty}\frac{(2+\frac{4}{x^2})(-3-\frac{1}{x^2})}{x(7-\frac{4}{x^5})}$$
$$=\frac{(2+0)(-3-0)}{(-\infty)(7-0)}=0$$

ដូចនេះ $\boxed{\lim_{x\to-\infty}\frac{(2x^3+4x)(-3x^2-1)}{7x^6-4x}=0}$

ង. $\lim_{x\to+\infty}\frac{(x^2+4)(-2-3x^3)}{(x^2-2)(4x^2+2020)}$, រាងមិនកំណត់ $\frac{\infty}{\infty}$

យើងបាន $\lim_{x\to+\infty}\frac{(x^2+4)(-2-3x^3)}{(x^2-2)(4x^2+2020)}=\lim_{x\to+\infty}\frac{x^2(1+\frac{4}{x^2})x^3(-\frac{2}{x^3}-3)}{x^2(1-\frac{2}{x^2})x^2(4+\frac{2020}{x^2})}$

$$=\lim_{x\to+\infty}\frac{x^5(1+\frac{4}{x^2})(-\frac{2}{x^3}-3)}{x^4(1-\frac{2}{x^2})(4+\frac{2020}{x^2})}$$
$$=\lim_{x\to+\infty}\frac{x(1+\frac{4}{x^2})(-\frac{2}{x^3}-3)}{(1-\frac{2}{x^2})(4+\frac{2020}{x^2})}$$
$$=\frac{(+\infty)(1+0)(0-3)}{(1-0)(4+0)}$$
$$=-\infty$$

ដូចនេះ $\boxed{\lim_{x\to+\infty}\frac{(x^2+4)(-2-3x^3)}{(x^2-2)(4x^2+2020)}=-\infty}$

ច. $\lim_{x\to-\infty}\frac{(2x-1)(3x-4)(1-x)}{(x-4)(2020-3x)}$, រាងមិនកំណត់ $\frac{\infty}{\infty}$

យើងបាន $\lim\limits_{x\to-\infty}\frac{(2x-1)(3x-4)(1-x)}{(x-4)(2020-3x)}=\lim\limits_{x\to-\infty}\frac{x(2-\frac{1}{x})x(3-\frac{4}{x})x(\frac{1}{x}-1)}{x(1-\frac{4}{x})x(\frac{2020}{x}-3)}$

$=\lim\limits_{x\to-\infty}\frac{x^3\left(2-\frac{1}{x}\right)\left(3-\frac{4}{x}\right)\left(\frac{1}{x}-1\right)}{x^2\left(1-\frac{4}{x}\right)\left(\frac{2020}{x}-3\right)}$

$=\lim\limits_{x\to-\infty}\frac{x\left(2-\frac{1}{x}\right)\left(3-\frac{4}{x}\right)\left(\frac{1}{x}-1\right)}{\left(1-\frac{4}{x}\right)\left(\frac{2020}{x}-3\right)}$

$=\lim\limits_{x\to-\infty}\frac{(-\infty)(2-0)(3-0)(0-1)}{(1-0)(0-3)}$

$=-\infty$

ដូចនេះ $\boxed{\lim\limits_{x\to-\infty}\frac{(2x-1)(3x-4)(1-x)}{(x-4)(2020-3x)}=-\infty}$

លំហាត់គំរូទី១២

ដោះស្រាយលីមីតខាងក្រោម៖

ក. $\lim\limits_{x\to+\infty}\frac{(x-3)(x^2+4)(2-3x)}{(2x-1)(1-3x^3)}$

ខ. $\lim\limits_{x\to-\infty}\frac{\frac{2}{3}x^{20}-x^5}{7x^{20}-3x^{15}}$

គ. $\lim\limits_{x\to+\infty}\frac{3-x^6+2x^{13}}{x^3-7x^2+\frac{x^{13}}{4}}$

ឃ. $\lim\limits_{x\to+\infty}\frac{2x^2+5-3x^{2020}}{5x+27x^{2020}-8}$

ដំណោះស្រាយ

ដោះស្រាយលីមីតខាងក្រោម៖

ក. $\lim\limits_{x\to+\infty}\frac{(x-3)(x^2+4)(2-3x)}{(2x-1)(1-3x^3)}$, រាងមិនកំណត់ $\frac{\infty}{\infty}$

យើងបាន $\lim\limits_{x\to+\infty}\frac{(x-3)(x^2+4)(2-3x)}{(2x-1)(1-3x^3)}=\lim\limits_{x\to+\infty}\frac{x(1-\frac{3}{x})x^2(1+\frac{4}{x^2})x(\frac{2}{x}-3)}{x(2-\frac{1}{x})x^3(\frac{1}{x^3}-3)}$

$=\lim\limits_{x\to+\infty}\frac{x^4(1-\frac{3}{x})(1+\frac{4}{x^2})(\frac{2}{x}-3)}{x^4(2-\frac{1}{x})(\frac{1}{x^3}-3)}$

$=\frac{(1-0)(1+0)(0-3)}{(2-0)(0-3)}=\frac{1}{2}$

ដូចនេះ $\boxed{\lim\limits_{x\to+\infty}\frac{(x-3)(x^2+4)(2-3x)}{(2x-1)(1-3x^3)}=\frac{1}{2}}$

ខ. $\lim\limits_{x\to-\infty}\frac{\frac{2}{3}x^{20}-x^5}{7x^{20}-3x^{15}}$, រាងមិនកំណត់ $\frac{\infty}{\infty}$

យើងបាន $\lim\limits_{x\to-\infty}\frac{\frac{2}{3}x^{20}-x^5}{7x^{20}-3x^{15}}=\lim\limits_{x\to-\infty}\frac{x^{20}(\frac{2}{3}-\frac{1}{x^{15}})}{x^{20}(7-\frac{3}{x^5})}$

$=\lim\limits_{x\to-\infty}\frac{(\frac{2}{3}-\frac{1}{x^{15}})}{(7-\frac{3}{x^5})}$

$=\frac{(\frac{2}{3}-0)}{(7-0)}=\frac{2}{21}$

ដូចនេះ $\boxed{\lim\limits_{x\to-\infty}\frac{\frac{2}{3}x^{20}-x^5}{7x^{20}-3x^{15}}=\frac{2}{21}}$

គ. $\lim\limits_{x\to+\infty}\frac{3-x^6+2x^{13}}{x^3-7x^2+\frac{x^{13}}{4}}$, រាងមិនកំណត់ $\frac{\infty}{\infty}$

យើងបាន $\lim\limits_{x\to+\infty}\frac{3-x^6+2x^{13}}{x^3-7x^2+\frac{x^{13}}{4}}=\lim\limits_{x\to+\infty}\frac{x^{13}\left(\frac{3}{x^{13}}-\frac{1}{x^7}+2\right)}{x^{13}\left(\frac{1}{x^{10}}-\frac{7}{x^{11}}+\frac{1}{4}\right)}$

$=\lim\limits_{x\to+\infty}\frac{\frac{3}{x^{13}}-\frac{1}{x^7}+2}{\frac{1}{x^{10}}-\frac{7}{x^{11}}+\frac{1}{4}}$

$=\frac{0-0+2}{0-0+\frac{1}{4}}$

$=\frac{2}{\frac{1}{4}}$

$=8$

ដូចនេះ $\boxed{\lim\limits_{x\to+\infty}\frac{3-x^6+2x^{13}}{x^3-7x^2+\frac{x^{13}}{4}}=8}$

ឃ. $\lim\limits_{x\to+\infty}\frac{2x^2+5-3x^{2020}}{5x+27x^{2020}-8}$, រាងមិនកំណត់ $\frac{\infty}{\infty}$

យើងបាន $\lim\limits_{x\to+\infty}\frac{2x^2+5-3x^{2020}}{5x+27x^{2020}-8}=\lim\limits_{x\to+\infty}\frac{x^{2020}\left(\frac{2}{x^{1998}}+\frac{5}{x^{2020}}-3\right)}{x^{2020}\left(\frac{5}{x^{1999}}+27-\frac{8}{x^{2020}}\right)}$

$=\lim\limits_{x\to+\infty}\frac{\frac{2}{x^{1998}}+\frac{5}{x^{2020}}-3}{\frac{5}{x^{1999}}+27-\frac{8}{x^{2020}}}$

$=\frac{0+0-3}{0+27-0}$

$=\frac{-3}{27}$

$=-\frac{1}{9}$

ដូចនេះ $\boxed{\lim\limits_{x\to+\infty}\frac{2x^2+5-3x^{2020}}{5x+27x^{2020}-8}=-\frac{1}{9}}$

លំហាត់គំរូទី១៣

ដោះស្រាយលីមីតខាងក្រោម៖

ក. $\lim\limits_{x\to+\infty}\left(1+x-x^5\right)$ ខ. $\lim\limits_{x\to-\infty}\left(3x^3-x^2+9\right)$ គ. $\lim\limits_{x\to\infty}\left(\sqrt{x^2+x}-x^2\right)$

ឃ. $\lim\limits_{x\to+\infty}\left(\sqrt{2x^2+1}-\sqrt{x+x^2}\right)$ ង. $\lim\limits_{x\to+\infty}\left(\sqrt{x^2+x}-\sqrt[3]{x^2-x}\right)$ ច. $\lim\limits_{x\to0}\left(\frac{1}{\sqrt{x}}-\frac{x+1}{\sqrt{x}}\right)$

ឆ. $\lim\limits_{x\to1}\left(\frac{2}{x^2-1}-\frac{1}{x-1}\right)$ ជ. $\lim\limits_{x\to-2}\left(\frac{1}{x+2}-\frac{12}{x^3+8}\right)$ ឈ. $\lim\limits_{x\to+\infty}\left(\sqrt{x+\sqrt{x+\sqrt{x}}}-\sqrt{x}\right)$

ញ. $\lim\limits_{x\to\infty}\left(\sqrt[3]{x^3+6x^2}-\sqrt[3]{x^3-6x^2}\right)$

ដំណោះស្រាយ

ដោះស្រាយលីមីតខាងក្រោម៖

ក. $\lim\limits_{x\to+\infty}\left(1+x-x^5\right)$, រាងមិនកំណត់ $+\infty-\infty$

យើងបាន $\lim\limits_{x\to+\infty}\left(1+x-x^5\right)=\lim\limits_{x\to+\infty}x^5\left(\frac{1}{x^5}+\frac{1}{x^4}-1\right)$

$$=+\infty(0+0-1)$$
$$=-\infty$$

ដូចនេះ $\boxed{\lim\limits_{x\to+\infty}\left(1+x-x^5\right)=-\infty}$

ខ. $\lim\limits_{x\to-\infty}\left(3x^3-x^2+9\right)$, រាងមិនកំណត់ $+\infty-\infty$

យើងបាន $\lim\limits_{x\to-\infty}\left(3x^3-x^2+9\right)=\lim\limits_{x\to-\infty}x^3\left(3-\frac{1}{x}+\frac{9}{x^3}\right)$

$$=-\infty(3-0+0)$$
$$=-\infty$$

ដូចនេះ $\boxed{\lim\limits_{x\to-\infty}\left(3x^3-x^2+9\right)=-\infty}$

គ. $\lim\limits_{x\to\infty}\left(\sqrt{x^2+x}-x^2\right)$, រាងមិនកំណត់ $+\infty-\infty$

យើងបាន $\lim\limits_{x\to\infty}\left(\sqrt{x^2+x}-x^2\right)=\lim\limits_{x\to\infty}\left(\sqrt{x^2(1+\frac{1}{x})}-x^2\right)$

$$=\lim_{x\to\infty}\left(|x|\sqrt{1+\frac{1}{x}}-x^2\right)$$

• ករណី $x\to+\infty\Rightarrow|x|=x$

$$\lim_{x\to+\infty}\left(|x|\sqrt{1+\frac{1}{x}}-x^2\right)=\lim_{x\to+\infty}\left(x\sqrt{1+\frac{1}{x}}-x^2\right)$$
$$=\lim_{x\to+\infty}\left[x^2\left(\frac{1}{x}\sqrt{1+\frac{1}{x}}-1\right)\right]$$
$$=+\infty(0-1)$$
$$=-\infty$$

នោះ $\lim\limits_{x\to+\infty}\left(|x|\sqrt{1+\frac{1}{x}}-x^2\right)=-\infty$

• ករណី $x\to-\infty\Rightarrow|x|=-x$

$$\lim_{x\to-\infty}\left(|x|\sqrt{1+\frac{1}{x}}-x^2\right)=\lim_{x\to+\infty}\left(-x\sqrt{1+\frac{1}{x}}-x^2\right)$$
$$=\lim_{x\to-\infty}\left[x^2\left(-\frac{1}{x}\sqrt{1+\frac{1}{x}}-1\right)\right]$$
$$=+\infty(0-1)$$
$$=-\infty$$

នោះ $\lim\limits_{x\to-\infty}\left(|x|\sqrt{1+\frac{1}{x}}-x^2\right)=-\infty$

ដូចនេះ $\boxed{\lim\limits_{x\to\infty}\left(\sqrt{x^2+x}-x^2\right)=-\infty}$

ឃ. $\lim\limits_{x\to+\infty}\left(\sqrt{2x^2+1}-\sqrt{x+x^2}\right)$, រាងមិនកំណត់ $+\infty-\infty$

តាង $A=\lim\limits_{x\to+\infty}\left(\sqrt{2x^2+1}-\sqrt{x+x^2}\right)$

យើងបាន $A=\lim\limits_{x\to+\infty}\left[\left(\sqrt{2x^2+1}-\sqrt{x+x^2}\right)\times\frac{\sqrt{2x^2+1}+\sqrt{x+x^2}}{\sqrt{2x^2+1}+\sqrt{x+x^2}}\right]$

$$A=\lim_{x\to+\infty}\frac{x^2-x+1}{\sqrt{2x^2+1}+\sqrt{x+x^2}}$$
$$A=\lim_{x\to+\infty}\frac{x^2-x+1}{\sqrt{x^2(2+\frac{1}{x^2})}+\sqrt{x^2(\frac{1}{x}+1)}}$$
$$A=\lim_{x\to+\infty}\frac{x^2(1-\frac{1}{x}+\frac{1}{x^2})}{x\left(\sqrt{2+\frac{1}{x^2}}+\sqrt{\frac{1}{x}+1}\right)}$$
$$A=\lim_{x\to+\infty}\frac{x(1-\frac{1}{x}+\frac{1}{x^2})}{\sqrt{2+\frac{1}{x^2}}+\sqrt{\frac{1}{x}+1}}$$
$$A=\frac{x(1-\frac{1}{x}+\frac{1}{x^2})}{\sqrt{2+\frac{1}{x^2}}+\sqrt{\frac{1}{x}+1}}$$
$$A=\frac{+\infty}{\sqrt{2}+1}=+\infty$$

ដូចនេះ $\boxed{\lim\limits_{x\to+\infty}\left(\sqrt{2x^2+1}-\sqrt{x+x^2}\right)=+\infty}$

ង. $\lim\limits_{x\to+\infty}\left(\sqrt{x^2+x}-\sqrt[3]{x^2-x}\right)$, រាងមិនកំណត់ $+\infty-\infty$

យើងបាន $\lim\limits_{x\to+\infty}\left(\sqrt{x^2+x}-\sqrt[3]{x^2-x}\right)=\lim\limits_{x\to+\infty}\left[\sqrt{x^2\left(1+\frac{1}{x}\right)}-\sqrt[3]{x^3\left(\frac{1}{x}-\frac{1}{x^2}\right)}\right]$

$$=\lim_{x\to+\infty}\left(x\sqrt{1+\frac{1}{x}}-x\sqrt[3]{\frac{1}{x}-\frac{1}{x^2}}\right)$$

$$=\lim_{x\to+\infty}\left[x\left(\sqrt{1+\frac{1}{x}}-\sqrt[3]{\frac{1}{x}-\frac{1}{x^2}}\right)\right]$$

$$=+\infty(1-0)=+\infty$$

ដូចនេះ $\boxed{\lim\limits_{x\to+\infty}\left(\sqrt{x^2+x}-\sqrt[3]{x^2-x}\right)=+\infty}$

ប. $\lim\limits_{x\to0}\left(\frac{1}{\sqrt{x}}-\frac{x+1}{\sqrt{x}}\right)$, រាងមិនកំណត់ $+\infty-\infty$

យើងបាន $\lim\limits_{x\to0}\left(\frac{1}{\sqrt{x}}-\frac{x+1}{\sqrt{x}}\right)=\lim\limits_{x\to0}\frac{1-x-1}{\sqrt{x}}$

$$=\lim_{x\to0}\frac{-x}{\sqrt{x}}$$

$$=\lim_{x\to0}\frac{-\sqrt{x}\times\sqrt{x}}{\sqrt{x}}$$

$$=\lim_{x\to0}-\sqrt{x}$$

$$=0$$

ដូចនេះ $\boxed{\lim\limits_{x\to0}\left(\frac{1}{\sqrt{x}}-\frac{x+1}{\sqrt{x}}\right)=0}$

ឆ. $\lim\limits_{x\to1}\left(\frac{2}{x^2-1}-\frac{1}{x-1}\right)$, រាងមិនកំណត់ $+\infty-\infty$

យើងបាន $\lim\limits_{x\to1}\left(\frac{2}{x^2-1}-\frac{1}{x-1}\right)=\lim\limits_{x\to1}\left(\frac{2}{x^2-1}-\frac{x+1}{x^2-1}\right)$

$$=\lim_{x\to1}\frac{2-(x+1)}{x^2-1}$$

$$=\lim_{x\to1}\frac{-x+1}{x^2-1}$$

$$=\lim_{x\to1}\frac{-(x-1)}{(x-1)(x+1)}$$

$$=\lim_{x\to1}\frac{-1}{x+1}$$

$$=\frac{-1}{1+1}$$

$$=-\frac{1}{2}$$

ដូចនេះ $\boxed{\lim\limits_{x\to1}\left(\frac{2}{x^2-1}-\frac{1}{x-1}\right)=-\frac{1}{2}}$

ជ. $\lim\limits_{x\to-2}\left(\frac{1}{x+2}-\frac{12}{x^3+8}\right)$, រាងមិនកំណត់ $+\infty-\infty$

យើងបាន $\lim\limits_{x\to-2}\left(\frac{1}{x+2}-\frac{12}{x^3+8}\right)=\lim\limits_{x\to-2}\left[\frac{x^2-2x+4}{(x+2)(x^2-2x+4)}-\frac{12}{x^3+8}\right]$

$$=\lim_{x\to-2}\frac{x^2-2x+4-12}{(x+2)(x^2-2x+4)}$$

$$=\lim_{x\to-2}\frac{x^2-2x-8}{(x+2)(x^2-2x+4)}$$

$$=\lim_{x\to-2}\frac{(x+2)(x-4)}{(x+2)(x^2-2x+4)}$$

$$=\lim_{x\to-2}\frac{x-4}{x^2-2x+4}$$

$$=\frac{-2-4}{4+4+4}$$

$$=-\frac{1}{2}$$

ដូចនេះ $\boxed{\lim\limits_{x\to-2}\left(\frac{1}{x+2}-\frac{12}{x^3+8}\right)=-\frac{1}{2}}$

ឈ. $\lim\limits_{x\to+\infty}\left(\sqrt{x+\sqrt{x+\sqrt{x}}}-\sqrt{x}\right)$, រាងមិនកំណត់ $+\infty-\infty$

តាង $A=\lim\limits_{x\to+\infty}\left(\sqrt{x+\sqrt{x+\sqrt{x}}}-\sqrt{x}\right)$

យើងអាចសរសេរ

$$A=\lim_{x\to+\infty}\left[\left(\sqrt{x+\sqrt{x+\sqrt{x}}}-\sqrt{x}\right)\times\frac{\sqrt{x+\sqrt{x+\sqrt{x}}}+\sqrt{x}}{\sqrt{x+\sqrt{x+\sqrt{x}}}+\sqrt{x}}\right]$$

$$A=\lim_{x\to+\infty}\frac{\sqrt{x+x\sqrt{\frac{1}{x}}}}{\sqrt{x+x\sqrt{\frac{1}{x}+\sqrt{\frac{1}{x^3}}}}+\sqrt{x}}$$

$$A=\lim_{x\to+\infty}\frac{\sqrt{x}\sqrt{1+\frac{1}{\sqrt{x}}}}{\sqrt{x}\left(\sqrt{1+\sqrt{\frac{1}{x}+\sqrt{\frac{1}{x^3}}}}\right)+\sqrt{x}}$$

$$A=\lim_{x\to+\infty}\frac{\sqrt{x}\sqrt{1+\frac{1}{\sqrt{x}}}}{\sqrt{x}\left(\sqrt{1+\sqrt{\frac{1}{x}+\sqrt{\frac{1}{x^3}}}}+1\right)}$$

$$A=\frac{\sqrt{1+\frac{1}{\infty}}}{\sqrt{1+\sqrt{\frac{1}{\infty}+\sqrt{\frac{1}{\infty}}}}+1}$$

ដូចនេះ $\boxed{\lim\limits_{x\to+\infty}\left(\sqrt{x+\sqrt{x+\sqrt{x}}}-\sqrt{x}\right)=\frac{1}{2}}$

ញ. $\lim\limits_{x\to\infty}\left(\sqrt[3]{x^3+6x^2}-\sqrt[3]{x^3-6x^2}\right)$, រាងមិនកំណត់ $+\infty-\infty$

តាង $A=\lim\limits_{x\to\infty}\left(\sqrt[3]{x^3+6x^2}-\sqrt[3]{x^3-6x^2}\right)$ អាចសរសេរ

$$A=\lim_{x\to\infty}\frac{(x^3+6x^2)-(x^3-6x^2)}{\sqrt[3]{(x^3+6x^2)^2}+\sqrt[3]{(x^3+6x^2)(x^3-6x^2)}+\sqrt[3]{(x^3-6x^2)^2}}$$

$$A=\lim_{x\to\infty}\frac{12x^2}{\sqrt[3]{x^6\left(1+\frac{6}{x}\right)^2}+\sqrt[3]{x^6\left(1+\frac{6}{x}\right)\left(1-\frac{6}{x}\right)}+\sqrt[3]{x^6\left(1-\frac{6}{x}\right)^2}}$$

$$A=\lim_{x\to\infty}\frac{12x^2}{x^2\left[\sqrt[3]{\left(1+\frac{6}{x}\right)^2}+\sqrt[3]{\left(1+\frac{6}{x}\right)\left(1-\frac{6}{x}\right)}+\sqrt[3]{\left(1-\frac{6}{x}\right)^2}\right]}$$

$$A=\lim_{x\to\infty}\frac{12}{\sqrt[3]{\left(1+\frac{6}{x}\right)^2}+\sqrt[3]{\left(1+\frac{6}{x}\right)\left(1-\frac{6}{x}\right)}+\sqrt[3]{\left(1-\frac{6}{x}\right)^2}}$$

$$A=\frac{12}{1+1+1}$$

$$A=4$$

ដូចនេះ $\boxed{\lim\limits_{x\to\infty}\left(\sqrt[3]{x^3+6x^2}-\sqrt[3]{x^3-6x^2}\right)=4}$

មេរៀនទី 3 លំហាត់ និង ដំណោះស្រាយ

លំហាត់គំរូទី១៤

ដោះស្រាយលីមីតខាងក្រោម៖

ក. $\lim_{x\to\frac{\pi}{6}}(\sin x-1)^2$ ខ. $\lim_{x\to\pi}\tan x$ គ. $\lim_{x\to 7\pi}\cos\frac{x}{3}$

ឃ. $\lim_{x\to\frac{13\pi}{4}}(\cot x-\tan x)$ ង. $\lim_{x\to 0}\frac{\sin x+\cos^2 x}{x^2+1}$ ច. $\lim_{x\to\frac{\pi}{4}}\frac{\cos x-\tan x}{\sin^2 x}$

ឆ. $\lim_{x\to\frac{\pi}{3}}\frac{\tan x-\sin x}{\cos x-1}$ ជ. $\lim_{x\to\frac{\pi}{6}}\frac{\sin^2 x+\cot^2 x}{2\sin x}$ ឈ. $\lim_{x\to 0}\frac{\cos x+\tan x}{\tan^2 x+3}$

ញ. $\lim_{x\to\frac{\pi}{4}}\frac{\cos x+\sin x}{\cos^2 2x}$

ដំណោះស្រាយ

ក. $\lim_{x\to\frac{\pi}{6}}(\sin x-1)^2$

យើងបាន $\lim_{x\to\frac{\pi}{6}}(\sin x-1)^2=\left(\sin\frac{\pi}{6}-1\right)^2$

$=\left(\frac{1}{2}-1\right)^2$

$=\left(-\frac{1}{2}\right)^2$

$=\frac{1}{4}$

ដូចនេះ $\boxed{\lim_{x\to\frac{\pi}{6}}(\sin x-1)^2=\frac{1}{4}}$

ខ. $\lim_{x\to\pi}\tan x$

យើងបាន $\lim_{x\to\pi}\tan x=\tan\pi$

$=0$

ដូចនេះ $\boxed{\lim_{x\to\pi}\tan x=0}$

គ. $\lim_{x\to 7\pi}\cos\frac{x}{3}$

យើងបាន $\lim_{x\to 7\pi}\cos\frac{x}{3}=\cos\frac{7\pi}{3}$

$=\cos\left(2\pi+\frac{\pi}{3}\right)$

$=\cos\frac{\pi}{3}$

$=\frac{1}{2}$

ដូចនេះ $\boxed{\lim_{x\to 7\pi}\cos\frac{x}{3}=\frac{1}{2}}$

ឃ. $\lim_{x\to\frac{13\pi}{4}}(\cot x-\tan x)$

យើងបាន $\lim_{x\to\frac{13\pi}{4}}(\cot x-\tan x)=\cot\frac{13\pi}{4}-\tan\frac{13\pi}{4}$

$=\cot\left(3\pi+\frac{\pi}{4}\right)-\tan\left(3\pi+\frac{\pi}{4}\right)$

$=\cot\frac{\pi}{4}-\tan\frac{\pi}{4}$

$=1-1$

$=0$

ដូចនេះ $\boxed{\lim_{x\to\frac{13\pi}{4}}(\cot x-\tan x)=0}$

ង. $\lim_{x\to 0}\frac{\sin x+\cos^2 x}{x^2+1}$

យើងបាន $\lim_{x\to 0}\frac{\sin x+\cos^2 x}{x^2+1}=\frac{\sin 0+\cos 0}{0+1}$

$=\frac{0+1}{1}$

$=1$

ដូចនេះ $\boxed{\lim_{x\to 0}\frac{\sin x+\cos^2 x}{x^2+1}=1}$

ច. $\lim_{x\to\frac{\pi}{4}}\frac{\cos x-\tan x}{\sin^2 x}$ យើងបាន

$\lim_{x\to\frac{\pi}{4}}\frac{\cos x-\tan x}{\sin^2 x}=\frac{\cos\frac{\pi}{4}-\tan\frac{\pi}{4}}{\sin^2 x}$

$=\frac{\frac{\sqrt{2}}{2}-1}{\left(\frac{\sqrt{2}}{2}\right)^2}$

$=\sqrt{2}-2$

ដូចនេះ $\boxed{\lim_{x\to\frac{\pi}{4}}\frac{\cos x-\tan x}{\sin^2 x}=\sqrt{2}-2}$

ឆ. $\lim_{x\to\frac{\pi}{3}}\frac{\tan x-\sin x}{\cos x-1}$

យើងបាន $\lim\limits_{x\to\frac{\pi}{3}} \frac{\tan x - \sin x}{\cos x - 1} = \frac{\tan\frac{\pi}{3} - \sin\frac{\pi}{3}}{\cos\frac{\pi}{3} - 1} = \frac{\sqrt{3} - \frac{\sqrt{3}}{2}}{\frac{1}{2} - 1} = \frac{\frac{\sqrt{3}}{2}}{-\frac{1}{2}} = -\sqrt{3}$

ដូចនេះ $\boxed{\lim\limits_{x\to\frac{\pi}{3}} \frac{\tan x - \sin x}{\cos x - 1} = -\sqrt{3}}$

ជ. $\lim\limits_{x\to\frac{\pi}{6}} \frac{\sin^2 x + \cot^2 x}{2\sin x}$

យើងបាន $\lim\limits_{x\to\frac{\pi}{6}} \frac{\sin^2 x + \cot^2 x}{2\sin x} = \frac{\sin^2\frac{\pi}{6} + \cot^2\frac{\pi}{6}}{2\sin\frac{\pi}{6}} = \frac{\left(\frac{1}{2}\right)^2 + \left(\sqrt{3}\right)^2}{2\times\frac{1}{2}} = \frac{13}{4}$

ដូចនេះ $\boxed{\lim\limits_{x\to\frac{\pi}{6}} \frac{\sin^2 x + \cot^2 x}{2\sin x} = \frac{13}{4}}$

ឈ. $\lim\limits_{x\to 0} \frac{\cos x + \tan x}{\tan^2 x + 3}$

យើងបាន $\lim\limits_{x\to 0} \frac{\cos x + \tan x}{\tan^2 x + 3} = \frac{\cos 0 + \tan 0}{\tan^2 0 + 3} = \frac{1+0}{0+3} = \frac{1}{3}$

ដូចនេះ $\boxed{\lim\limits_{x\to 0} \frac{\cos x + \tan x}{\tan^2 x + 3} = \frac{1}{3}}$

ញ. $\lim\limits_{x\to\frac{\pi}{4}} \frac{\cos x + \sin x}{\cos^2 2x}$

យើងបាន $\lim\limits_{x\to\frac{\pi}{4}} \frac{\cos x + \sin x}{\cos^2 2x} = \frac{\cos\frac{\pi}{4} + \sin\frac{\pi}{4}}{\cos^2 2\frac{\pi}{4}} = \frac{\cos\frac{\pi}{4} + \sin\frac{\pi}{4}}{\cos^2\frac{\pi}{2}} = \frac{\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}}{0^+} = \frac{\sqrt{2}}{0^+} = +\infty$

ដូចនេះ $\boxed{\lim\limits_{x\to\frac{\pi}{4}} \frac{\cos x + \sin x}{\cos^2 2x} = +\infty}$

លំហាត់គំរូទី១៥

ដោះស្រាយលីមីតខាងក្រោម៖

ក. $\lim\limits_{x\to\frac{\pi}{4}} \frac{2\sin\left(x - \frac{\pi}{4}\right)}{\frac{\pi}{4} - x}$

ខ. $\lim\limits_{x\to 0} \frac{-2\sin 5x}{\sqrt{5} - \sqrt{x+5}}$

គ. $\lim\limits_{x\to 0} \frac{1 - \cos^2 3x}{-2x^2}$

ឃ. $\lim\limits_{x\to -\frac{\pi}{2}} \frac{\sin^2 - 1}{1 + \sin x}$

ង. $\lim\limits_{x\to 0} \frac{\cos x - 1}{\sin^2 x}$

ច. $\lim\limits_{x\to 0} \frac{3\sin 3x}{x}$

ឆ. $\lim\limits_{x\to 0} \frac{5\sin 5x}{x}$

ជ. $\lim\limits_{x\to 0} \frac{\sin 3x}{-x}$

ឈ. $\lim\limits_{x\to 0} \frac{\sqrt{2+x} - \sqrt{2-x}}{\sin x}$

ញ. $\lim\limits_{x\to 0} \frac{-2x}{\sin 3x}$

ដ. $\lim\limits_{x\to\frac{\pi}{3}} \frac{\sin x - \sqrt{3}\cos x}{\pi - 3x}$

ឋ. $\lim\limits_{x\to 0} \frac{6x}{\sin(-3x)}$

ឌ. $\lim\limits_{x\to\frac{\pi}{6}} \frac{2(\pi - 6x)}{\sin x - \frac{1}{\sqrt{3}}\cos x}$

ឍ. $\lim\limits_{x\to 0} \frac{-2x\sin x}{1 - \cos^2 x}$

ណ. $\lim\limits_{x\to -\frac{\pi}{2}} \frac{1 + \sin x}{\sin^4 - 1}$

ត. $\lim\limits_{x\to\frac{\pi}{4}} \frac{1 - \tan^3 x}{\tan^2 x - 1}$

ថ. $\lim\limits_{x\to 0} \frac{\sin 6x + \sin 9x}{2x}$

ទ. $\lim\limits_{x\to 0} \frac{x + 2\sin 3x}{x + 3\sin 2x}$

ធ. $\lim\limits_{x\to 0} \frac{2019\sin x}{2018x}$

ន. $\lim\limits_{x\to 0} \frac{\sin 2018x}{\sin 2019x}$

ដំណោះស្រាយ

ដោះស្រាយលីមីតខាងក្រោម៖

ក. $\lim\limits_{x\to\frac{\pi}{4}} \frac{2\sin\left(x - \frac{\pi}{4}\right)}{\frac{\pi}{4} - x}$, រាងមិនកំណត់ $\frac{0}{0}$

យើងបាន $\lim\limits_{x\to\frac{\pi}{4}} \frac{2\sin\left(x - \frac{\pi}{4}\right)}{\frac{\pi}{4} - x} = \lim\limits_{x\to\frac{\pi}{4}} \frac{2\sin\left(x - \frac{\pi}{4}\right)}{-\left(x - \frac{\pi}{4}\right)} = -2\times 1$

ដូចនេះ $\boxed{\lim\limits_{x\to\frac{\pi}{4}}\frac{2\sin\left(x-\frac{\pi}{4}\right)}{\frac{\pi}{4}-x}=-2}$

ខ. $\lim\limits_{x\to 0}\frac{-2\sin 5x}{\sqrt{5}-\sqrt{x+5}}$, រាងមិនកំណត់$\frac{0}{0}$

យើងបាន $\lim\limits_{x\to 0}\frac{-2\sin 5x}{\sqrt{5}-\sqrt{x+5}}=\lim\limits_{x\to 0}\left[\frac{-2\sin 5x}{\sqrt{5}-\sqrt{x+5}}\times\frac{\sqrt{5}+\sqrt{x+5}}{\sqrt{5}+\sqrt{x+5}}\right]$

$$=\lim_{x\to 0}\frac{-2\sin 5x\left(\sqrt{5}+\sqrt{x+5}\right)}{5-(x+5)}$$

$$=\lim_{x\to 0}\frac{-2\sin 5x\left(\sqrt{5}+\sqrt{x+5}\right)}{-x}$$

$$=\lim_{x\to 0}\left[2\left(\sqrt{5}+\sqrt{x+5}\right)\frac{\sin 5x}{5x}\times 5\right]$$

$$=2\left(\sqrt{5}+\sqrt{0+5}\right)\times 1\times 5\ ,\left(\lim_{x\to 0}\frac{\sin 5x}{5x}=1\right)$$

$$=20\sqrt{5}$$

ដូចនេះ $\boxed{\lim\limits_{x\to 0}\frac{-2\sin 5x}{\sqrt{5}-\sqrt{x+5}}=20\sqrt{5}}$

គ. $\lim\limits_{x\to 0}\frac{1-\cos^2 3x}{-2x^2}$,រាងមិនកំណត់$\frac{0}{0}$

យើងបាន $\lim\limits_{x\to 0}\frac{1-\cos^2 3x}{-2x^2}=\lim\limits_{x\to 0}\frac{\sin^2 3x}{-2x^2}$

$$=-\frac{1}{2}\lim_{x\to 0}\frac{\sin^2 3x}{(3x)^2}\times 9$$

$$=-\frac{9}{2}\lim_{x\to 0}\left(\frac{\sin 3x}{3x}\right)^2$$

$$=-\frac{9}{2}\times 1$$

$$=-\frac{9}{2}$$

ដូចនេះ $\boxed{\lim\limits_{x\to 0}\frac{1-\cos^2 3x}{-2x^2}=-\frac{9}{2}}$

ឃ. $\lim\limits_{x\to-\frac{\pi}{2}}\frac{\sin^2 x-1}{1+\sin x}$,រាងមិនកំណត់$\frac{0}{0}$

យើងបាន $\lim\limits_{x\to-\frac{\pi}{2}}\frac{\sin^2 x-1}{1+\sin x}=\lim\limits_{x\to-\frac{\pi}{2}}\frac{(\sin x-1)\cancel{(\sin x+1)}}{\cancel{1+\sin x}}$

$$=\lim_{x\to-\frac{\pi}{2}}(\sin x-1)$$

$$=\sin\left(-\frac{\pi}{2}\right)-1$$

$$=-1-1$$

$$=-2$$

ដូចនេះ $\boxed{\lim\limits_{x\to-\frac{\pi}{2}}\frac{\sin^2 x-1}{1+\sin x}=-2}$

ង. $\lim\limits_{x\to 0}\frac{\cos x-1}{\sin^2 x}$,រាងមិនកំណត់$\frac{0}{0}$

យើងបាន $\lim\limits_{x\to 0}\frac{\cos x-1}{\sin^2 x}=\lim\limits_{x\to 0}\frac{-2\sin^2\frac{x}{2}}{\sin^2 x}$, $\left(1-\cos x=2\sin^2\frac{x}{2}\right)$

$$=\lim_{x\to 0}\left[\frac{-2\sin^2\frac{x}{2}}{\left(\frac{x}{2}\right)^2}\times\frac{x^2}{4\sin^2 x}\right]$$

$$=-2\times\frac{1}{4}$$

$$=-\frac{1}{2}$$

ដូចនេះ $\boxed{\lim\limits_{x\to 0}\frac{\cos x-1}{\sin^2 x}=-\frac{1}{2}}$

ច. $\lim\limits_{x\to 0}\frac{3\sin 3x}{x}$,រាងមិនកំណត់$\frac{0}{0}$

យើងបាន $\lim\limits_{x\to 0}\frac{3\sin 3x}{x}=3\lim\limits_{x\to 0}\frac{\sin 3x}{3x}\times 3$

$$=3\times 1\times 3\quad ,\left(\lim_{x\to 0}\frac{\sin 3x}{3x}=1\right)$$

$$=9$$

ដូចនេះ $\boxed{\lim\limits_{x\to 0}\frac{3\sin 3x}{x}=9}$

ឆ. $\lim\limits_{x\to 0}\frac{5\sin 5x}{x}$,រាងមិនកំណត់$\frac{0}{0}$

យើងបាន $\lim\limits_{x\to 0}\frac{5\sin 5x}{x}=5\lim\limits_{x\to 0}\frac{\sin 5x}{5x}\times 5$

$$=5\times 1\times 5$$

$$=25$$

ដូចនេះ $\boxed{\lim\limits_{x\to 0}\frac{5\sin 5x}{x}=25}$

ជ. $\lim\limits_{x\to 0}\frac{\sin 3x}{-x}$,រាងមិនកំណត់$\frac{0}{0}$

យើងបាន $\lim\limits_{x\to 0}\frac{\sin 3x}{-x}=\lim\limits_{x\to 0}\frac{\sin 3x}{-3x}\times 3$

$$=-1\times 3=-3$$

ដូចនេះ $\boxed{\lim\limits_{x\to 0}\frac{\sin 3x}{-x}=-3}$

ឈ. $\lim\limits_{x\to 0}\frac{\sqrt{2+x}-\sqrt{2-x}}{\sin x}$,រាងមិនកំណត់$\frac{0}{0}$

យើងបាន $\lim\limits_{x\to 0}\frac{\sqrt{2+x}-\sqrt{2-x}}{\sin x}=\lim\limits_{x\to 0}\left[\frac{\sqrt{2+x}-\sqrt{2-x}}{\sin x}\times\frac{\sqrt{2+x}+\sqrt{2-x}}{\sqrt{2+x}+\sqrt{2-x}}\right]$

$$=\lim_{x\to 0}\frac{(2+x)-(2-x)}{\sin x\left(\sqrt{2+x}+\sqrt{2-x}\right)}$$

$$=\lim_{x\to 0}\frac{2x}{\sin x\left(\sqrt{2+x}+\sqrt{2-x}\right)}$$

$$=2\lim_{x\to 0}\left[\frac{x}{\sin x}\times\frac{1}{\sqrt{2+x}+\sqrt{2-x}}\right]$$

$$=2\times 1\times\frac{1}{2\sqrt{2}}$$

$$=\frac{\sqrt{2}}{2}$$

ដូចនេះ $\boxed{\lim\limits_{x\to 0}\frac{\sqrt{2+x}-\sqrt{2-x}}{\sin x}=\frac{\sqrt{2}}{2}}$

ញ. $\lim\limits_{x\to 0}\frac{-2x}{\sin 3x}$,រាងមិនកំណត់$\frac{0}{0}$

យើងបាន $\lim\limits_{x\to 0}\frac{-2x}{\sin 3x}=-2\lim\limits_{x\to 0}\frac{3x}{\sin 3x}\times\frac{1}{3}$

$$=-2\times 1\times\frac{1}{3}$$

$$=-\frac{2}{3}$$

ដូចនេះ $\boxed{\lim\limits_{x\to 0}\frac{-2x}{\sin 3x}=-\frac{2}{3}}$

ដ. $\lim\limits_{x\to\frac{\pi}{3}}\frac{\sin x-\sqrt{3}\cos x}{\pi-3x}$,រាងមិនកំណត់$\frac{0}{0}$

យើងបាន $\lim\limits_{x\to\frac{\pi}{3}}\dfrac{\sin x-\sqrt{3}\cos x}{\pi-3x}=\lim\limits_{x\to\frac{\pi}{3}}\dfrac{2\left(\frac{1}{2}\sin x-\frac{\sqrt{3}}{2}\cos x\right)}{-3\left(x-\frac{\pi}{3}\right)}$

$=\lim\limits_{x\to\frac{\pi}{3}}\dfrac{2\left(\cos\frac{\pi}{3}\sin x-\cos x\sin\frac{\pi}{3}\right)}{-3\left(x-\frac{\pi}{3}\right)}$

$=\lim\limits_{x\to\frac{\pi}{3}}\dfrac{2\sin\left(x-\frac{\pi}{3}\right)}{-3\left(x-\frac{\pi}{3}\right)}$

$=-\dfrac{2}{3}\lim\limits_{x\to\frac{\pi}{3}}\dfrac{\sin\left(x-\frac{\pi}{3}\right)}{\left(x-\frac{\pi}{3}\right)}$

$=-\dfrac{2}{3}$

ដូចនេះ $\boxed{\lim\limits_{x\to\frac{\pi}{3}}\frac{\sin x-\sqrt{3}\cos x}{\pi-3x}=-\frac{2}{3}}$

ឋ. $\lim\limits_{x\to0}\frac{6x}{\sin(-3x)}$,រាងមិនកំណត់ $\frac{0}{0}$

យើងបាន $\lim\limits_{x\to0}\dfrac{6x}{\sin(-3x)}=\lim\limits_{x\to0}\dfrac{-3x}{\sin(-3x)}\times(-2)$

$=1\times(-2)$

$=-2$

ដូចនេះ $\boxed{\lim\limits_{x\to0}\frac{6x}{\sin(-3x)}=-2}$

ឌ. $\lim\limits_{x\to\frac{\pi}{6}}\frac{2(\pi-6x)}{\sin x-\frac{1}{\sqrt{3}}\cos x}$,រាងមិនកំណត់ $\frac{0}{0}$

យើងបាន $\lim\limits_{x\to\frac{\pi}{6}}\dfrac{2(\pi-6x)}{\sin x-\frac{1}{\sqrt{3}}\cos x}=\lim\limits_{x\to\frac{\pi}{6}}\dfrac{-12\left(x-\frac{\pi}{6}\right)}{\frac{2}{\sqrt{3}}\left(\frac{\sqrt{3}}{2}\sin x-\frac{1}{2}\cos x\right)}$

$=\lim\limits_{x\to\frac{\pi}{6}}\dfrac{-6\sqrt{3}\left(x-\frac{\pi}{6}\right)}{\left(\sin x\cos\frac{\pi}{6}-\sin\frac{\pi}{6}\cos x\right)}$

$=\lim\limits_{x\to\frac{\pi}{6}}\dfrac{-6\sqrt{3}\left(x-\frac{\pi}{6}\right)}{\sin\left(x-\frac{\pi}{6}\right)}$

$=-6\sqrt{3}\lim\limits_{x\to\frac{\pi}{6}}\dfrac{\left(x-\frac{\pi}{6}\right)}{\sin\left(x-\frac{\pi}{6}\right)}$

តាង $t=x-\frac{\pi}{6}$

បើ $x\to\frac{\pi}{6}$ នោះ $t\to0$

យើងបាន $-6\sqrt{3}\lim\limits_{x\to\frac{\pi}{6}}\dfrac{\left(x-\frac{\pi}{6}\right)}{\sin\left(x-\frac{\pi}{6}\right)}=-6\sqrt{3}\lim\limits_{t\to0}\dfrac{t}{\sin t}$

$=-6\sqrt{3}\times1$

$=-6\sqrt{3}$

ដូចនេះ $\boxed{\lim\limits_{x\to\frac{\pi}{6}}\frac{2(\pi-6x)}{\sin x-\frac{1}{\sqrt{3}}\cos x}=-6\sqrt{3}}$

ឍ. $\lim\limits_{x\to0}\frac{-2x\sin x}{1-\cos^2x}$, រាងមិនកំណត់ $\frac{0}{0}$

យើងបាន $\lim\limits_{x\to0}\dfrac{-2x\sin x}{1-\cos^2x}=\lim\limits_{x\to0}\dfrac{-2x\sin x}{\sin^2x}$

$=\lim\limits_{x\to0}\dfrac{-2x}{\sin x}$

$=-2\lim\limits_{x\to0}\dfrac{x}{\sin x}$

$=-2\times1$

$=-2$

ដូចនេះ $\boxed{\lim\limits_{x\to0}\frac{-2x\sin x}{1-\cos^2x}=-2}$

ណ. $\lim\limits_{x\to-\frac{\pi}{2}}\frac{1+\sin x}{\sin^4-1}$, រាងមិនកំណត់ $\frac{0}{0}$

យើងបាន $\lim\limits_{x\to-\frac{\pi}{2}}\dfrac{1+\sin x}{\sin^4-1}=\lim\limits_{x\to-\frac{\pi}{2}}\dfrac{1+\sin x}{\left(\sin^2x-1\right)\left(\sin^2x+1\right)}$

$=\lim\limits_{x\to-\frac{\pi}{2}}\dfrac{1+\sin x}{(\sin x-1)(\sin x+1)\left(\sin^2+1\right)}$

$=\lim\limits_{x\to-\frac{\pi}{2}}\dfrac{1}{(\sin x-1)\left(\sin^2x+1\right)}$

$=\lim\limits_{x\to-\frac{\pi}{2}}\dfrac{1}{\left[\sin\left(-\frac{\pi}{2}\right)-1\right]\left[\sin^2\left(-\frac{\pi}{2}\right)+1\right]}$

$=\dfrac{1}{(-1-1)(1+1)}$

$=\dfrac{1}{-2\times2}$

$=-\dfrac{1}{4}$

ដូចនេះ $\boxed{\lim\limits_{x\to-\frac{\pi}{2}}\frac{1+\sin x}{\sin^4-1}=-\frac{1}{4}}$

ត. $\lim\limits_{x\to\frac{\pi}{4}}\frac{1-\tan^3x}{\tan^2x-1}$, រាងមិនកំណត់ $\frac{0}{0}$

យើងបាន $\lim\limits_{x\to\frac{\pi}{4}}\dfrac{1-\tan^3x}{\tan^2x-1}=\lim\limits_{x\to\frac{\pi}{4}}\dfrac{(1-\tan x)\left(1^2+\tan x+\tan^2x\right)}{(\tan x-1)(\tan x+1)}$

$=\lim\limits_{x\to\frac{\pi}{4}}\dfrac{-(\tan x-1)\left(1^2+\tan x+\tan^2x\right)}{(\tan x-1)(\tan x+1)}$

$=\lim\limits_{x\to\frac{\pi}{4}}\dfrac{-\left(1^2+\tan x+\tan^2x\right)}{\tan x+1}$

$=\dfrac{-(1+1+1)}{1+1}$

$=-\dfrac{3}{2}$

ដូចនេះ $\boxed{\lim\limits_{x\to\frac{\pi}{4}}\frac{1-\tan^3x}{\tan^2x-1}=-\frac{3}{2}}$

ថ. $\lim\limits_{x\to0}\frac{\sin6x+\sin9x}{2x}$, រាងមិនកំណត់ $\frac{0}{0}$

យើងបាន $\lim\limits_{x\to0}\dfrac{\sin6x+\sin9x}{2x}=\lim\limits_{x\to0}\dfrac{\sin6x}{2x}+\lim\limits_{x\to0}\dfrac{\sin9x}{2x}$

$=\lim\limits_{x\to0}\dfrac{\sin6x}{6x}\times3+\lim\limits_{x\to0}\dfrac{\sin9x}{9x}\times\dfrac{9}{2}$

$=1\times3+1\times\dfrac{9}{2}$

$=\dfrac{15}{2}$

ដូចនេះ $\boxed{\lim\limits_{x\to0}\frac{\sin6x+\sin9x}{2x}=\frac{15}{2}}$

ទ. $\lim\limits_{x\to0}\frac{x+2\sin3x}{x+3\sin2x}$, រាងមិនកំណត់ $\frac{0}{0}$

យើងបាន $\lim\limits_{x\to0}\dfrac{x+2\sin3x}{x+3\sin2x}=\lim\limits_{x\to0}\dfrac{\frac{x}{x}+\frac{2\sin3x}{x}}{\frac{x}{x}+\frac{3\sin2x}{x}}$

$=\lim\limits_{x\to0}\dfrac{1+\frac{2\sin3x}{3x}\times3}{1+\frac{3\sin2x}{2x}\times2}$

$=\dfrac{1+2\times3}{1+3\times2}$

$=\dfrac{1+6}{1+6}$

$=1$

ដូចនេះ $\boxed{\lim\limits_{x\to0}\frac{x+2\sin3x}{x+3\sin2x}=1}$

ធ. $\lim\limits_{x\to0}\frac{2019\sin x}{2018x}$, រាងមិនកំណត់ $\frac{0}{0}$

យើងបាន $\lim\limits_{x\to0}\dfrac{2019\sin x}{2018x}=\dfrac{2019}{2018}\lim\limits_{x\to0}\dfrac{\sin x}{x}$

$=\dfrac{2019}{2018}\times1$

ដូចនេះ $\boxed{\lim\limits_{x\to0}\frac{2019\sin x}{2018x}=\frac{2019}{2018}}$

ន. $\lim\limits_{x\to0}\frac{\sin2018x}{\sin2019x}$, រាងមិនកំណត់ $\frac{0}{0}$

យើងបាន $\lim\limits_{x\to0}\dfrac{\sin2018x}{\sin2019x}=\lim\limits_{x\to0}\left[\dfrac{\sin2018x}{2018x}\times\dfrac{2019x}{\sin2019x}\times\dfrac{2018}{2019}\right]$

$=1\times1\times\dfrac{2018}{2019}$

$=\dfrac{2018}{2019}$

ដូចនេះ $\boxed{\lim\limits_{x\to0}\frac{\sin2018x}{\sin2019x}=\frac{2018}{2019}}$

លំហាត់គំរូទី១៦

ដោះស្រាយលីមីតខាងក្រោម៖

ក. $\lim\limits_{x\to0}\frac{\cos x-\cos^2x}{x}$

ខ. $\lim\limits_{x\to0}\frac{\cos3x-\cos^23x}{6x}$

គ. $\lim\limits_{x\to0}\frac{1-\cos4x}{x^2}$

ឃ. $\lim\limits_{x\to0}\frac{\sin x-\sin3x}{x}$

ង. $\lim\limits_{x\to0}\frac{\sin2x-3x}{\sin3x-2x}$

ច. $\lim\limits_{x\to0}\frac{\sin x-2x}{x-2\sin x}$

ឆ. $\lim\limits_{x\to0}\frac{x+\sin x}{2-3\sin2x}$

ជ. $\lim\limits_{x\to0}\frac{1-\cos^2x}{2x^2}$

ឈ. $\lim\limits_{x\to0}\frac{1-\cos5x}{x^2}$

ញ. $\lim\limits_{x\to0}\frac{\sin(3x+1989\pi)}{\sin(4x+1987\pi)}$

ដ. $\lim\limits_{x\to0}\frac{1-\cos x}{x}$

ឋ. $\lim\limits_{x\to0}\left(\frac{2}{\sin^2x}-\frac{1}{1-\cos x}\right)$

ឌ. $\lim\limits_{x\to0}\frac{\sin x+1-\cos x}{\sin^2x}$

ឍ. $\lim\limits_{x\to0}\frac{x\sin x-2\sin4x}{\tan x}$

ណ. $\lim\limits_{x\to\frac{\pi}{6}}\frac{2\sin^2x-3\sin x+1}{4\sin^2-1}$

ត. $\lim\limits_{x\to0}\frac{1-\cos nx}{1-\cos mx}$

ថ. $\lim\limits_{x\to0}\frac{3x+\sin2x}{3x-\sin2x}$

ទ. $\lim\limits_{x\to0}\frac{(1+x^2)-\cos x}{\tan^2x}$

ធ. $\lim\limits_{x\to0}\frac{\sin(a+x)-\sin(a-x)}{x}$

ន. $\lim\limits_{x\to0}\frac{\tan x-\sin x}{x}$

ដំណោះស្រាយ

ក. $\lim\limits_{x\to0}\frac{\cos x-\cos^2x}{x}$, រាងមិនកំណត់ $\frac{0}{0}$

យើងបាន $\lim\limits_{x\to0}\dfrac{\cos x-\cos^2x}{x}=\lim\limits_{x\to0}\dfrac{\cos x(1-\cos x)}{x}$

$=\lim\limits_{x\to0}\cos x\times\lim\limits_{x\to0}\dfrac{(1-\cos x)}{x}$

$=1\times0=0$

ដូចនេះ $\boxed{\lim\limits_{x\to0}\frac{\cos x-\cos^2x}{x}=0}$

ខ. $\lim\limits_{x\to0}\frac{\cos3x-\cos^23x}{6x}$, រាងមិនកំណត់ $\frac{0}{0}$

យើងបាន $\lim\limits_{x\to0}\dfrac{\cos3x-\cos^23x}{6x}=\lim\limits_{x\to0}\dfrac{\cos3x(1-\cos3x)}{6x}$

$=\lim\limits_{x\to0}\left[\dfrac{\cos3x}{2}\times\dfrac{(1-\cos3x)}{3x}\right]$

$=\dfrac{1}{2}\times0=0$

ដូចនេះ $\boxed{\lim\limits_{x\to0}\frac{\cos3x-\cos^23x}{6x}=0}$

គ. $\lim\limits_{x\to0}\frac{1-\cos4x}{x^2}$, រាងមិនកំណត់ $\frac{0}{0}$

យើងបាន $\lim\limits_{x\to 0}\dfrac{1-\cos 4x}{x^2}=\lim\limits_{x\to 0}\dfrac{2\sin^2 2x}{x^2}$

$=\lim\limits_{x\to 0}\dfrac{2\sin^2 2x}{(2x)^2}\times 4$

$=8\lim\limits_{x\to 0}\dfrac{\sin^2 2x}{(2x)^2}$

$=8\times 1$

$=8$

ដូចនេះ $\boxed{\lim\limits_{x\to 0}\frac{1-\cos 3x}{x^2}=8}$

ឃ. $\lim\limits_{x\to 0}\frac{\sin x-\sin 3x}{x}$, រាងមិនកំណត់ $\frac{0}{0}$

យើងបាន $\lim\limits_{x\to 0}\dfrac{\sin x-\sin 3x}{x}=\lim\limits_{x\to 0}\left(\dfrac{\sin x}{x}-\dfrac{\sin 3x}{x}\right)$

$=\lim\limits_{x\to 0}\dfrac{\sin x}{x}-\lim\limits_{x\to 0}\dfrac{\sin 3x}{3x}\times 3$

$=1-1\times 3=-2$

ដូចនេះ $\boxed{\lim\limits_{x\to 0}\frac{\sin x-\sin 3x}{x}=-2}$

ង. $\lim\limits_{x\to 0}\frac{\sin 2x-3x}{\sin 3x-2x}$, រាងមិនកំណត់ $\frac{0}{0}$

យើងបាន $\lim\limits_{x\to 0}\dfrac{\sin 2x-3x}{\sin 3x-2x}=\lim\limits_{x\to 0}\dfrac{\frac{\sin 2x-3x}{x}}{\frac{\sin 3x-2x}{x}}$

$=\lim\limits_{x\to 0}\dfrac{\frac{\sin 2x}{x}-3}{\frac{\sin 3x}{x}-2}$

$=\lim\limits_{x\to 0}\dfrac{\frac{\sin 2x}{2x}\times 2-3}{\frac{\sin 3x}{3x}\times 3-2}$

$=\dfrac{1\times 2-3}{1\times 3-2}$

$=-1$

ដូចនេះ $\boxed{\lim\limits_{x\to 0}\frac{\sin 2x-3x}{\sin 3x-2x}=-1}$

ច. $\lim\limits_{x\to 0}\frac{\sin x-2x}{x-2\sin x}$, រាងមិនកំណត់ $\frac{0}{0}$

យើងបាន $\lim\limits_{x\to 0}\dfrac{\sin x-2x}{x-2\sin x}=\lim\limits_{x\to 0}\dfrac{\frac{\sin x-2x}{x}}{\frac{x-2\sin x}{x}}$

$=\lim\limits_{x\to 0}\dfrac{\frac{\sin x}{x}-2}{1-2\frac{\sin x}{x}}$

$=\dfrac{1-2}{1-2\times 1}=1$

ដូចនេះ $\boxed{\lim\limits_{x\to 0}\frac{\sin x-2x}{x-2\sin x}=1}$

ឆ. $\lim\limits_{x\to 0}\frac{x+\sin x}{2-3\sin 2x}$, រាងមិនកំណត់ $\frac{0}{0}$

យើងបាន $\lim\limits_{x\to 0}\dfrac{x+\sin x}{2-3\sin 2x}=\lim\limits_{x\to 0}\dfrac{\frac{x+\sin x}{x}}{\frac{2x-3\sin 2x}{x}}$

$=\lim\limits_{x\to 0}\dfrac{1+\frac{\sin x}{x}}{2-\frac{3\sin 2x}{x}}$

$=\lim\limits_{x\to 0}\dfrac{1+\frac{\sin x}{x}}{2-\frac{\sin 2x}{2x}\times 6}$

$=\dfrac{1+1}{2-1\times 6}=-\dfrac{1}{2}$

ដូចនេះ $\boxed{\lim\limits_{x\to 0}\frac{x+\sin x}{2-3\sin 2x}=-\frac{1}{2}}$

ជ. $\lim\limits_{x\to 0}\frac{1-\cos^2 x}{2x^2}$, រាងមិនកំណត់ $\frac{0}{0}$

យើងបាន $\lim\limits_{x\to 0}\dfrac{1-\cos^2 x}{2x^2}=\lim\limits_{x\to 0}\dfrac{\sin^2 x}{3x^2}$

$=\lim\limits_{x\to 0}\dfrac{\sin^2 x}{x^2}\times\dfrac{1}{3}$

$=1\times\dfrac{1}{3}=\dfrac{1}{3}$

ដូចនេះ $\boxed{\lim\limits_{x\to 0}\frac{1-\cos^2 x}{2x^2}=\frac{1}{3}}$

ឈ. $\lim\limits_{x\to 0}\frac{1-\cos 5x}{x^2}$, រាងមិនកំណត់ $\frac{0}{0}$

យើងបាន $\lim\limits_{x\to 0}\dfrac{1-\cos 5x}{x^2}=\lim\limits_{x\to 0}\dfrac{2\sin^2\frac{5x}{2}}{x^2}$

$=2\lim\limits_{x\to 0}\dfrac{\sin^2\frac{5x}{2}}{\left(\frac{5x}{2}\right)^2}\times\dfrac{25}{4}$

$=2\times 1\times\dfrac{25}{4}$

$=\dfrac{25}{2}$

ដូចនេះ $\boxed{\lim\limits_{x\to 0}\frac{1-\cos 5x}{x^2}=\frac{25}{2}}$

ញ. $\lim\limits_{x\to 0}\frac{\sin(3x+1989\pi)}{\sin(4x+1987\pi)}$, រាងមិនកំណត់ $\frac{0}{0}$

យើងបាន $\lim\limits_{x\to 0}\dfrac{\sin(3x+1989\pi)}{\sin(4x+1987\pi)}=\lim\limits_{x\to 0}\dfrac{\sin(3x+\pi)}{\sin(4x+\pi)}$

$=\lim\limits_{x\to 0}\dfrac{\sin 3x}{\sin 4x}$

$=\lim\limits_{x\to 0}\left[\dfrac{\sin 3x}{3x}\times\dfrac{4x}{\sin 4x}\times\dfrac{3}{4}\right]$

$=1\times 1\times\dfrac{3}{4}=\dfrac{3}{4}$

ដូចនេះ $\boxed{\lim\limits_{x\to 0}\frac{\sin(3x+1989\pi)}{\sin(4x+1987\pi)}=\frac{3}{4}}$

ដ. $\lim\limits_{x\to 0}\frac{1-\cos x}{\sin^2 x}$, រាងមិនកំណត់ $\frac{0}{0}$

យើងបាន $\lim\limits_{x\to 0}\dfrac{1-\cos x}{\sin^2 x}=\lim\limits_{x\to 0}\dfrac{1-\cos x}{1-\cos^2 x}$

$=\lim\limits_{x\to 0}\dfrac{1-\cos x}{(1-\cos x)(1+\cos x)}$

$=\lim\limits_{x\to 0}\dfrac{1}{1+\cos x}$

$=\dfrac{1}{1+1}=\dfrac{1}{2}$

ដូចនេះ $\boxed{\lim\limits_{x\to 0}\frac{1-\cos x}{\sin^2 x}=\frac{1}{2}}$

ឋ. $\lim\limits_{x\to 0}\left(\frac{2}{\sin^2 x}-\frac{1}{1-\cos x}\right)$, រាងមិនកំណត់ $\infty-\infty$

យើងបាន $\lim\limits_{x\to 0}\left(\dfrac{2}{\sin^2 x}-\dfrac{1}{1-\cos x}\right)=\lim\limits_{x\to 0}\dfrac{2-(1+\cos x)}{1-\cos^2 x}$

$$=\lim_{x\to 0}\frac{1-\cos x}{(1-\cos x)(1+\cos x)}$$

$$=\lim_{x\to 0}\frac{1}{1+\cos x}$$

$$=\frac{1}{2}$$

ដូចនេះ $\boxed{\lim\limits_{x\to 0}\left(\frac{2}{\sin^2 x}-\frac{1}{1-\cos x}\right)=\frac{1}{2}}$

ឌ. $\lim\limits_{x\to 0}\frac{\sin x+1-\cos x}{x}$, រាងមិនកំណត់ $\frac{0}{0}$

យើងបាន $\lim\limits_{x\to 0}\dfrac{\sin x+1-\cos x}{\sin^2 x}=\lim\limits_{x\to 0}\left(\dfrac{\sin x}{x}+\dfrac{1-\cos x}{x}\right)$

$$=\lim_{x\to 0}\frac{\sin x}{x}+\lim_{x\to 0}\frac{1-\cos x}{x}$$

$$=1+0=1$$

ដូចនេះ $\boxed{\lim\limits_{x\to 0}\frac{\sin x+1-\cos x}{x}=1}$

ឍ. $\lim\limits_{x\to 0}\frac{x\sin x-2\sin 4x}{\tan x}$, រាងមិនកំណត់ $\frac{0}{0}$

យើងបាន $\lim\limits_{x\to 0}\dfrac{x\sin x-2\sin 4x}{\tan x}=\lim\limits_{x\to 0}\dfrac{\frac{x\sin x-2\sin 4x}{x}}{\frac{\tan x}{x}}$

$$=\lim_{x\to 0}\frac{\sin x-8\frac{\sin 4x}{4x}}{\frac{\tan x}{x}}$$

$$=\frac{0-8\times 1}{1}=-8$$

ដូចនេះ $\boxed{\lim\limits_{x\to 0}\frac{x\sin x-2\sin 4x}{\tan x}=-8}$

ណ. $\lim\limits_{x\to\frac{\pi}{6}}\frac{2\sin^2 x-3\sin x+1}{4\sin^2-1}$, រាងមិនកំណត់ $\frac{0}{0}$

យើងបាន $\lim\limits_{x\to\frac{\pi}{6}}\dfrac{2\sin^2 x-3\sin x+1}{4\sin^2-1}=\lim\limits_{x\to\frac{\pi}{6}}\dfrac{(\sin x-1)(2\sin x-1)}{(2\sin x-1)(2\sin x+1)}$

$$=\lim_{x\to\frac{\pi}{6}}\frac{\sin x-1}{2\sin x+1}$$

$$=\frac{\frac{1}{2}-1}{2\times\frac{1}{2}+1}$$

$$=-\frac{1}{4}$$

ដូចនេះ $\boxed{\lim\limits_{x\to\frac{\pi}{6}}\frac{2\sin^2 x-3\sin x+1}{4\sin^x-1}=-\frac{1}{4}}$

ត. $\lim\limits_{x\to 0}\frac{1-\cos nx}{1-\cos mx}$, រាងមិនកំណត់ $\frac{0}{0}$

យើងបាន $\lim\limits_{x\to 0}\dfrac{1-\cos nx}{1-\cos mx}=\lim\limits_{x\to 0}\dfrac{2\sin^2\frac{nx}{2}}{2\sin^2\frac{mx}{2}}$

$$=\lim_{x\to 0}\frac{\sin^2\frac{nx}{2}}{\left(\frac{nx}{2}\right)^2}\times\frac{\left(\frac{mx}{2}\right)^2}{\sin^2\frac{mx}{2}}\times\frac{n^2}{m^2}$$

$$=1\times 1\times\frac{n^2}{m^2}=\frac{n^2}{m^2}$$

ដូចនេះ $\boxed{\lim\limits_{x\to 0}\frac{1-\cos nx}{1-\cos mx}=\frac{n^2}{m^2}}$

ថ. $\lim\limits_{x\to 0}\frac{3x+\sin 2x}{3x-\sin 2x}$, រាងមិនកំណត់ $\frac{0}{0}$

យើងបាន $\lim\limits_{x\to 0}\dfrac{3x+\sin 2x}{3x-\sin 2x}=\lim\limits_{x\to 0}\dfrac{\frac{3x+\sin 2x}{x}}{\frac{3x-\sin 2x}{x}}$

$$=\frac{3+1\times 2}{3-1\times 2}=5$$

ដូចនេះ $\boxed{\lim\limits_{x\to 0}\frac{3x+\sin 2x}{3x-\sin 2x}=5}$

ទ. $\lim\limits_{x\to 0}\frac{(1+x^2)-\cos x}{\tan^2 x}$, រាងមិនកំណត់ $\frac{0}{0}$

យើងបាន $\lim\limits_{x\to 0}\dfrac{(1+x^2)-\cos x}{\tan^2 x}=\lim\limits_{x\to 0}\dfrac{x^2+(1-\cos x)}{\tan^2 x}$

$$=\lim_{x\to 0}\frac{x^2}{\tan^2 x}+\lim_{x\to 0}\frac{2\sin^2\frac{x}{2}}{\frac{\sin^2 x}{\cos^2 x}}$$

$$=\lim_{x\to 0}\frac{x^2}{\tan^2 x}+\lim_{x\to 0}\frac{2\sin^2\frac{x}{2}\times\cos^2 x}{\sin^2 x}$$

$$=\lim_{x\to 0}\frac{x^2}{\tan^2 x}+\lim_{x\to 0}\frac{2\sin^2\frac{x}{2}\times\cos^2 x}{4\sin^2\frac{x}{2}\cos^2\frac{x}{2}}$$

$$=\lim_{x\to 0}\frac{x^2}{\tan^2 x}+\lim_{x\to 0}\frac{\cos^2 x}{2\cos^2\frac{x}{2}}=1+\frac{1}{2}=\frac{3}{2}$$

ដូចនេះ $\boxed{\lim\limits_{x\to 0}\frac{(1+x^2)-\cos x}{\tan^2 x}=\frac{3}{2}}$

ធ. $\lim\limits_{x\to 0}\frac{\sin(a+x)-\sin(a-x)}{x}$, រាងមិនកំណត់ $\frac{0}{0}$

តាង $A=\lim\limits_{x\to 0}\frac{\sin(a+x)-\sin(a-x)}{x}$

យើងបាន $A=\lim\limits_{x\to 0}\dfrac{(\sin a\cos x+\cos a\sin x)-(\sin a\cos x-\cos a\sin x)}{x}$

$$=\lim_{x\to 0}\frac{\sin a\cos x+\cos a\sin x-\sin a\cos x+\cos a\sin x}{x}$$

$$=\lim_{x\to 0}\frac{\cos a\sin x+\cos a\sin x}{x}$$

$$=\lim_{x\to 0}\frac{2\cos a\sin x}{x}$$

$$=2\lim_{x\to 0}\frac{\sin x}{x}\times\cos a$$

$$=2\cos a$$

ដូចនេះ $\boxed{\lim\limits_{x\to 0}\frac{\sin(a+x)-\sin(a-x)}{x}=2\cos a}$

ន. $\lim\limits_{x\to 0}\frac{\tan x-\sin x}{x}$, រាងមិនកំណត់ $\frac{0}{0}$

ដោយ $\tan x=\frac{\sin x}{\cos x}\Rightarrow\sin x=\cos x.\tan x$

យើងបាន $\lim\limits_{x\to 0}\dfrac{\tan x-\sin x}{x}=\lim\limits_{x\to 0}\dfrac{\tan x(1-\cos x)}{x}$

$$=\lim_{x\to 0}\frac{\tan x}{x}\times(1-\cos x)$$

$$=1\times(1-1)$$

$$=0$$

ដូចនេះ $\boxed{\lim\limits_{x\to 0}\frac{\tan x-\sin x}{x}=0}$

លំហាត់គំរូទី១៧

ដោះស្រាយលីមីតខាងក្រោម៖

ក. $\lim\limits_{x\to 0}\frac{1+\sin x-\cos x}{1-\sin x-\cos x}$

ខ. $\lim\limits_{x\to 0}\frac{\tan x-\sin x}{x^3}$

គ. $\lim\limits_{x\to 0}\frac{x(a-b)}{\sin ax-\sin bx}\ ;a,b\neq 0$

ឃ. $\lim\limits_{x\to 0}\frac{\sin(\sin(\sin x))}{x}$

ង. $\lim\limits_{x\to 0}\frac{1+\sin x-\cos x}{1+\sin nx-\cos nx}$

ច. $\lim\limits_{x\to 0}\frac{(1-\cos x)\sin x}{\tan^3 x}$

ឆ. $\lim\limits_{x\to 0}\frac{\sqrt{1+x}-\sqrt{1-x}}{\sin 3x}$

ជ. $\lim\limits_{x\to 0}\frac{\sqrt{x+4}-2}{\sin 5x}$

ឈ. $\lim\limits_{x\to 0}\frac{\sqrt{x+9}-3}{\sin 7x}$

ញ. $\lim\limits_{x\to 0}\frac{2\sin 2x}{\sqrt{x+2}-\sqrt{2}}$

ដ. $\lim\limits_{x\to 0}\frac{\sqrt{2}-\sqrt{1+\cos x}}{\sin^2 x}$

ឋ. $\lim\limits_{x\to 0}\frac{1+x\sin x-\cos 2x}{\sin^2 x}$

ឌ. $\lim\limits_{x\to 0}\frac{\sin^2 x}{\sqrt{1+x\sin x}-\cos x}$

ឍ. $\lim\limits_{x\to 0}\frac{2x-\sin x}{\sqrt{1-\cos x}}$

ណ. $\lim\limits_{x\to 0}\frac{\sqrt{3x+1}-\sqrt{2x+1}}{\sin x}$

ត. $\lim\limits_{x\to 0}\frac{\sqrt{1+\sin x}-\sqrt{\cos 2x}}{\cot^2\left(\frac{\pi}{2}-x\right)}$

ថ. $\lim\limits_{x\to 0}\frac{\left(\sqrt[3]{x-1}+\sqrt[3]{x+1}\right)\sin x}{1-\cos \pi x}$

ទ. $\lim\limits_{x\to 0}\frac{\sqrt{1+\sin x}-\sqrt{1-\sin x}}{\tan x}$

ធ. $\lim\limits_{x\to 0}\frac{\sin^2 x}{1-\sqrt{\cos x}}$

ន. $\lim\limits_{x\to 0}\frac{\sin 2x}{\sqrt{x+1}-1}$

ដំណោះស្រាយ

ក. $\lim\limits_{x\to 0}\frac{1+\sin x-\cos x}{1-\sin x-\cos x}$, រាងមិនកំណត់ $\frac{0}{0}$

យើងបាន $\lim\limits_{x\to 0}\frac{1+\sin x-\cos x}{1-\sin x-\cos x}=\lim\limits_{x\to 0}\frac{1-\cos x+\sin x}{1-\cos x-\sin x}$

$$=\lim_{x\to 0}\frac{2\sin^2\frac{x}{2}+2\sin\frac{x}{2}\cos\frac{x}{2}}{2\sin^2\frac{x}{2}-2\sin\frac{x}{2}\cos\frac{x}{2}}$$

$$=\lim_{x\to 0}\frac{2\sin\frac{x}{2}\left(\sin\frac{x}{2}+\cos\frac{x}{2}\right)}{2\sin\frac{x}{2}\left(\sin\frac{x}{2}-\cos\frac{x}{2}\right)}$$

$$=\lim_{x\to 0}\frac{\sin\frac{x}{2}+\cos\frac{x}{2}}{\sin\frac{x}{2}-\cos\frac{x}{2}}$$

$$=\frac{0+1}{0-1}=-1$$

ដូចនេះ៖ $\boxed{\lim\limits_{x\to 0}\frac{1+\sin x-\cos x}{1-\sin x-\cos x}=-1}$

ខ. $\lim\limits_{x\to 0}\frac{\tan x-\sin x}{x^3}$, រាងមិនកំណត់ $\frac{0}{0}$

ដោយ $\tan x=\frac{\sin x}{\cos x}\Rightarrow \sin x=\cos x.\tan x$

$$\lim_{x\to 0}\frac{\tan x-\sin x}{x^3}=\lim_{x\to 0}\frac{\tan x(1-\cos x)}{x^3}$$

$$=\lim_{x\to 0}\frac{\tan x.2\sin^2\frac{x}{2}}{x^3}$$

$$=\lim_{x\to 0}\frac{\tan x}{x}\times 2\lim_{x\to 0}\frac{\sin^2\frac{x}{2}}{x^2}$$

$$=\lim_{x\to 0}\frac{\tan x}{x}\times 2\lim_{x\to 0}\frac{\sin^2\frac{x}{2}}{\left(\frac{x}{2}\right)^2}\times\frac{1}{4}$$

$$=1\times 2\times\frac{1}{4}=\frac{1}{2}$$

ដូចនេះ៖ $\boxed{\lim\limits_{x\to 0}\frac{\tan x-\sin x}{x^3}=\frac{1}{2}}$

គ. $\lim\limits_{x\to 0}\frac{x(a-b)}{\sin ax-\sin bx}\ ;a,b\neq 0$, រាងមិនកំណត់ $\frac{0}{0}$

យើងបាន $\frac{x(a-b)}{\sin ax-\sin bx}=\lim\limits_{x\to 0}\frac{\frac{x(a-b)}{x}}{\frac{\sin ax-\sin bx}{x}}$

$$=\lim_{x\to 0}\frac{a-b}{\frac{\sin ax}{x}-\frac{\sin bx}{x}}$$

$$=\lim_{x\to 0}\frac{a-b}{\frac{\sin ax}{ax}\times a-\frac{\sin bx}{bx}\times b}$$

$$=\frac{a-b}{1\times a-1\times b}$$

$$=\frac{a-b}{a-b},a\neq b$$

$$=1$$

ដូចនេះ៖ $\boxed{\frac{x(a-b)}{\sin ax-\sin bx}=1}$

ឃ. $\lim\limits_{x\to 0}\frac{\sin(\sin(\sin x))}{x}$, រាងមិនកំណត់ $\frac{0}{0}$

យើងបាន $\lim\limits_{x\to 0}\frac{\sin(\sin(\sin x))}{x}=\lim\limits_{x\to 0}\left[\frac{\sin(\sin(\sin x))}{\sin(\sin x)}\times\frac{\sin(\sin x)}{\sin x}\times\frac{\sin x}{x}\right]$

$$=\lim_{x\to 0}\frac{\sin(\sin(\sin x))}{\sin(\sin x)}\times\lim_{x\to 0}\frac{\sin(\sin x)}{\sin x}\times\lim_{x\to 0}\frac{\sin x}{x}$$

$$=1\times 1\times 1$$

$$=1$$

ដូចនេះ៖ $\boxed{\lim\limits_{x\to 0}\frac{\sin(\sin(\sin x))}{x}=1}$

ង. $\lim\limits_{x\to 0}\frac{1+\sin x-\cos x}{1+\sin nx-\cos nx}$, រាងមិនកំណត់ $\frac{0}{0}$

យើងសង្កេតឃើញថា៖

- $1+\sin x-\cos x=1-\cos x+\sin x$
- $1+\sin nx-\cos nx=1-\cos nx+\sin nx$

យើងបាន $\displaystyle\lim_{x\to0}\frac{1+\sin x-\cos x}{1+\sin nx-\cos nx}=\lim_{x\to0}\frac{(1-\cos x)+\sin x}{(1-\cos nx)+\sin nx}$

$$=\lim_{x\to0}\frac{2\sin^2\frac{x}{2}+2\sin\frac{x}{2}\cos\frac{x}{2}}{2\sin^2\frac{nx}{2}+2\sin\frac{nx}{2}\cos\frac{nx}{2}}$$

$$=\lim_{x\to0}\frac{2\sin\frac{x}{2}\left(\sin\frac{x}{2}+\cos\frac{x}{2}\right)}{2\sin\frac{nx}{2}\left(\sin\frac{nx}{2}+\cos\frac{nx}{2}\right)}$$

$$=\lim_{x\to0}\frac{\sin\frac{x}{2}}{\frac{x}{2}}\times\frac{\frac{nx}{2}}{\sin\frac{nx}{2}}\times\frac{1}{n}\times\frac{\sin\frac{x}{2}+\cos\frac{x}{2}}{\sin\frac{nx}{2}+\cos\frac{nx}{2}}$$

$$=1\times1\times\frac{1}{n}\times\frac{0+1}{0+1}$$

$$=\frac{1}{n}$$

ដូចនេះ $\boxed{\lim_{x\to0}\frac{1+\sin x-\cos x}{1+\sin nx-\cos nx}=\frac{1}{n}}$

ប. $\displaystyle\lim_{x\to0}\frac{(1-\cos x)\sin x}{\tan^3 x}$, រាងមិនកំណត់ $\frac{0}{0}$

យើងបាន $\displaystyle\lim_{x\to0}\frac{(1-\cos x)\sin x}{\tan^3 x}=\lim_{x\to0}\frac{(1-\cos x)\sin x}{\frac{\sin^3 x}{\cos^3 x}}$

$$=\lim_{x\to0}\frac{(1-\cos x)\sin x.\cos^3 x}{\sin^3 x}$$

$$=\lim_{x\to0}\frac{(1-\cos x)\cos^3 x}{\sin^2 x}$$

$$=\lim_{x\to0}\frac{2\sin^2\frac{x}{2}\cos^3 x}{\sin^2 x}$$

$$=\lim_{x\to0}\frac{2\sin^2\frac{x}{2}\cos^3 x}{4\sin^2\frac{x}{2}\cos^2\frac{x}{2}}$$

$$=\lim_{x\to0}\frac{\cos^3 x}{2\cos^2\frac{x}{2}}$$

$$=\frac{1}{2\times1}=\frac{1}{2}$$

ដូចនេះ $\boxed{\lim_{x\to0}\frac{(1-\cos x)\sin x}{\tan^3 x}=\frac{1}{2}}$

ឆ. $\displaystyle\lim_{x\to0}\frac{\sqrt{1+x}-\sqrt{1-x}}{\sin 3x}$, រាងមិនកំណត់ $\frac{0}{0}$

យើងបាន $\displaystyle\lim_{x\to0}\frac{\sqrt{1+x}-\sqrt{1-x}}{\sin 3x}=\lim_{x\to0}\frac{\left(\sqrt{1+x}-\sqrt{1-x}\right)\left(\sqrt{1+x}+\sqrt{1-x}\right)}{\sin 3x\left(\sqrt{1+x}+\sqrt{1-x}\right)}$

$$=\lim_{x\to0}\frac{1+x-1+x}{\sin 3x\left(\sqrt{1+x}+\sqrt{1-x}\right)}$$

$$=\lim_{x\to0}\frac{2x}{\sin 3x\left(\sqrt{1+x}+\sqrt{1-x}\right)}$$

$$=\lim_{x\to0}\frac{3x}{\sin 3x}\times\frac{2}{3\left(\sqrt{1+x}+\sqrt{1-x}\right)}$$

$$=1\times\frac{2}{3(1+1)}=\frac{1}{3}$$

ដូចនេះ $\boxed{\lim_{x\to0}\frac{\sqrt{1+x}-\sqrt{1-x}}{\sin 3x}=\frac{1}{3}}$

ជ. $\displaystyle\lim_{x\to0}\frac{\sqrt{x+4}-2}{\sin 5x}$, រាងមិនកំណត់

យើងបាន $\displaystyle\lim_{x\to0}\frac{\sqrt{x+4}-2}{\sin 5x}=\lim_{x\to0}\frac{\left(\sqrt{x+4}-2\right)\left(\sqrt{x+4}+2\right)}{\sin 5x\left(\sqrt{x+4}+2\right)}$

$$=\lim_{x\to0}\frac{x+4-4}{\sin 5x\left(\sqrt{x+4}+2\right)}$$

$$=\lim_{x\to0}\frac{x}{\sin 5x\left(\sqrt{x+4}+2\right)}$$

$$=\lim_{x\to0}\frac{5x}{\sin 5x}\times\frac{1}{5}\times\frac{1}{\sqrt{x+4}+2}$$

$$=1\times\frac{1}{5}\times\frac{1}{\sqrt{0+4}+2}$$

$$=\frac{1}{20}$$

ដូចនេះ $\boxed{\lim_{x\to0}\frac{\sqrt{x+4}-2}{\sin 5x}=\frac{1}{20}}$

ឈ. $\displaystyle\lim_{x\to0}\frac{\sqrt{x+9}-3}{\sin 7x}$, រាងមិនកំណត់ $\frac{0}{0}$

យើងបាន $\displaystyle\lim_{x\to0}\frac{\sqrt{x+9}-3}{\sin 7x}=\lim_{x\to0}\frac{\left(\sqrt{x+9}-3\right)\left(\sqrt{x+9}+3\right)}{\sin 7x\left(\sqrt{x+9}+3\right)}$

$$=\lim_{x\to0}\frac{x+9-9}{\sin 7x\left(\sqrt{x+9}+3\right)}$$

$$=\lim_{x\to0}\frac{x}{\sin 7x\left(\sqrt{x+9}+3\right)}$$

$$=\lim_{x\to0}\left[\frac{7x}{\sin 7x}\times\frac{1}{7}\times\frac{1}{\sqrt{x+9}+3}\right]$$

$$=1\times\frac{1}{7}\times\frac{1}{\sqrt{0+9}+3}$$

$$=1\times\frac{1}{7}\times\frac{1}{6}$$

$$=\frac{1}{42}$$

ដូចនេះ $\boxed{\lim_{x\to0}\frac{\sqrt{x+9}-3}{\sin 7x}=\frac{1}{42}}$

ញ. $\displaystyle\lim_{x\to0}\frac{2\sin 2x}{\sqrt{x+2}-\sqrt{2}}$, រាងមិនកំណត់ $\frac{0}{0}$

យើងបាន $\displaystyle\lim_{x\to0}\frac{2\sin 2x}{\sqrt{x+2}-\sqrt{2}}=\lim_{x\to0}\frac{2\sin 2x\left(\sqrt{x+2}+\sqrt{2}\right)}{\left(\sqrt{x+2}-\sqrt{2}\right)\left(\sqrt{x+2}+\sqrt{2}\right)}$

$$=\lim_{x\to0}\frac{2\sin 2x\left(\sqrt{x+2}+2\right)}{x+2-2}$$

$$=\lim_{x\to0}\frac{2\sin 2x\left(\sqrt{x+2}+2\right)}{x}$$

$$=2\lim_{x\to0}\left[\frac{\sin 2x}{2x}\times2\times\left(\sqrt{x+2}+2\right)\right]$$

$$=2\times1\times2\times(2+2)=8$$

ដូចនេះ $\boxed{\lim_{x\to0}\frac{2\sin 2x}{\sqrt{x+2}-\sqrt{2}}=8}$

ដ. $\displaystyle\lim_{x\to0}\frac{\sqrt{2}-\sqrt{1+\cos x}}{\sin^2 x}$, រាងមិនកំណត់ $\frac{0}{0}$

យើងបាន $\lim\limits_{x\to 0}\dfrac{\sqrt{2}-\sqrt{1+\cos x}}{\sin^2 x}=\lim\limits_{x\to 0}\dfrac{1-\cos x}{\sin^2 x\left(\sqrt{2}+\sqrt{1+\cos x}\right)}$

$$=\lim_{x\to 0}\frac{2\sin^2\frac{x}{2}}{\sin^2 x\left(\sqrt{2}+\sqrt{1+\cos x}\right)}$$

$$=\lim_{x\to 0}\frac{2\sin^2\frac{x}{2}}{4\sin^2\frac{x}{2}\cos^2\frac{x}{2}\left(\sqrt{2}+\sqrt{1+\cos x}\right)}$$

$$=\lim_{x\to 0}\frac{1}{2\cos^2\frac{x}{2}\left(\sqrt{2}+\sqrt{1+\cos x}\right)}$$

$$=\frac{1}{2\left(\sqrt{2}+\sqrt{2}\right)}$$

$$=\frac{\sqrt{2}}{8}$$

ដូចនេះ $\boxed{\lim\limits_{x\to 0}\frac{\sqrt{2}-\sqrt{1+\cos x}}{\sin^2 x}=\frac{\sqrt{2}}{8}}$

ឋ. $\lim\limits_{x\to 0}\frac{1+x\sin x-\cos 2x}{\sin^2 x}$, រាងមិនកំណត់ $\frac{0}{0}$

យើងបាន $\lim\limits_{x\to 0}\dfrac{1+x\sin x-\cos 2x}{\sin^2 x}=\lim\limits_{x\to 0}\dfrac{(1-\cos 2x)+x\sin x}{\sin^2 x}$

$$=\lim_{x\to 0}\frac{2\sin^2 x+x\sin x}{\sin^2 x}$$

$$=\lim_{x\to 0}\frac{\sin x\,(2\sin x+x)}{\sin^2 x}$$

$$=\lim_{x\to 0}\frac{2\sin x+x}{\sin x}$$

$$=2\times\lim_{x\to 0}1+\lim_{x\to 0}\frac{x}{\sin x}$$

$$=2\times 1+1$$

$$=3$$

ដូចនេះ $\boxed{\lim\limits_{x\to 0}\frac{1+x\sin x-\cos 2x}{\sin^2 x}=3}$

ឌ. $\lim\limits_{x\to 0}\frac{\sin^2 x}{\sqrt{1+x\sin x}-\cos x}$, រាងមិនកំណត់ $\frac{0}{0}$

យើងបាន $\lim\limits_{x\to 0}\dfrac{\sin^2 x}{\sqrt{1+x\sin x}-\cos x}=\lim\limits_{x\to 0}\dfrac{\sin^2 x\left(\sqrt{1+x\sin x}+\cos x\right)}{\left(\sqrt{1+x\sin x}-\cos x\right)\left(\sqrt{1+x\sin x}+\cos x\right)}$

$$=\lim_{x\to 0}\frac{\sin^2 x\left(\sqrt{1+x\sin x}+\cos x\right)}{1+x\sin x-\cos^2 x}$$

$$=\lim_{x\to 0}\frac{\sin^2 x\left(\sqrt{1+x\sin x}+\cos x\right)}{\sin^2 x+x\sin x}$$

$$=\lim_{x\to 0}\frac{\sin^2 x\left(\sqrt{1+x\sin x}+\cos x\right)}{\sin^2 x\left(1+\frac{x}{\sin x}\right)}$$

$$=\lim_{x\to 0}\frac{\sqrt{1+x\sin x}+\cos x}{1+\frac{x}{\sin x}}$$

$$=\frac{1+1}{1+1}$$

$$=1$$

ដូចនេះ $\boxed{\lim\limits_{x\to 0}\frac{\sin^2 x}{\sqrt{1+x\sin x}-\cos x}=1}$

ឍ. $\lim\limits_{x\to 0}\frac{2x-\sin x}{\sqrt{1-\cos x}}$, រាងមិនកំណត់ $\frac{0}{0}$

យើងបាន $\lim\limits_{x\to 0}\dfrac{2x-\sin x}{\sqrt{1-\cos x}}=\lim\limits_{x\to 0}\dfrac{2x-\sin x}{\sqrt{2\sin^2\frac{x}{2}}}$

$$=\lim_{x\to 0}\frac{2x-2\sin\frac{x}{2}\cos\frac{x}{2}}{\left|\sin\frac{x}{2}\right|\sqrt{2}}$$

$$=\lim_{x\to 0}\frac{2\left(x-\sin\frac{x}{2}\cos\frac{x}{2}\right)}{\left|\sin\frac{x}{2}\right|\sqrt{2}}$$

$$=\lim_{x\to 0}\frac{\sqrt{2}\left(x-\sin\frac{x}{2}\cos\frac{x}{2}\right)}{\left|\sin\frac{x}{2}\right|}$$

• ករណី $x\to 0^+$ នោះ $\left|\sin\frac{x}{2}\right|=\sin\frac{x}{2}$

យើងបាន $\lim\limits_{x\to 0^+}\dfrac{\sqrt{2}\left(x-\sin\frac{x}{2}\cos\frac{x}{2}\right)}{\left|\sin\frac{x}{2}\right|}=\lim\limits_{x\to 0}\dfrac{\sqrt{2}\left(x-\sin\frac{x}{2}\cos\frac{x}{2}\right)}{\sin\frac{x}{2}}$

$$=\sqrt{2}\lim_{x\to 0^+}\left(\frac{x}{\sin\frac{x}{2}}-\cos\frac{x}{2}\right)$$

$$=\sqrt{2}\lim_{x\to 0^+}\left(\frac{\frac{x}{2}}{\sin\frac{x}{2}}\times 2-\cos\frac{x}{2}\right)$$

$$=\sqrt{2}(1\times 2-1)=\sqrt{2}$$

ដូចនេះ $\boxed{\lim\limits_{x\to 0^+}\frac{2x-\sin x}{\sqrt{1-\cos x}}=\sqrt{2}}$

• ករណី $x\to 0^-$ នោះ $\left|\sin\frac{x}{2}\right|=\sin\frac{x}{2}$

យើងបាន $\lim\limits_{x\to 0}\dfrac{\sqrt{2}\left(x-\sin\frac{x}{2}\cos\frac{x}{2}\right)}{\left|\sin^2\frac{x}{2}\right|}=\lim\limits_{x\to 0^-}\dfrac{\sqrt{2}\left(x-\sin\frac{x}{2}\cos\frac{x}{2}\right)}{-\sin^2\frac{x}{2}}$

$$=\sqrt{2}\lim_{x\to 0^-}\left(-\frac{x}{\sin\frac{x}{2}}+\cos\frac{x}{2}\right)$$

$$=\sqrt{2}\lim_{x\to 0^-}\left(-\frac{\frac{x}{2}}{\sin\frac{x}{2}}\times 2+\cos\frac{x}{2}\right)$$

$$=\sqrt{2}(-1\times 2+1)=-\sqrt{2}$$

ដូចនេះ $\boxed{\lim\limits_{x\to 0^-}\frac{2x-\sin x}{\sqrt{1-\cos x}}=-\sqrt{2}}$

ណ. $\lim\limits_{x\to 0}\frac{\sqrt{3x+1}-\sqrt{2x+1}}{\sin x}$, រាងមិនកំណត់ $\frac{0}{0}$

យើងបាន $\lim\limits_{x\to 0}\dfrac{\sqrt{3x+1}-\sqrt{2x+1}}{\sin x}=\lim\limits_{x\to 0}\dfrac{\left(\sqrt{3x+1}-\sqrt{2x+1}\right)\left(\sqrt{3x+1}+\sqrt{2x+1}\right)}{\sin x\left(\sqrt{3x+1}+\sqrt{2x+1}\right)}$

$$=\lim_{x\to 0}\frac{(3x+1)-(2x+1)}{\sin x\left(\sqrt{3x+1}+\sqrt{2x+1}\right)}$$

$$=\lim_{x\to 0}\frac{x}{\sin x\left(\sqrt{3x+1}+\sqrt{2x+1}\right)}$$

$$=\lim_{x\to 0}\left[\frac{x}{\sin x}\times\frac{1}{\sqrt{3x+1}+\sqrt{2x+1}}\right]$$

$$=1\times\frac{1}{1+1}$$

$$=\frac{1}{2}$$

ដូចនេះ $\boxed{\lim\limits_{x\to 0}\frac{\sqrt{3x+1}-\sqrt{2x+1}}{\sin x}=\frac{1}{2}}$

ត. $\lim\limits_{x\to 0}\frac{\sqrt{1+\sin x}-\sqrt{\cos 2x}}{\cot^2\left(\frac{\pi}{2}-x\right)}$, រាងមិនកំណត់ $\frac{0}{0}$

មេរៀនទី ៣ លំហាត់ និង ដំណោះស្រាយ

$$\lim_{x\to0}\frac{\sqrt{1+\sin x}-\sqrt{\cos 2x}}{\cot^2\left(\frac{\pi}{2}-x\right)}=\lim_{x\to0}\frac{\left(\sqrt{1+x\sin x}-\sqrt{\cos 2x}\right)\left(\sqrt{1+x\sin x}+\sqrt{\cos 2x}\right)}{\cot^2\left(\frac{\pi}{2}-x\right)\left(\sqrt{1+x\sin x}+\sqrt{\cos 2x}\right)}$$
$$=\lim_{x\to0}\frac{1+x\sin x-\cos 2x}{\cot^2\left(\frac{\pi}{2}-x\right)\left(\sqrt{1+x\sin x}+\sqrt{\cos 2x}\right)}$$
$$=\lim_{x\to0}\frac{x\sin x+2\sin^2 x}{\frac{\sin^2 x}{\cos^2 x}\left(\sqrt{1+x\sin x}+\sqrt{\cos 2x}\right)}$$
$$=\lim_{x\to0}\frac{\frac{\left(x\sin x+2\sin^2 x\right)\cos^2 x}{\sin^2 x}}{\frac{\sin^2 x\left(\sqrt{1+x\sin x}+\sqrt{\cos 2x}\right)}{\sin^2 x}}$$
$$=\lim_{x\to0}\frac{\left(\frac{x}{\sin x}+2\right)\cos^2 x}{\sqrt{1+x\sin x}+\sqrt{\cos 2x}}$$
$$=\frac{(1+2)\,1}{1+1}=\frac{3}{2}$$

ដូចនេះ $\boxed{\lim_{x\to0}\frac{\sqrt{1+\sin x}-\sqrt{\cos 2x}}{\cot^2\left(\frac{\pi}{2}-x\right)}=\frac{3}{2}}$

ច. $\lim_{x\to0}\frac{\left(\sqrt[3]{x-1}+\sqrt[3]{x+1}\right)\sin x}{1-\cos\pi x}$, រាងមិនកំណត់ $\frac{0}{0}$

តាង $A=\lim_{x\to0}\frac{\left(\sqrt[3]{x-1}+\sqrt[3]{x+1}\right)\sin x}{1-\cos\pi x}$

យើងបាន$A=\lim_{x\to0}\frac{\sin x\left(\sqrt[3]{x-1}+\sqrt[3]{x+1}\right)\left[\sqrt[3]{(x-1)^2}+\sqrt[3]{x^2-1}+\sqrt[3]{(x+1)^2}\right]}{(1-\cos\pi x)\left[\sqrt[3]{(x-1)^2}+\sqrt[3]{x^2-1}+\sqrt[3]{(x+1)^2}\right]}$

$$=\lim_{x\to0}\frac{\sin x\,(x-1+x+1)}{(1-\cos\pi x)\left[\sqrt[3]{(x-1)^2}+\sqrt[3]{x^2-1}+\sqrt[3]{(x+1)^2}\right]}$$
$$=\lim_{x\to0}\frac{2x\sin x}{2\sin^2\frac{\pi x}{2}\left[\sqrt[3]{(x-1)^2}+\sqrt[3]{x^2-1}+\sqrt[3]{(x+1)^2}\right]}$$
$$=\lim_{x\to0}\frac{x\sin x}{\sin^2\frac{\pi x}{2}\left[\sqrt[3]{(x-1)^2}+\sqrt[3]{x^2-1}+\sqrt[3]{(x+1)^2}\right]}$$
$$=\lim_{x\to0}\frac{\frac{x\sin x}{x^2}}{\frac{\sin^2\frac{\pi x}{2}}{x^2}\left[\sqrt[3]{(x-1)^2}+\sqrt[3]{x^2-1}+\sqrt[3]{(x+1)^2}\right]}$$
$$=\lim_{x\to0}\frac{\frac{x\sin x}{x^2}}{\frac{\sin^2\frac{\pi x}{2}}{\left(\frac{\pi x}{2}\right)^2}\times\frac{\pi^2}{4}\left[\sqrt[3]{(x-1)^2}+\sqrt[3]{x^2-1}+\sqrt[3]{(x+1)^2}\right]}$$
$$=\lim_{x\to0}\frac{\frac{\sin x}{x}}{\frac{\sin^2\frac{\pi x}{2}}{\left(\frac{\pi x}{2}\right)^2}\times\frac{\pi^2}{4}\left[\sqrt[3]{(x-1)^2}+\sqrt[3]{x^2-1}+\sqrt[3]{(x+1)^2}\right]}$$
$$=\frac{1}{1\times\frac{\pi^2}{4}(1+1+1)}$$
$$=\frac{4}{3\pi^2}$$

ដូចនេះ $\boxed{\lim_{x\to0}\frac{\left(\sqrt[3]{x-1}+\sqrt[3]{x+1}\right)\sin x}{1-\cos\pi x}=\frac{4}{3\pi^2}}$

ឆ. $\lim_{x\to0}\frac{\sqrt{1+\sin x}-\sqrt{1-\sin x}}{\tan x}$, រាងមិនកំណត់ $\frac{0}{0}$

យើងបាន $\lim_{x\to0}\frac{\sqrt{1+\sin x}-\sqrt{1-\sin x}}{\tan x}=\lim_{x\to0}\frac{\sqrt{(1+\sin x)^2}-\sqrt{(1-\sin x)^2}}{\tan x\left(\sqrt{1+\sin x}+\sqrt{1-\sin x}\right)}$

$$=\lim_{x\to0}\frac{(1+\sin x)-(1-\sin x)}{\tan x\left(\sqrt{1+\sin x}+\sqrt{1-\sin x}\right)}$$
$$=\lim_{x\to0}\frac{2\sin x}{\frac{\sin x}{\cos x}\left(\sqrt{1+\sin x}+\sqrt{1-\sin x}\right)}$$
$$=\lim_{x\to0}\frac{2\sin x\cos x}{\sin x\left(\sqrt{1+\sin x}+\sqrt{1-\sin x}\right)}$$
$$=\lim_{x\to0}\frac{2\cos x}{\sqrt{1+\sin x}+\sqrt{1-\sin x}}$$
$$=\frac{2}{\sqrt{1+0}+\sqrt{1-0}}$$
$$=1$$

ដូចនេះ $\boxed{\lim_{x\to0}\frac{\sqrt{1+\sin x}-\sqrt{1-\sin x}}{\tan x}=1}$

ជ. $\lim_{x\to0}\frac{\sin^2 x}{1-\sqrt{\cos x}}$, រាងមិនកំណត់ $\frac{0}{0}$

យើងបាន $\lim_{x\to0}\frac{\sin^2 x}{1-\sqrt{\cos x}}=\lim_{x\to0}\frac{\sin^2 x\left(1+\sqrt{\cos x}\right)}{\left(1-\sqrt{\cos x}\right)\left(1+\sqrt{\cos x}\right)}$

$$=\lim_{x\to0}\frac{\sin^2 x\left(1+\sqrt{\cos x}\right)}{1-\cos x}$$
$$=\lim_{x\to0}\frac{4\sin^2\frac{x}{2}\cos^2\frac{x}{2}\left(1+\sqrt{\cos x}\right)}{2\sin^2\frac{x}{2}}$$
$$=\lim_{x\to0}\left[2\sin^2\frac{x}{2}\cos^2\frac{x}{2}\left(1+\sqrt{\cos x}\right)\right]$$
$$=2\times1\times1\,(1+1)$$
$$=4$$

ដូចនេះ $\boxed{\lim_{x\to0}\frac{\sin^2 x}{1-\sqrt{\cos x}}=4}$

ឈ. $\lim_{x\to0}\frac{\sin 2x}{\sqrt{x+1}-1}$, រាងមិនកំណត់ $\frac{0}{0}$

យើងបាន $\lim_{x\to0}\frac{\sin 2x}{\sqrt{x+1}-1}=\lim_{x\to0}\frac{\sin 2x\left(\sqrt{x+1}+1\right)}{\left(\sqrt{x+1}-1\right)\left(\sqrt{x+1}+1\right)}$

$$=\lim_{x\to0}\frac{\sin 2x\left(\sqrt{x+1}+1\right)}{x}$$
$$=\lim_{x\to0}\left[\frac{\sin 2x}{2x}\times2\times\left(\sqrt{x+1}+1\right)\right]$$
$$=1\times2\,(1+1)=4$$

ដូចនេះ $\boxed{\lim_{x\to0}\frac{\sin 2x}{\sqrt{x+1}-1}=4}$

លំហាត់គំរូទី១៨

ដោះស្រាយលីមីតខាងក្រោម៖

ក. $\lim_{x\to0}\frac{\sqrt{1+\tan x}-\sqrt{1+\sin x}}{x^3}$

ខ. $\lim_{x\to0}\frac{\sin x}{\sqrt[3]{x}}$

គ. $\lim_{x\to0^+}\frac{1-\cos\sqrt{x}}{\sin x}$

ឃ. $\lim_{x\to0}\frac{\cos ax-\cos bx}{x^2}$

ង. $\lim_{x\to0}\frac{\sqrt{1+\sin^2 x}-\cos x}{\sin^2 x}$

ច. $\lim_{x\to0}\frac{1-\cos x\cos 2x}{x^2}$

ឆ. $\lim_{x\to0}\frac{1-\cos x\sqrt{\cos x}}{x^2}$

ជ. $\lim_{x\to0}\frac{2x+\sin 2019x-\tan 2020x}{9x-\sin 2021x+\tan 2022x}$

ឈ. $\lim_{x\to0}\frac{(1-\cos x)^2}{\tan^3 x-\sin^3 x}$

ញ. $\lim\limits_{x\to 0}\frac{1-\cos 4x}{2x\tan 2x}$ ដ. $\lim\limits_{x\to 0}\frac{2\sin^2 x-2(1-\cos x)}{5x^2}$ ឋ. $\lim\limits_{x\to 0}\frac{\cos 3x-\cos x}{\sin 5x+\sin 3x}$

ឌ. $\lim\limits_{x\to 0}\frac{1-\cos 2x}{x^2(1+\sqrt{\cos x})}$ ឍ. $\lim\limits_{x\to 0}\frac{\cos x-\sqrt{\cos 2x}}{\sin^2 x}$ ណ. $\lim\limits_{x\to 0}\frac{\tan x}{\sqrt[3]{(1-\cos x)^2}}$

ត. $\lim\limits_{x\to 0}\frac{1-\cos(1-\cos x)}{x^4}$ ថ. $\lim\limits_{x\to 0}\frac{2x-3\arcsin x}{2\arcsin x}$ ទ. $\lim\limits_{x\to 2}\frac{x^3-8+\tan\pi x}{x-2}$

ធ. $\lim\limits_{x\to 0}\frac{x^2-4+\sin\pi x}{x-2}$ ន. $\lim\limits_{x\to 1}\frac{\sqrt[3]{x}-1+\tan\pi x}{x-1}$

ដំណោះស្រាយ

ក. $\lim\limits_{x\to 0}\frac{\sqrt{1+\tan x}-\sqrt{1+\sin x}}{x^3}$, រាងមិនកំណត់ $\frac{0}{0}$

តាង $A=\lim\limits_{x\to 0}\frac{\sqrt{1+\tan x}-\sqrt{1+\sin x}}{x^3}$

យើងបាន $A=\lim\limits_{x\to 0}\frac{\left(\sqrt{1+\tan x}-\sqrt{1+\sin x}\right)\left(\sqrt{1+\tan x}+\sqrt{1+\sin x}\right)}{x^3\left(\sqrt{1+\tan x}+\sqrt{1+\sin x}\right)}$

$$=\lim_{x\to 0}\frac{\tan x-\sin x}{x^3\left(\sqrt{1+\tan x}+\sqrt{1+\sin x}\right)}$$
$$=\lim_{x\to 0}\frac{\tan x(1-\cos x)}{x^3\left(\sqrt{1+\tan x}+\sqrt{1+\sin x}\right)}$$
$$=\lim_{x\to 0}\frac{\tan x.2\sin^2\frac{x}{2}}{x^3\left(\sqrt{1+\tan x}+\sqrt{1+\sin x}\right)}$$
$$=2\lim_{x\to 0}\left[\frac{\tan x}{x}\times\frac{\sin^2\frac{x}{2}}{(\frac{x}{2})^2}\times\frac{1}{4}\times\frac{1}{\sqrt{1+\tan x}+\sqrt{1+\sin x}}\right]$$
$$=2\times 1\times 1\times\frac{1}{4}\times\frac{1}{\sqrt{1+0}+\sqrt{1+0}}$$
$$=\frac{1}{4}$$

ដូចនេះ $\boxed{\lim\limits_{x\to 0}\frac{\sqrt{1+\tan x}-\sqrt{1+\sin x}}{x^3}=\frac{1}{4}}$

ខ. $\lim\limits_{x\to 0}\frac{\sin x}{\sqrt[3]{x}}$,រាងមិនកំណត់ $\frac{0}{0}$

យើងបាន $\lim\limits_{x\to 0}\frac{\sin x}{\sqrt[3]{x}}=\lim\limits_{x\to 0}\frac{\sin x}{\sqrt[3]{x}}\times\frac{\sqrt[3]{x^2}}{\sqrt[3]{x^2}}$

$$=\lim_{x\to 0}\frac{\sqrt[3]{x^2}\sin x}{x}$$
$$=\lim_{x\to 0}\frac{\sin x}{x}\times\sqrt[3]{x^2}$$
$$=1\times 0=0$$

ដូចនេះ $\boxed{\lim\limits_{x\to 0}\frac{\sin x}{\sqrt[3]{x}}=0}$

គ. $\lim\limits_{x\to 0^+}\frac{1-\cos\sqrt{x}}{\sin x}$,រាងមិនកំណត់ $\frac{0}{0}$

យើងបាន $\lim\limits_{x\to 0^+}\frac{1-\cos\sqrt{x}}{\sin x}=\lim\limits_{x\to 0^+}\frac{2\sin^2\frac{\sqrt{x}}{2}}{\sin x}$

$$=2\lim_{x\to 0^+}\frac{\sin^2\frac{\sqrt{x}}{2}}{\left(\frac{\sqrt{x}}{2}\right)^2}\times\frac{x}{\sin x}\times\frac{1}{4}$$
$$=2\times 1\times 1\times\frac{1}{4}$$
$$=\frac{1}{2}$$

ដូចនេះ $\boxed{\lim\limits_{x\to 0^+}\frac{1-\cos\sqrt{x}}{\sin x}=\frac{1}{2}}$

ឃ. $\lim\limits_{x\to 0}\frac{\cos ax-\cos bx}{x^2}$,រាងមិនកំណត់ $\frac{0}{0}$

យើងបាន $\lim\limits_{x\to 0}\frac{\cos ax-\cos bx}{x^2}=\lim\limits_{x\to 0}\frac{\cos ax-1+1-\cos bx}{x^2}$

$$=\lim_{x\to 0}\frac{(1-\cos bx)-(1-\cos ax)}{x^2}$$
$$=\lim_{x\to 0}\left[\frac{(1-\cos bx)}{x^2}-\frac{(1-\cos ax)}{x^2}\right]$$
$$=\lim_{x\to 0}\frac{1-\cos bx}{x^2}-\lim_{x\to 0}\frac{1-\cos ax}{x^2}$$
$$=\lim_{x\to 0}\frac{2\sin^2\frac{bx}{2}}{x^2}-\lim_{x\to 0}\frac{2\sin^2\frac{ax}{2}}{x^2}$$
$$=2\lim_{x\to 0}\frac{\sin^2\frac{bx}{2}}{(\frac{bx}{2})^2}\times\frac{b^2}{4}-2\lim_{x\to 0}\frac{\sin^2\frac{ax}{2}}{(\frac{ax}{2})^2}\times\frac{a^2}{4}$$
$$=2\times 1\times\frac{b^2}{4}-2\times 1\times\frac{a^2}{4}$$
$$=\frac{b^2-a^2}{2}$$

ដូចនេះ $\boxed{\lim\limits_{x\to 0}\frac{\cos ax-\cos bx}{x^2}=\frac{b^2-a^2}{2}}$

ង. $\lim\limits_{x\to 0}\frac{\sqrt{1+\sin^2 x}-\cos x}{\sin^2 x}$,រាងមិនកំណត់ $\frac{0}{0}$

យើងបាន $\lim\limits_{x\to0}\dfrac{\sqrt{1+\sin^2x}-\cos x}{\sin^2x}=\lim\limits_{x\to0}\dfrac{\left(\sqrt{1+\sin^2x}-\cos x\right)\left(\sqrt{1+\sin^2x}+\cos x\right)}{\sin^2x\left(\sqrt{1+\sin^2x}+\cos x\right)}$

$$=\lim_{x\to0}\frac{\left(\sqrt{1+\sin^2x}\right)^2-\cos^2x}{\sin^2x\left(\sqrt{1+\sin^2x}+\cos x\right)}$$

$$=\lim_{x\to0}\frac{1+\sin^2x-\cos^2x}{\sin^2x\left(\sqrt{1+\sin^2x}+\cos x\right)}$$

$$=\lim_{x\to0}\frac{\sin^2x+(1-\cos^2x)}{\sin^2x\left(\sqrt{1+\sin^2x}+\cos x\right)}$$

$$=\lim_{x\to0}\frac{\sin^2x+\sin^2x}{\sin^2x\left(\sqrt{1+\sin^2x}+\cos x\right)}$$

$$=\lim_{x\to0}\frac{2\sin^2x}{\sin^2x\left(\sqrt{1+\sin^2x}+\cos x\right)}$$

$$=\lim_{x\to0}\frac{2}{\sqrt{1+\sin^2x}+\cos x}$$

$$=\frac{2}{1+1}$$

$$=1$$

ដូចនេះ $\boxed{\lim\limits_{x\to0}\frac{\sqrt{1+\sin^2x}-\cos x}{\sin^2x}=1}$

ច. $\lim\limits_{x\to0}\frac{1-\cos x\cos2x}{x^2}$, រាងមិនកំណត់ $\frac{0}{0}$ យើងបាន

$$\lim_{x\to0}\frac{1-\cos x\cos2x}{x^2}=\lim_{x\to0}\frac{1-\cos x\left(\cos^2x-\sin^2x\right)}{x^2}$$

$$=\lim_{x\to0}\frac{\left(1-\cos^3x\right)+\cos x\sin^2x}{x^2}$$

$$=\lim_{x\to0}\frac{(1-\cos x)\left(1+\cos x+\cos^2x\right)+\cos x\sin^2x}{x^2}$$

$$=\lim_{x\to0}\frac{2\sin^2\frac{x}{2}\left[\left(1+\cos x+\cos^2x\right)+2\cos x\cos^2\frac{x}{2}\right]}{x^2}$$

$$=2\lim_{x\to0}\frac{\sin^2\frac{x}{2}}{\left(\frac{x}{2}\right)^2}\times\frac{1}{4}\times\left[\left(1+\cos x+\cos^2x\right)+2\cos x\cos^2\frac{x}{2}\right]$$

$$=2\times1\times\frac{1}{4}\times(3+2)$$

$$=\frac{5}{2}$$

ដូចនេះ $\boxed{\lim\limits_{x\to0}\frac{1-\cos x\cos2x}{x^2}=\frac{5}{2}}$

ឆ. $\lim\limits_{x\to0}\frac{1-\cos x\sqrt{\cos x}}{x^2}$,រាងមិនកំណត់ $\frac{0}{0}$

យើងបាន $\lim\limits_{x\to0}\dfrac{1-\cos x\sqrt{\cos x}}{x^2}=\lim\limits_{x\to0}\dfrac{\left(1-\cos x\sqrt{\cos x}\right)\left(1+\cos x\sqrt{\cos x}\right)}{x^2\left(1+\cos x\sqrt{\cos 2x}\right)}$

$$=\lim_{x\to0}\frac{1-\cos^3x}{x^2\left(1+\cos x\sqrt{\cos x}\right)}$$

$$=\lim_{x\to0}\frac{(1-\cos x)\left(1+\cos x+\cos^2x\right)}{x^2\left(1+\cos x\sqrt{\cos x}\right)}$$

$$=\lim_{x\to0}\frac{2\sin^2\frac{x}{2}\left(1+\cos x+\cos^2x\right)}{x^2\left(1+\cos x\sqrt{\cos x}\right)}$$

$$=2\lim_{x\to0}\frac{\sin^2\frac{x}{2}}{\left(\frac{x}{2}\right)^2}\times\frac{1}{4}\times\frac{1+\cos x+\cos^2x}{1+\cos x\sqrt{\cos x}}$$

$$=2\times1\times\frac{1}{4}\times\frac{1+1+1}{1+1}=\frac{3}{4}$$

ដូចនេះ $\boxed{\lim\limits_{x\to0}\frac{1-\cos x\sqrt{\cos 2x}}{x^2}=\frac{3}{4}}$

ជ. $\lim\limits_{x\to0}\frac{2x+\sin2019x-\tan2020x}{9x-\sin2021x+\tan2022x}$,រាងមិនកំណត់ $\frac{0}{0}$

យើងបាន $\lim\limits_{x\to0}\dfrac{2x+\sin2019x-\tan2020x}{9x-\sin2021x+\tan2022x}=\lim\limits_{x\to0}\dfrac{\frac{2x+\sin2019x-\tan2020x}{x}}{\frac{9x-\sin2021x+\tan2022x}{x}}$

$$=\lim_{x\to0}\frac{2+\frac{\sin2019x}{x}-\frac{\tan2020x}{x}}{9-\frac{\sin2021x}{x}+\frac{\tan2022x}{x}}$$

$$=\lim_{x\to0}\frac{2+\frac{\sin2019x}{2019x}\times2019-\frac{\tan2020x}{2020x}\times2020}{9-\frac{\sin2021x}{2021x}\times2021+\frac{\tan2022x}{2022x}\times2022}$$

$$=\frac{2+2019-2020}{9-2021+2022}$$

$$=\frac{1}{10}$$

ដូចនេះ $\boxed{\lim\limits_{x\to0}\frac{2x+\sin2019x-\tan2020x}{9x-\sin2021x+\tan2022x}=\frac{1}{10}}$

ឈ. $\lim\limits_{x\to0}\frac{(1-\cos x)^2}{\tan^3x-\sin^3x}$,រាងមិនកំណត់ $\frac{0}{0}$

យើងបាន $\lim\limits_{x\to0}\dfrac{(1-\cos x)^2}{\tan^3x-\sin^3x}=\lim\limits_{x\to0}\dfrac{(1-\cos x)^2}{\frac{\sin^3x}{\cos^3x}(1-\cos x)\left(1+\cos x+\cos^2x\right)}$

$$=\lim_{x\to0}\frac{(1-\cos x)\cos^3x}{\sin^3x\left(1+\cos x+\cos^2x\right)}$$

$$=\lim_{x\to0}\frac{2\sin^2\frac{x}{2}\cos^3x}{8\sin^3\frac{x}{2}\cos^3\frac{x}{2}\left(1+\cos x+\cos^2x\right)}$$

$$=\lim_{x\to0}\frac{\cos^3x}{4\sin\frac{x}{2}\cos^3\frac{x}{2}\left(1+\cos x+\cos^2x\right)}$$

$$=\lim_{x\to0}\frac{\cos^3x}{4\sin\frac{x}{2}\cos^3\frac{x}{2}\left(1+\cos x+\cos^2x\right)}$$

$$=\frac{1}{4\times0\times1\times(1+1+1)}$$

$$=\infty$$

ដូចនេះ $\boxed{\lim\limits_{x\to0}\frac{(1-\cos x)^2}{\tan^3x-\sin^3x}=\infty}$

ញ. $\lim\limits_{x\to0}\frac{1-\cos4x}{2x\tan2x}$,រាងមិនកំណត់ $\frac{0}{0}$

យើងបាន $\lim\limits_{x\to0}\dfrac{1-\cos4x}{2x\tan2x}=\lim\limits_{x\to0}\dfrac{2\sin^22x}{2x\times\frac{\sin2x}{\cos2x}}$

$$=\lim_{x\to0}\frac{2\sin^22x}{2x\times\sin2x\frac{1}{\cos2x}}$$

$$=\lim_{x\to0}\frac{\sin2x}{x\times\frac{1}{\cos2x}}$$

$$=\lim_{x\to0}\frac{\sin2x}{2x}\times2\times\cos2x$$

$$=1\times2\times1$$

$$=2$$

ដូចនេះ $\boxed{\lim\limits_{x\to0}\frac{1-\cos4x}{2x\tan2x}=2}$

ដ. $\lim\limits_{x\to0}\frac{2\sin^2x-2(1-\cos x)}{5x^2}$,រាងមិនកំណត់ $\frac{0}{0}$

យើងបាន $\lim\limits_{x\to0}\dfrac{2\sin^2x-2(1-\cos x)}{5x^2}=\lim\limits_{x\to0}\dfrac{2\cdot4\sin^2\frac{x}{2}\cos^2\frac{x}{2}-2\cdot2\sin^2\frac{x}{2}}{5x^2}$

$$=\lim_{x\to0}\frac{4\sin^2\frac{x}{2}\left(2\cos^2\frac{x}{2}-1\right)}{5x^2}$$

$$=4\lim_{x\to0}\frac{\sin^2\frac{x}{2}}{\left(\frac{x}{2}\right)^2}\times\frac{1}{4\times5}\left(2\cos^2\frac{x}{2}-1\right)$$

$$=4\times\frac{1}{4\times5}\times(2-1)$$

$$=\frac{1}{5}$$

ដូចនេះ $\boxed{\lim\limits_{x\to 0}\frac{2\sin^2 x-2(1-\cos x)}{5x^2}=\frac{1}{5}}$

ឋ. $\lim\limits_{x\to 0}\frac{\cos 3x-\cos x}{\sin 5x+\sin 3x}$, រាងមិនកំណត់ $\frac{0}{0}$

យើងបាន $\lim\limits_{x\to 0}\dfrac{\cos 3x-\cos x}{\sin 5x+\sin 3x}=\lim\limits_{x\to 0}\dfrac{\cos 3x-1+1-\cos x}{\sin 5x+\sin 3x}$

$$=\lim_{x\to 0}\frac{(1-\cos x)-(1-\cos 3x)}{\sin 5x+\sin 3x}$$
$$=\lim_{x\to 0}\frac{2\sin^2\frac{x}{2}-2\sin^2\frac{3x}{2}}{\sin 5x+\sin 3x}$$
$$=2\lim_{x\to 0}\frac{\sin^2\frac{x}{2}-\sin^2\frac{3x}{2}}{\sin 5x+\sin 3x}$$
$$=2\lim_{x\to 0}\frac{\frac{\sin^2\frac{x}{2}}{x^2}-\frac{\sin^2\frac{3x}{2}}{x^2}}{\frac{1}{x}\left(\frac{\sin 5x}{x}+\frac{\sin 3x}{x}\right)}$$
$$=2\lim_{x\to 0}\frac{x\left(\frac{\sin^2\frac{x}{2}}{x^2}-\frac{\sin^2\frac{3x}{2}}{x^2}\right)}{\frac{\sin 5x}{x}+\frac{\sin 3x}{x}}$$
$$=2\lim_{x\to 0}\frac{x\left[\frac{\sin^2\frac{x}{2}}{\left(\frac{x}{2}\right)^2}\times\frac{1}{4}-\frac{\sin^2\frac{3x}{2}}{\left(\frac{3x}{2}\right)^2}\times\frac{9}{4}\right]}{\frac{\sin 5x}{5x}\times 5+\frac{\sin 3x}{3x}\times 3}$$
$$=2\times\frac{0\left(\frac{1}{4}-\frac{9}{4}\right)}{5+3}=0$$

ដូចនេះ $\boxed{\lim\limits_{x\to 0}\frac{\cos 3x-\cos x}{\sin 5x+\sin 3x}=0}$

ឌ. $\lim\limits_{x\to 0}\frac{1-\cos 2x}{x^2(1+\sqrt{\cos x})}$, រាងមិនកំណត់ $\frac{0}{0}$

យើងបាន $\lim\limits_{x\to 0}\dfrac{1-\cos 2x}{x^2(1+\sqrt{\cos x})}=\lim\limits_{x\to 0}\dfrac{2\sin^2\frac{x}{2}}{x^2(1+\sqrt{\cos x})}$

$$=\lim_{x\to 0}\frac{2\sin^2\frac{x}{2}}{x^2}\times\left(\frac{1}{1+\sqrt{\cos x}}\right)$$
$$=2\lim_{x\to 0}\frac{\sin^2\frac{x}{2}}{\left(\frac{x}{2}\right)^2}\times\frac{1}{4}\times\left(\frac{1}{1+\sqrt{\cos x}}\right)$$
$$=2\times\frac{1}{4}\times\frac{1}{1+1}=\frac{1}{4}$$

ដូចនេះ $\boxed{\lim\limits_{x\to 0}\frac{1-\cos 2x}{x^2(1+\sqrt{\cos x})}=\frac{1}{4}}$

ឍ. $\lim\limits_{x\to 0}\frac{\cos x-\sqrt{\cos 2x}}{\sin^2 x}$,រាងមិនកំណត់ $\frac{0}{0}$

យើងបាន $\lim\limits_{x\to 0}\dfrac{1-\cos 2x}{x^2(1+\sqrt{\cos x})}=\lim\limits_{x\to 0}\dfrac{\left(\cos x-\sqrt{\cos 2x}\right)\left(\cos x+\sqrt{\cos 2x}\right)}{\sin^2 x\left(\cos x+\sqrt{\cos 2x}\right)}$

$$=\lim_{x\to 0}\frac{\cos^2 x-\left(\sqrt{\cos 2x}\right)}{\sin^2 x\left(\cos x+\sqrt{\cos 2x}\right)}$$
$$=\lim_{x\to 0}\frac{\cos^2 x-\cos 2x}{\sin^2 x\left(\cos x+\sqrt{\cos 2x}\right)}$$
$$=\lim_{x\to 0}\frac{\cos^2 x-\cos 2x}{\sin^2 x\left(\cos x+\sqrt{\cos 2x}\right)}$$
$$=\lim_{x\to 0}\frac{\cos^2 x-\left(\cos^2 x-\sin^2 x\right)}{\sin^2 x\left(\cos x+\sqrt{\cos 2x}\right)}$$
$$=\lim_{x\to 0}\frac{\cos^2 x-\cos^2 x+\sin^2 x}{\sin^2 x\left(\cos x+\sqrt{\cos 2x}\right)}$$
$$=\lim_{x\to 0}\frac{\sin^2 x}{\sin^2 x\left(\cos x+\sqrt{\cos 2x}\right)}$$
$$=\lim_{x\to 0}\frac{1}{\cos x+\sqrt{\cos 2x}}$$
$$=\frac{1}{\cos 0+\sqrt{\cos 0}}$$
$$=\frac{1}{1+1}$$
$$=\frac{1}{2}$$

ដូចនេះ $\boxed{\lim\limits_{x\to 0}\frac{1-\cos 2x}{x^2(1+\sqrt{\cos x})}=\frac{1}{2}}$

ណ. $\lim\limits_{x\to 0}\frac{\tan x}{\sqrt[3]{(1-\cos x)^2}}$,រាងមិនកំណត់ $\frac{0}{0}$

យើងបាន $\lim\limits_{x\to 0}\dfrac{\tan x}{\sqrt[3]{(1-\cos x)^2}}=\lim\limits_{x\to 0}\sqrt[3]{\dfrac{\tan^3 x}{(1-\cos x)^2}}$

$$=\lim_{x\to 0}\sqrt[3]{\frac{\sin^3 x}{\cos^3 x(1-\cos x)^2}}$$
$$=\lim_{x\to 0}\sqrt[3]{\frac{8\sin^3\frac{x}{2}\cos^3\frac{x}{2}}{\cos^3 x\left(2\sin^2\frac{x}{2}\right)^2}}$$
$$=\lim_{x\to 0}\sqrt[3]{\frac{8\sin^3\frac{x}{2}\cos^3\frac{x}{2}}{\cos^3\cdot 4\sin^4\frac{x}{2}}}$$
$$=\lim_{x\to 0}\sqrt[3]{\frac{2\cos^3\frac{x}{2}}{\cos^3 x\cdot\sin\frac{x}{2}}}$$
$$=\sqrt[3]{\frac{2}{0}}$$
$$=\infty$$

ដូចនេះ $\boxed{\lim\limits_{x\to 0}\frac{\tan x}{\sqrt[3]{(1-\cos x)^2}}=\infty}$

ត. $\lim\limits_{x\to 0}\frac{1-\cos(1-\cos x)}{x^4}$, រាងមិនកំណត់ $\frac{0}{0}$

$$\text{យើងបាន} \lim_{x\to 0}\frac{1-\cos(1-\cos x)}{x^4}=\lim_{x\to 0}\frac{1-\cos\left(2\sin^2\frac{x}{2}\right)}{x^4}$$
$$=\lim_{x\to 0}\frac{2\sin^2\left(\sin^2\frac{x}{2}\right)}{x^4}$$
$$=2\lim_{x\to 0}\frac{\sin^2\left(\sin^2\frac{x}{2}\right)}{\left(\sin^2\frac{x}{2}\right)^2}\times\frac{\sin^4\frac{x}{2}}{\left(\frac{x}{2}\right)^4}\times\frac{1}{16}$$
$$=2\times 1\times 1\times\frac{1}{16}$$
$$=\frac{1}{8}$$

ដូចនេះ $\boxed{\lim_{x\to 0}\frac{1-\cos(1-\cos x)}{x^4}=\frac{1}{8}}$

ថ. $\lim_{x\to 0}\frac{2x-3\arcsin x}{2\arcsin x}$,រាងមិនកំណត់ $\frac{0}{0}$

តាង $k=\arcsin x\Rightarrow x=\sin k$ បើ $x\to 0$ នោះ $k\to 0$

$$\text{យើងបាន} \lim_{x\to 0}\frac{2x-3\arcsin x}{2\arcsin x}=\lim_{k\to 0}\frac{2\sin k-3k}{k}$$
$$=\lim_{k\to 0}\left(\frac{2\sin k}{k}-\frac{3k}{k}\right)$$
$$=\lim_{k\to 0}\frac{2\sin k}{k}-\lim_{k\to 0}\frac{3k}{k}$$
$$=2\lim_{k\to 0}\frac{\sin k}{k}-3$$
$$=2\times 1-3=-1$$

ដូចនេះ $\boxed{\lim_{x\to 0}\frac{2x-3\arcsin x}{2\arcsin x}=-1}$

ទ. $\lim_{x\to 2}\frac{x^3-8+\tan\pi x}{x-2}$,រាងមិនកំណត់ $\frac{0}{0}$

$$\text{យើងបាន} \lim_{x\to 2}\frac{x^3-8+\tan\pi x}{x-2}=\lim_{x\to 2}\left[\frac{(x-2)\left(x^2+2x+4\right)+\tan\pi x}{(x-2)}\right]$$
$$=\lim_{x\to 2}\left[\left(x^2+2x+4\right)+\frac{\tan\pi x}{(x-2)}\right]$$
$$=\lim_{x\to 2}\left(x^2+2x+4\right)+\lim_{x\to 2}\frac{\tan\pi x}{(x-2)}$$
$$=12+\lim_{x\to 2}\frac{\tan\pi x}{(x-2)}$$

តាង $k=x-2\Rightarrow x=2+k$ បើ $x\to 2$ នោះ $k\to 0$

$$\text{យើងបាន} \lim_{x\to 2}\frac{\tan\pi x}{(x-2)}=\lim_{k\to 0}\frac{\tan\pi(2+k)}{k}$$
$$=\lim_{k\to 0}\frac{\tan(2\pi+k\pi)}{k}$$
$$=\lim_{k\to 0}\frac{\tan k\pi}{k}$$
$$=\lim_{k\to 0}\frac{\tan k\pi}{\pi k}\times\pi=1\times\pi=\pi$$

ដូចនេះ $\boxed{\lim_{x\to 0}\frac{2x-3\arcsin x}{2\arcsin x}=12+\pi}$

ធ. $\lim_{x\to 0}\frac{x^2-4+\sin\pi x}{x-2}$,រាងមិនកំណត់ $\frac{0}{0}$

$$\text{យើងបាន} \lim_{x\to 0}\frac{x^2-4+\sin\pi x}{x-2}=\lim_{x\to 2}\frac{(x-2)(x+2)+\sin\pi x}{x-2}$$
$$=\lim_{x\to 2}\left[(x+2)+\frac{\sin\pi x}{x-2}\right]$$
$$=4+\lim_{x\to 2}\frac{\sin\pi x}{x-2}$$

តាង $k=x-2\Rightarrow x=k+2$ បើ $x\to 2$ នោះ $k\to 0$

$$\text{យើងបាន} \lim_{x\to 2}\frac{\sin\pi x}{x-2}=\lim_{k\to 0}\frac{\sin\pi(k+2)}{k}$$
$$=\lim_{k\to 0}\frac{\sin(k\pi+2\pi)}{k}$$
$$=\lim_{k\to 0}\frac{\sin k\pi}{k}$$
$$=\lim_{k\to 0}\frac{\sin k\pi}{\pi k}\times\pi=\pi$$

ដូចនេះ $\boxed{\lim_{x\to 2}\frac{x^3-8+\tan\pi x}{x-2}=4+\pi}$

ន. $\lim_{x\to 1}\frac{\sqrt[3]{x}-1+\tan\pi x}{x-1}$,រាងមិនកំណត់ $\frac{0}{0}$

$$\text{យើងបាន} \lim_{x\to 1}\frac{\sqrt[3]{x}-1+\tan\pi x}{x-1}=\lim_{x\to 1}\left[\frac{\sqrt[3]{x}-1}{x-1}+\frac{\tan\pi x}{x-1}\right]$$
$$=\lim_{x\to 1}\left[\frac{(\sqrt[3]{x}-1)\left(\sqrt[3]{x^2}+\sqrt[3]{x}+1\right)}{(x-1)\left(\sqrt[3]{x^2}+\sqrt[3]{x}+1\right)}+\frac{\tan\pi x}{x-1}\right]$$
$$=\lim_{x\to 1}\left[\frac{x-1}{(x-1)\left(\sqrt[3]{x^2}+\sqrt[3]{x}+1\right)}+\frac{\tan\pi x}{x-1}\right]$$
$$=\lim_{x\to 1}\left[\frac{1}{\sqrt[3]{x^2}+\sqrt[3]{x}+1}+\frac{\tan\pi x}{x-1}\right]$$
$$=\lim_{x\to 1}\frac{1}{\sqrt[3]{x^2}+\sqrt[3]{x}+1}+\lim_{x\to 1}\frac{\tan\pi x}{x-1}$$
$$=\frac{1}{3}+\lim_{x\to 1}\frac{\tan\pi x}{x-1}$$

តាង $k=x-1\Rightarrow x=k+1$

បើ $x\to 1$ នោះ $k\to 0$

$$\text{យើងបាន} \lim_{x\to 1}\frac{\tan\pi x}{x-1}=\lim_{x\to 1}\frac{\tan\pi x}{x-1}$$
$$=\lim_{k\to 0}\frac{\tan\pi(k+1)}{k}$$
$$=\lim_{k\to 0}\frac{\tan(\pi k+\pi)}{k}$$
$$=\lim_{k\to 0}\frac{\tan\pi k}{k}=\lim_{k\to 0}\frac{\tan\pi k}{k}\times\pi$$
$$=1\times\pi$$
$$=\pi$$

ដូចនេះ $\boxed{\lim_{x\to 1}\frac{\sqrt[3]{x}-1+\tan\pi x}{x-1}=\frac{1}{3}+\pi}$

លំហាត់គំរូទី១៩

ដោះស្រាយលីមីតខាងក្រោម៖

ក. $\lim_{x\to+\infty}e^{-x}$

ខ. $\lim_{x\to-\infty}e^{2-3x}$

គ. $\lim_{x\to 0}\left(e^x+2x^2-6\right)$

ឃ. $\lim_{x\to+\infty}\left(e^x-e^{-x}\right)$

ង. $\lim_{x\to+\infty}\frac{xe^x+2}{x^2-1}$

ច. $\lim_{x\to 0}\frac{e^x+x+1}{2e^x+9}$

ឆ. $\lim\limits_{x\to+\infty}\frac{2e^x-x}{1+e^x}$ ជ. $\lim\limits_{x\to+\infty}\frac{e^{2x}-3e^x+2}{2e^{2x}+5e^x-7}$ ឈ. $\lim\limits_{x\to+\infty}\frac{e^x-e^{-x}}{e^x+e^{-x}}$

ញ. $\lim\limits_{x\to+\infty}\frac{1+e^x-xe^{2x}}{x^2e^x+e^{2x}}$

ដំណោះស្រាយ

ដោះស្រាយលីមីតខាងក្រោម៖

ក. $\lim\limits_{x\to+\infty}e^{-x}$

យើងបាន $\lim\limits_{x\to+\infty}e^{-x}=e^{-(+\infty)}=0$

ដូចនេះ $\boxed{\lim\limits_{x\to+\infty}e^{-x}=0}$

ខ. $\lim\limits_{x\to-\infty}e^{2-3x}$

យើងបាន $\lim\limits_{x\to-\infty}e^{2-3x}=+\infty$

ដូចនេះ $\boxed{\lim\limits_{x\to-\infty}e^{2-3x}=+\infty}$

គ. $\lim\limits_{x\to 0}(e^x+2x^2-6)$

យើងបាន
$$\lim_{x\to 0}(e^x+2x^2-6)=e^0+2(0)^2-6=-5$$

ដូចនេះ $\boxed{\lim\limits_{x\to 0}(e^x+2x^2-6)=-5}$

ឃ. $\lim\limits_{x\to+\infty}(e^x-e^{-x})$

យើងបាន
$$\lim_{x\to+\infty}(e^x-e^{-x})=e^{+\infty}-e^{-(+\infty)}=+\infty$$

ដូចនេះ $\boxed{\lim\limits_{x\to+\infty}(e^x-e^{-x})=+\infty}$

ង. $\lim\limits_{x\to+\infty}\frac{xe^x+2}{x^2-1}$ រាងមិនកំណត់ $\frac{\infty}{\infty}$

យើងបាន
$$\begin{aligned}\lim_{x\to+\infty}\frac{xe^x+2}{x^2-1}&=\lim_{x\to+\infty}\frac{xe^x\left(1+\frac{2}{xe^x}\right)}{x^2\left(1-\frac{1}{x^2}\right)}\\&=\lim_{x\to+\infty}\frac{e^x\left(1+\frac{2}{xe^x}\right)}{x\left(1-\frac{1}{x^2}\right)}\\&=\lim_{x\to+\infty}\frac{e^x}{x}\left(\frac{1+\frac{2}{xe^x}}{1-\frac{1}{x^2}}\right)\\&=+\infty\left(\frac{1+0}{1-0}\right)\\&=+\infty\end{aligned}$$

ដូចនេះ $\boxed{\lim\limits_{x\to+\infty}\frac{xe^x+2}{x^2-1}=+\infty}$

ច. $\lim\limits_{x\to 0}\frac{e^x+x+1}{2e^x+9}$

យើងបាន
$$\begin{aligned}\lim_{x\to 0}\frac{e^x+x+1}{2e^x+9}&=\frac{e^0+0+1}{2e^0+9}\\&=\frac{1+0+1}{2+9}\\&=\frac{2}{11}\end{aligned}$$

ដូចនេះ $\boxed{\lim\limits_{x\to 0}\frac{e^x+x+1}{2e^x+9}=\frac{2}{11}}$

ឆ. $\lim\limits_{x\to+\infty}\frac{2e^x-x}{1+e^x}$ រាងមិនកំណត់ $\frac{\infty}{\infty}$

យើងបាន
$$\begin{aligned}\lim_{x\to+\infty}\frac{2e^x-x}{1+e^x}&=\lim_{x\to+\infty}\frac{e^x\left(2-\frac{x}{e^x}\right)}{e^x\left(\frac{1}{e^x}+1\right)}\\&=\lim_{x\to+\infty}\frac{2-\frac{x}{e^x}}{\frac{1}{e^x}+1}\\&=\frac{2-0}{0+1}\\&=2\end{aligned}$$

ដូចនេះ $\boxed{\lim\limits_{x\to+\infty}\frac{2e^x-x}{1+e^x}=2}$

ជ. $\lim\limits_{x\to+\infty}\frac{e^{2x}-3e^x+2}{2e^{2x}+5e^x-7}$ រាងមិនកំណត់ $\frac{\infty}{\infty}$

យើងបាន
$$\begin{aligned}\lim_{x\to+\infty}\frac{e^{2x}-3e^x+2}{2e^{2x}+5e^x-7}&=\lim_{x\to+\infty}\frac{e^{2x}\left(1-\frac{3}{e^x}+\frac{2}{e^{2x}}\right)}{e^{2x}\left(2+\frac{5}{e^x}-\frac{7}{e^{2x}}\right)}\\&=\lim_{x\to+\infty}\frac{1-\frac{3}{e^x}+\frac{2}{e^{2x}}}{2+\frac{5}{e^x}-\frac{7}{e^{2x}}}\\&=\frac{1-0+0}{2+0-0}\\&=\frac{1}{2}\end{aligned}$$

ដូចនេះ $\boxed{\lim\limits_{x\to+\infty}\frac{e^{2x}-3e^x+2}{2e^{2x}+5e^x-7}=\frac{1}{2}}$

ឈ. $\lim\limits_{x\to+\infty}\frac{e^x-e^{-x}}{e^x+e^{-x}}$ រាងមិនកំណត់ $\frac{\infty}{\infty}$

យើងបាន
$$\begin{aligned}\lim_{x\to+\infty}\frac{e^x-e^{-x}}{e^x+e^{-x}}&=\lim_{x\to+\infty}\frac{e^x\left(1-e^{2x}\right)}{e^x\left(1+e^{2x}\right)}\\&=\lim_{x\to+\infty}\frac{1-e^{2x}}{1+e^{2x}}\\&=\frac{1-0}{1+0}\\&=1\end{aligned}$$

ដូចនេះ $\boxed{\lim\limits_{x\to+\infty}\frac{e^x-e^{-x}}{e^x+e^{-x}}=1}$

ញ. $\lim\limits_{x\to+\infty}\frac{1+e^x-xe^{2x}}{x^2e^x+e^{2x}}$ រាងមិនកំណត់ $\frac{\infty}{\infty}$

យើងបាន
$$\lim_{x\to+\infty}\frac{1+e^x-xe^{2x}}{x^2e^x+e^{2x}}=\lim_{x\to+\infty}\frac{xe^{2x}\left(\frac{1}{xe^{2x}}+\frac{1}{xe^x}-1\right)}{xe^{2x}\left(\frac{x}{e^x}+\frac{1}{x}\right)}$$
$$=\lim_{x\to+\infty}\frac{\frac{1}{xe^{2x}}+\frac{1}{xe^x}-1}{\frac{x}{e^x}+\frac{1}{x}}$$
$$=\frac{0+0-1}{0^+}$$
$$=-\infty$$

ដូចនេះ $\boxed{\lim\limits_{x\to+\infty}\frac{1+e^x-xe^{2x}}{x^2e^x+e^{2x}}=-\infty}$

លំហាត់គំរូទី២០

ដោះស្រាយលីមីតខាងក្រោម៖

ក. $\lim\limits_{x\to0}\frac{e^{ax}-e^{bx}}{\sin ax-\sin bx}, a\neq b$

ខ. $\lim\limits_{x\to0}\frac{e^x+\sin x-1}{x}$

គ. $\lim\limits_{x\to0}\frac{e^{2x}-1}{e^{3x}-1}$

ឃ. $\lim\limits_{x\to0}\frac{e^{\sin x}+2\sin x-1}{\sin x}$

ង. $\lim\limits_{x\to0}\frac{\left(e^{-3x+2}-e^2\right)\sin\pi x}{4x^2}$

ច. $\lim\limits_{x\to0}\frac{\sin x+e^x-1}{x^2+x}$

ឆ. $\lim\limits_{x\to1}\frac{1-x}{e^x-e}$

ជ. $\lim\limits_{x\to0}\frac{5^x-4^x}{x^2-x}$

ឈ. $\lim\limits_{x\to0}\frac{2-(e^x+e^{-x})}{x^2}$

ញ. $\lim\limits_{x\to0}\frac{e^{\sin x}-e^{\sin 3x}}{\sin 2x}$

ដ. $\lim\limits_{x\to0}\frac{e^{ax}-e^{bx}}{x}$

ឋ. $\lim\limits_{x\to0}\frac{e^{\sin^2 2x}-\cos(\sin 3x)}{x^2}$

ឌ. $\lim\limits_{x\to0}\frac{e^{x^2}-\cos x}{x^2}$

ឍ. $\lim\limits_{x\to0}\frac{e^{x+1}-e}{\sin\pi x}$

ណ. $\lim\limits_{x\to\pi}\frac{\sin 2x}{e^{x+\pi}-e^{2\pi}}$

ត. $\lim\limits_{x\to0}\frac{e^x+\sin x-1}{1-\sqrt{x+1}}$

ថ. $\lim\limits_{x\to0}\frac{(e^x-1)(1-\cos 2x)\sin 7x}{x^4}$

ទ. $\lim\limits_{x\to0}\frac{x^2}{e^{x^2}-4^{x^2}}$

ធ. $\lim\limits_{x\to0}\frac{e^{2x}-e^{-2x}}{x}$

ន. $\lim\limits_{x\to0}\frac{e^x+e^{-x}-2}{x^2}$

ប. $\lim\limits_{x\to0}\frac{2e^{2x}-3e^x+1}{x}$

ផ. $\lim\limits_{x\to0}\frac{e^x-e^{-x}}{\sin 2x}$

ព. $\lim\limits_{x\to0}\frac{2e^x+3e^{2x}-5}{e^{3x}-1}$

ភ. $\lim\limits_{x\to0}\frac{e^{-x^2}-\cos 2x}{x^2}$

ម. $\lim\limits_{x\to0}\frac{e^{-2\sin x}-e^{\tan 3x}}{x^3+x}$

យ. $\lim\limits_{x\to0}\frac{xe^{-2x^2}+\sin x-\tan x-x}{x^3}$

រ. $\lim\limits_{x\to0}\frac{e^{3\sin^2 x}-\cos x\cos 3x}{-e^{2x^2}+\cos 2x}$

ល. $\lim\limits_{x\to0}\frac{4^x+2^x-2}{x}$

វ. $\lim\limits_{x\to0}\frac{7^{x^2}-\cos 2x}{x^2}$

ស. $\lim\limits_{x\to0}\frac{2^{\sin x}-3^{\tan x}}{x}$

ដំណោះស្រាយ

ដោះស្រាយលីមីតខាងក្រោម៖

ក. $\lim\limits_{x\to0}\frac{e^{ax}-e^{bx}}{\sin ax-\sin bx}$, រាងមិនកំណត់ $\frac{0}{0}$

យើងបាន
$$\lim_{x\to0}\frac{e^{ax}-e^{bx}}{\sin ax-\sin bx}=\lim_{x\to0}\frac{\left(e^{ax}-1\right)-\left(e^{bx}-1\right)}{\sin ax-\sin bx}$$
$$=\lim_{x\to0}\frac{\frac{e^{ax}-1}{ax}\times a-\frac{e^{bx}-1}{bx}\times b}{\frac{\sin ax}{ax}\times a-\frac{\sin bx}{bx}\times b}$$
$$=\frac{a-b}{a-b}$$
$$=1 \quad \text{ដោយ } a\neq b$$

ដូចនេះ $\boxed{\lim\limits_{x\to0}\frac{e^{ax}-e^{bx}}{\sin ax-\sin bx}=1}$

ខ. $\lim\limits_{x\to0}\frac{e^x+\sin x-1}{x}$ រាងមិនកំណត់ $\frac{0}{0}$

យើងបាន
$$\lim_{x\to0}\frac{e^x+\sin x-1}{x}=\lim_{x\to0}\left(\frac{e^x-1}{x}+\frac{\sin x}{x}\right)$$
$$=\lim_{x\to0}\frac{e^x-1}{x}+\lim_{x\to0}\frac{\sin x}{x}$$
$$=1+1=2$$

ដូចនេះ $\boxed{\lim\limits_{x\to0}\frac{e^x+\sin x-1}{x}=2}$

គ. $\lim\limits_{x\to0}\frac{e^{2x}-1}{e^{3x}-1}$, រាងមិនកំណត់$\frac{0}{0}$

យើងបាន $\lim\limits_{x\to0}\dfrac{e^{2x}-1}{e^{3x}-1}=\lim\limits_{x\to0}\dfrac{\frac{e^{2x}-1}{2x}\times2}{\frac{e^{3x}-1}{3x}\times3}$
$=\dfrac{1\times2}{1\times3}$
$=\dfrac{2}{3}$

ដូចនេះ $\boxed{\lim\limits_{x\to0}\frac{e^{2x}-1}{e^{3x}-1}=\frac{2}{3}}$

ឃ. $\lim\limits_{x\to0}\frac{e^{\sin x}+2\sin x-1}{\sin x}$, រាងមិនកំណត់ $\frac{0}{0}$

យើងបាន $\lim\limits_{x\to0}\dfrac{e^{\sin x}+2\sin x-1}{\sin x}=\lim\limits_{x\to0}\dfrac{e^{\sin x}-1+\sin 2x}{\sin x}$
$=\lim\limits_{x\to0}\left(\dfrac{e^{\sin x}-1}{\sin x}+\dfrac{\sin 2x}{\sin x}\right)$
$=\lim\limits_{x\to0}\dfrac{e^{\sin x}-1}{\sin x}+\lim\limits_{x\to0}\dfrac{\sin 2x}{\sin x}$
$=\lim\limits_{x\to0}\dfrac{e^{\sin x}-1}{\sin x}+\lim\limits_{x\to0}\dfrac{2\sin x\cos x}{\sin x}$
$=\lim\limits_{x\to0}\dfrac{e^{\sin x}-1}{\sin x}+\lim\limits_{x\to0}2\cos x$
$=1+2$
$=3$

ដូចនេះ $\boxed{\lim\limits_{x\to0}\frac{e^{\sin x}+2\sin x-1}{\sin x}=3}$

ង. $\lim\limits_{x\to0}\frac{\left(e^{-3x+2}-e^2\right)\sin\pi x}{4x^2}$, រាងមិនកំណត់ $\frac{0}{0}$

យើងបាន $\lim\limits_{x\to0}\dfrac{\left(e^{-3x+2}-e^2\right)\sin\pi x}{4x^2}=\lim\limits_{x\to0}\left[\dfrac{e^2\left(e^{-3x}-1\right)}{4x}\times\dfrac{\sin\pi x}{x}\right]$
$=\lim\limits_{x\to0}\left(\dfrac{e^{-3x}-1}{-3x}\times\dfrac{-3e^2}{4}\times\dfrac{\sin\pi x}{\pi x}\times\pi\right)$
$=1\times\left(\dfrac{-3e^2}{4}\right)\times1\times\pi$
$=-\dfrac{3\pi e^2}{4}$

ដូចនេះ $\boxed{\lim\limits_{x\to0}\frac{\left(e^{-3x+2}-e^2\right)\sin\pi x}{4x^2}=-\frac{3\pi e^2}{4}}$

ច. $\lim\limits_{x\to0}\frac{\sin x+e^x-1}{x^2+x}$, រាងមិនកំណត់$\frac{0}{0}$

យើងបាន $\lim\limits_{x\to0}\dfrac{\sin x+e^x-1}{x^2+x}=\lim\limits_{x\to0}\dfrac{e^x-1+\sin x}{x(x+1)}$
$=\lim\limits_{x\to0}\left(\dfrac{e^x-1}{x}+\dfrac{\sin x}{x}\right)\times\dfrac{1}{x+1}$
$=\left(\lim\limits_{x\to0}\dfrac{e^x-1}{x}+\lim\limits_{x\to0}\dfrac{\sin x}{x}\right)\times\dfrac{1}{x+1}$
$=(1+1)\dfrac{1}{0+1}$
$=2$

ដូចនេះ $\boxed{\lim\limits_{x\to0}\frac{\sin x+e^x-1}{x^2+x}=2}$

ឆ. $\lim\limits_{x\to1}\frac{1-x}{e^x-e}$, រាងមិនកំណត់ $\frac{0}{0}$

តាង $u=x-1\Rightarrow x=u+1$

បើ $x\to0\Rightarrow u\to0$

យើងបាន $\lim\limits_{x\to1}\dfrac{1-x}{e^x-e}=\lim\limits_{u\to0}\dfrac{1-(u+1)}{e^{u+1}-e}$
$=\lim\limits_{u\to0}\dfrac{-u}{e\left(e^u-1\right)}$
$=-\dfrac{1}{e}\lim\limits_{u\to0}\dfrac{u}{e^u-1}$
$=-\dfrac{1}{e}\times1$
$=-\dfrac{1}{e}$

ដូចនេះ $\boxed{\lim\limits_{x\to1}\frac{1-x}{e^x-e}=-\frac{1}{e}}$

ជ. $\lim\limits_{x\to0}\frac{5^x-4^x}{x^2-x}$, រាងមិនកំណត់ $\frac{0}{0}$

យើងបាន $\lim\limits_{x\to0}\dfrac{5^x-4^x}{x^2-x}=\lim\limits_{x\to0}\dfrac{5^x-1+1-4^x}{x(x-1)}$
$=\lim\limits_{x\to0}\dfrac{(5^x-1)-(4^x-1)}{x(x-1)}$
$=\lim\limits_{x\to0}\left[\dfrac{(5^x-1)-(4^x-1)}{x}\right]\left(\dfrac{1}{x-1}\right)$
$=\left[\lim\limits_{x\to0}\dfrac{(5^x-1)}{x}-\lim\limits_{x\to0}\dfrac{(4^x-1)}{x}\right]\left(\dfrac{1}{x-1}\right)$
$=(\ln5-\ln4)(-1)$
$=-\ln\dfrac{5}{4}$

ដូចនេះ $\boxed{\lim\limits_{x\to0}\frac{5^x-4^x}{x^2-x}=-\ln\frac{5}{4}}$

ឈ. $\lim\limits_{x\to0}\frac{2-(e^x+e^{-x})}{x^2}$, រាងមិនកំណត់ $\frac{0}{0}$

យើងបាន $\lim\limits_{x\to0}\dfrac{2-(e^x+e^{-x})}{x^2}=\lim\limits_{x\to0}\dfrac{(2-e^x-e^{-x})e^x}{x^2e^x}$
$=\lim\limits_{x\to0}\dfrac{2e^x-e^{2x}-1}{x^2e^x}$
$=\lim\limits_{x\to0}\dfrac{-\left(e^{2x}-2e^x+1\right)}{x^2e^x}$
$=\lim\limits_{x\to0}\dfrac{-(e^x-1)^2}{x^2e^x}$
$=-\lim\limits_{x\to0}\left(\dfrac{e^x-1}{x}\right)^2\left(\dfrac{1}{e^x}\right)$
$=-(1)^2\left(\dfrac{1}{1}\right)$
$=-1$

ដូចនេះ $\boxed{\lim\limits_{x\to0}\frac{2-(e^x+e^{-x})}{x^2}=-1}$

ញ. $\lim\limits_{x\to0}\frac{e^{\sin x}-e^{\sin3x}}{\sin2x}$, រាងមិនកំណត់ $\frac{0}{0}$

យើងបាន $\displaystyle\lim_{x\to0}\frac{e^{\sin x}-e^{\sin 3x}}{\sin 2x}=\lim_{x\to0}\frac{e^{\sin x}-1+1-e^{\sin 3x}}{\sin 2x}$

$$=\lim_{x\to0}\frac{\left(e^{\sin x}-1\right)-\left(e^{\sin 3x}-1\right)}{\sin 2x}$$

$$=\lim_{x\to0}\frac{\frac{\left(e^{\sin x}-1\right)-\left(e^{\sin 3x}-1\right)}{x}}{\frac{\sin 2x}{x}}$$

$$=\lim_{x\to0}\frac{\frac{e^{\sin x}-1}{x}-\frac{e^{\sin 3x}-1}{x}}{\frac{\sin 2x}{x}}$$

$$=\lim_{x\to0}\frac{\frac{e^{\sin x}-1}{\sin x}\times\frac{\sin x}{x}-\frac{e^{\sin 3x}-1}{\sin 3x}\times\frac{\sin 3x}{3x}\times 3}{\frac{\sin 2x}{2x}\times 2}$$

$$=\frac{(1\times1)-(1\times1\times3)}{1\times2}$$

$$=-1$$

ដូចនេះ $\boxed{\lim_{x\to0}\frac{e^{\sin x}-e^{\sin 3x}}{\sin 2x}=-1}$

ង. $\lim_{x\to0}\frac{e^{ax}-e^{bx}}{x}$, រាងមិនកំណត់ $\frac{0}{0}$

យើងបាន $\displaystyle\lim_{x\to0}\frac{e^{ax}+e^{bx}}{x}=\lim_{x\to0}\frac{e^{ax}-1+1-e^{bx}}{x}$

$$=\lim_{x\to0}\left[\frac{\left(e^{ax}-1\right)}{x}-\frac{\left(e^{bx}-1\right)}{x}\right]$$

$$=\lim_{x\to0}\frac{e^{ax}-1}{x}-\lim_{x\to0}\frac{e^{bx}-1}{x}$$

$$=\lim_{x\to0}\frac{e^{ax}-1}{ax}\times a-\lim_{x\to0}\frac{e^{bx}-1}{x}\times b$$

$$=1\times a-1\times b$$

$$=a-b$$

ដូចនេះ $\boxed{\lim_{x\to0}\frac{e^{ax}-e^{bx}}{x}=a-b}$

ច. $\lim_{x\to0}\frac{e^{\sin^2 2x}-\cos(\sin 3x)}{x^2}$, រាងមិនកំណត់ $\frac{0}{0}$

យើងបាន $\displaystyle\lim_{x\to0}\frac{e^{\sin^2 2x}-\cos(\sin 3x)}{x^2}=\lim_{x\to0}\frac{e^{\sin^2 2x}-1+1-\cos(\sin 3x)}{x^2}$

$$=\lim_{x\to0}\frac{\left(e^{\sin^2 2x}-1\right)+[1-\cos(\sin 3x)]}{x^2}$$

$$=\lim_{x\to0}\frac{\left(e^{\sin^2 2x}-1\right)+2\sin^2\left(\frac{\sin 3x}{2}\right)}{x^2}$$

$$=\lim_{x\to0}\left[\frac{e^{\sin^2 2x}-1}{x^2}+\frac{2\sin^2\left(\frac{\sin 3x}{2}\right)}{x^2}\right]$$

$$=\lim_{x\to0}\frac{e^{\sin^2 2x}-1}{x^2}+\lim_{x\to0}\frac{2\sin^2\left(\frac{\sin 3x}{2}\right)}{x^2}$$

យើងបាន៖

$\bullet\ \displaystyle\lim_{x\to0}\frac{e^{\sin^2 x}-1}{x^2}=\lim_{x\to0}\left[\frac{e^{\sin^2 x}-1}{\sin^2 2x}\times\frac{\sin^2 2x}{(2x)^2}\times4\right]$

$$=1\times1\times4$$

$$=4$$

$\bullet\ \displaystyle\lim_{x\to0}\frac{2\sin^2\left(\frac{\sin 3x}{2}\right)}{x^2}=\lim_{x\to0}\left[\frac{2\sin^2\left(\frac{\sin 3x}{2}\right)}{\left(\frac{\sin 3x}{2}\right)^2}\times\frac{\sin^2 3x}{(3x)^2}\times\frac{9}{2}\right]$

$$=1\times1\times\frac{9}{2}$$

$$=\frac{9}{2}$$

នាំឱ្យ $\displaystyle\lim_{x\to0}\frac{e^{\sin^2 2x}-1}{x^2}+\lim_{x\to0}\frac{2\sin^2\left(\frac{\sin 3x}{2}\right)}{x^2}=4+\frac{9}{2}$

$$=\frac{17}{2}$$

ដូចនេះ $\boxed{\lim_{x\to0}\frac{e^{\sin^2 2x}-\cos(\sin 3x)}{x^2}=\frac{17}{2}}$

ឆ. $\lim_{x\to0}\frac{e^{x^2}-\cos x}{x^2}$, រាងមិនកំណត់ $\frac{0}{0}$

យើងបាន $\displaystyle\lim_{x\to0}\frac{e^{x^2}-\cos x}{x^2}=\lim_{x\to0}\frac{e^{x^2}-1+1-\cos x}{x^2}$

$$=\lim_{x\to0}\frac{\left(e^{x^2}-1\right)+(1-\cos x)}{x^2}$$

$$=\lim_{x\to0}\frac{\left(e^{x^2}-1\right)+2\sin^2\frac{x}{2}}{x^2}$$

$$=\lim_{x\to0}\left[\frac{\left(e^{x^2}-1\right)}{x^2}+\frac{2\sin^2\frac{x}{2}}{x^2}\right]$$

$$=\lim_{x\to0}\frac{e^{x^2}-1}{x^2}+\lim_{x\to0}\frac{2\sin^2\frac{x}{2}}{x^2}$$

$$=\lim_{x\to0}\frac{e^{x^2}-1}{x^2}+2\lim_{x\to0}\left[\frac{\sin^2\frac{x}{2}}{\left(\frac{x}{2}\right)^2}\times\frac{1}{4}\right]$$

$$=1+2\times\frac{1}{4}=\frac{3}{2}$$

ដូចនេះ $\boxed{\lim_{x\to0}\frac{e^{x^2}-\cos x}{x^2}=\frac{3}{2}}$

ជ. $\lim_{x\to0}\frac{e^{x+1}-e}{\sin\pi x}$, រាងមិនកំណត់ $\frac{0}{0}$

យើងបាន $\displaystyle\lim_{x\to0}\frac{e^{x+1}-e}{\sin\pi x}=\lim_{x\to0}\frac{e\left(e^{x}-1\right)}{\sin\pi x}$

$$=e\lim_{x\to0}\frac{e^{x}-1}{\sin\pi x}$$

$$=e\lim_{x\to0}\left[\frac{e^{x}-1}{x}\times\frac{\pi x}{\sin\pi x}\times\frac{1}{\pi}\right]$$

$$=e\times1\times1\times\frac{1}{\pi}$$

$$=\frac{e}{\pi}$$

ដូចនេះ $\boxed{\lim_{x\to0}\frac{e^{x+1}-e}{\sin\pi x}=\frac{e}{\pi}}$

ឈ. $\lim_{x\to\pi}\frac{\sin 2x}{e^{x+\pi}-e^{2\pi}}$, រាងមិនកំណត់ $\frac{0}{0}$

តាង $u=x-\pi\Rightarrow x=u+\pi$

បើ $x\to0\Rightarrow u\to0$ យើងបាន ៖

យើងបាន $\lim\limits_{x\to\pi}\dfrac{\sin 2x}{e^{x+\pi}-e^{2\pi}}=\lim\limits_{u\to 0}\dfrac{\sin 2(\pi+u)}{e^{\pi+u+\pi}-e^{2\pi}}$

$$=\lim_{u\to 0}\frac{\sin(2\pi+2u)}{e^{u+2\pi}-e^{2\pi}}$$
$$=\frac{1}{e^{2\pi}}\lim_{u\to 0}\frac{\sin 2u}{e^u-1}$$
$$=\frac{1}{e^{2\pi}}\lim_{u\to 0}\left[\frac{\sin 2u}{2u}\times\frac{u}{e^u-1}\times 2\right]$$
$$=\frac{1}{e^{2\pi}}\times 1\times 1\times 2$$
$$=\frac{2}{e^{2\pi}}$$

ដូចនេះ $\boxed{\lim\limits_{x\to\pi}\frac{\sin 2x}{e^{x+\pi}-e^{2\pi}}=\frac{2}{e^{2\pi}}}$

ត. $\lim\limits_{x\to 0}\frac{e^x+\sin x-1}{1-\sqrt{x+1}}$, រាងមិនកំណត់ $\frac{0}{0}$

យើងបាន $\lim\limits_{x\to 0}\dfrac{e^x+\sin x-1}{1-\sqrt{x+1}}=\lim\limits_{x\to 0}\dfrac{(e^x-1+\sin x)\left(1+\sqrt{x+1}\right)}{\left(1-\sqrt{x+1}\right)\left(1+\sqrt{x+1}\right)}$

$$=\lim_{x\to 0}\frac{(e^x-1+\sin x)\left(1+\sqrt{x+1}\right)}{1-x-1}$$
$$=\lim_{x\to 0}\frac{(e^x-1+\sin x)\left(1+\sqrt{x+1}\right)}{1-x-1}$$
$$=-\lim_{x\to 0}\left(\frac{e^x-1}{x}+\frac{\sin x}{x}\right)\left(1+\sqrt{x+1}\right)$$
$$=-\left(\lim_{x\to 0}\frac{e^x-1}{x}+\lim_{x\to 0}\frac{\sin x}{x}\right)\times\lim_{x\to 0}\left(1+\sqrt{x+1}\right)$$
$$=-(1+1)(1+1)$$
$$=-4$$

ដូចនេះ $\boxed{\lim\limits_{x\to 0}\frac{e^x+\sin x-1}{1-\sqrt{x+1}}=-4}$

ថ. $\lim\limits_{x\to 0}\frac{(e^x-1)(1-\cos 2x)\sin 7x}{x^4}$, រាងមិនកំណត់ $\frac{0}{0}$ យើងបាន

$$\lim_{x\to 0}\frac{(e^x-1)(1-\cos 2x)\sin 7x}{x^4}=\lim_{x\to 0}\frac{(e^x-1)\cdot 2\sin^2 x\cdot\sin 7x}{x^4}$$
$$=\lim_{x\to 0}\left[\frac{e^x-1}{x}\times\frac{2\sin^2 x}{x^2}\times\frac{\sin 7x}{7x}\times 7\right]$$
$$=\lim_{x\to 0}\frac{e^x-1}{x}\times 2\lim_{x\to 0}\frac{\sin^2 x}{x^2}\times\lim_{x\to 0}\frac{\sin 7x}{7x}\times 7$$
$$=1\times 2\times 1\times 7$$
$$=14$$

ដូចនេះ $\boxed{\lim\limits_{x\to 0}\frac{(e^x-1)(1-\cos 2x)\sin 7x}{x^4}=14}$

ទ. $\lim\limits_{x\to 0}\frac{x^2}{e^{x^2}-4^{x^2}}$, រាងមិនកំណត់ $\frac{0}{0}$

យើងបាន $\lim\limits_{x\to 0}\dfrac{x^2}{e^{x^2}-4^{x^2}}=\lim\limits_{x\to 0}\dfrac{x^2}{e^{x^2}-1+1-4^{x^2}}$

$$=\lim_{x\to 0}\frac{x^2}{\left(e^{x^2}-1\right)-\left(4^{x^2}-1\right)}$$
$$=\lim_{x\to 0}\frac{\frac{x^2}{x^2}}{\frac{\left(e^{x^2}-1\right)-\left(4^{x^2}-1\right)}{x^2}}$$
$$=\lim_{x\to 0}\frac{1}{\left(\frac{e^{x^2}-1}{x^2}\right)-\left(\frac{4^{x^2}-1}{x^2}\right)}$$
$$=\frac{1}{1-\ln 4}$$

ដូចនេះ $\boxed{\lim\limits_{x\to 0}\frac{x^2}{e^{x^2}-4^{x^2}}=\frac{1}{1-\ln 4}}$

ធ. $\lim\limits_{x\to 0}\frac{e^{2x}-e^{-2x}}{x}$, រាងមិនកំណត់ $\frac{0}{0}$

យើងបាន $\lim\limits_{x\to 0}\dfrac{e^{2x}-e^{-2x}}{x}=\lim\limits_{x\to 0}\dfrac{e^{2x}-1+1-e^{-2x}}{x}$

$$=\lim_{x\to 0}\frac{\left(e^{2x}-1\right)-\left(e^{-2x}-1\right)}{x}$$
$$=\lim_{x\to 0}\left(\frac{e^{2x}-1}{x}-\frac{e^{-2x}-1}{x}\right)$$
$$=\lim_{x\to 0}\frac{e^{2x}-1}{x}-\lim_{x\to 0}\frac{e^{-2x}-1}{x}$$
$$=\lim_{x\to 0}\frac{e^{2x}-1}{2x}\times 2-\lim_{x\to 0}\frac{e^{-2x}-1}{-2x}\times(-2)$$
$$=1\times 2-(-2)$$
$$=4$$

ដូចនេះ $\boxed{\lim\limits_{x\to 0}\frac{e^{2x}-e^{-2x}}{x}=4}$

ន. $\lim\limits_{x\to 0}\frac{e^x+e^{-x}-2}{x^2}$, រាងមិនកំណត់ $\frac{0}{0}$

យើងបាន $\lim\limits_{x\to 0}\dfrac{e^x+e^{-x}-2}{x^2}=\lim\limits_{x\to 0}\dfrac{e^x+\frac{1}{e^x}-2}{x^2}$

$$=\lim_{x\to 0}\frac{e^{2x}+1-2e^x}{x^2}$$
$$=\lim_{x\to 0}\frac{(e^x-1)^2}{x^2}$$
$$=\lim_{x\to 0}\left(\frac{e^x-1}{x}\right)^2$$
$$=1^2$$
$$=1$$

ដូចនេះ $\boxed{\lim\limits_{x\to 0}\frac{e^x+e^{-x}-2}{x^2}=1}$

ប. $\lim\limits_{x\to 0}\frac{2e^{2x}-3e^x+1}{x}$, រាងមិនកំណត់ $\frac{0}{0}$

យើងបាន $\lim\limits_{x\to 0}\dfrac{2e^{2x}-3e^x+1}{x}=\lim\limits_{x\to 0}\dfrac{2e^{2x}-2e^x-e^x+1}{x}$

$$=\lim_{x\to 0}\frac{2e^x(e^x-1)-(e^x-1)}{x}$$
$$=\lim_{x\to 0}\frac{(e^x-1)(2e^x-1)}{x}$$
$$=\lim_{x\to 0}\left(\frac{e^x-1}{x}\right)(2e^x-1)$$
$$=1(2-1)$$
$$=1$$

ដូចនេះ $\boxed{\lim\limits_{x\to 0}\frac{2e^{2x}-3e^x+1}{x}=1}$

ផ. $\lim\limits_{x\to 0}\frac{e^x-e^{-x}}{\sin 2x}$, រាងមិនកំណត់ $\frac{0}{0}$

យើងបាន $\lim\limits_{x\to0}\dfrac{e^x-e^{-x}}{\sin 2x}=\lim\limits_{x\to0}\dfrac{e^x-\frac{1}{e^x}}{\sin 2x}$

$$=\lim_{x\to0}\frac{\frac{e^{2x}-1}{e^x}}{\sin 2x}$$

$$=\lim_{x\to0}\frac{e^{2x}-1}{e^x\cdot\sin 2x}$$

$$=\lim_{x\to0}\left[\frac{e^{2x}-1}{2x}\times\frac{2x}{\sin 2x}\times\frac{1}{e^x}\right]$$

$$=1\times1\times\frac{1}{1}$$

$$=1$$

ដូចនេះ $\boxed{\lim\limits_{x\to0}\frac{e^x-e^{-x}}{\sin 2x}=1}$

ព. $\lim\limits_{x\to0}\frac{2e^x+3e^{2x}-5}{e^{3x}-1}$, រាងមិនកំណត់ $\frac{0}{0}$

យើងបាន $\lim\limits_{x\to0}\dfrac{2e^x+3e^{2x}-5}{e^{3x}-1}=\lim\limits_{x\to0}\dfrac{2e^x-2+3e^x-3}{e^{3x}-1}$

$$=\lim_{x\to0}\frac{2\left(e^x-1\right)+3\left(e^{2x}-1\right)}{e^{3x}-1}$$

$$=\lim_{x\to0}\frac{\frac{2(e^x-1)+3\left(e^{2x}-1\right)}{x}}{\frac{e^{3x}-1}{x}}$$

$$=\lim_{x\to0}\frac{\frac{2(e^x-1)}{x}+\frac{\left(e^{2x}-1\right)}{2x}\times6}{\frac{e^{3x}-1}{3x}\times3}$$

$$=\frac{2+6}{1\times3}$$

$$=\frac{8}{3}$$

ដូចនេះ $\boxed{\lim\limits_{x\to0}\frac{2e^x+3e^{2x}-5}{e^{3x}-1}=\frac{8}{3}}$

ភ. $\lim\limits_{x\to0}\frac{e^{-x^2}-\cos 2x}{x^2}$, រាងមិនកំណត់ $\frac{0}{0}$

យើងបាន $\lim\limits_{x\to0}\dfrac{e^{-x^2}-\cos 2x}{x^2}=\lim\limits_{x\to0}\dfrac{e^{-x^2}-\left(1-2\sin^2 x\right)}{x^2}$

$$=\lim_{x\to0}\frac{e^{-x^2}-1+2\sin^2 x}{x^2}$$

$$=\lim_{x\to0}\left(\frac{e^{-x^2}-1}{x^2}+\frac{2\sin^2 x}{x^2}\right)$$

$$=-\lim_{x\to0}\frac{e^{-x^2}-1}{-x^2}+2\lim_{x\to0}\frac{\sin^2 x}{x^2}$$

$$=-1+2$$

$$=1$$

ដូចនេះ $\boxed{\lim\limits_{x\to0}\frac{e^{-x^2}-\cos 2x}{x^2}=1}$

ម. $\lim\limits_{x\to0}\frac{e^{-2\sin x}-e^{\tan 3x}}{x^3+x}$, រាងមិនកំណត់ $\frac{0}{0}$

យើងបាន $\lim\limits_{x\to0}\dfrac{e^{-2\sin x}-e^{\tan 3x}}{x^3+x}=\lim\limits_{x\to0}\dfrac{e^{-2\sin x}-1+1-e^{\tan 3x}}{x\left(x^2+1\right)}$

$$=\lim_{x\to0}\frac{\left(e^{-2\sin x}-1\right)-\left(e^{\tan 3x}-1\right)}{x\left(x^2+1\right)}$$

$$=\lim_{x\to0}\frac{\left(e^{-2\sin x}-1\right)-\left(e^{\tan 3x}-1\right)}{x\left(x^2+1\right)}$$

$$=\lim_{x\to0}\left[\frac{e^{-2\sin x}-1}{x\left(x^2+1\right)}-\frac{e^{\tan 3x}-1}{x\left(x^2+1\right)}\right]$$

$$=\lim_{x\to0}\frac{e^{-2\sin x}-1}{x\left(x^2+1\right)}-\lim_{x\to0}\frac{e^{\tan 3x}-1}{x\left(x^2+1\right)}$$

យើងបាន

$\bullet\ \lim\limits_{x\to0}\dfrac{e^{2\sin x}-1}{x\left(x^2-1\right)}=\lim\limits_{x\to0}\left[\dfrac{e^{2\sin x}-1}{-2\sin x}\times\dfrac{\sin x}{x}\times\dfrac{-2}{x^2+1}\right]$

$$=1\times1\times\frac{-2}{1}$$

$$=-2$$

$\bullet\ \lim\limits_{x\to0}\dfrac{e^{\tan x}-1}{x\left(x^2-1\right)}=\lim\limits_{x\to0}\left[\dfrac{e^{\tan x}-1}{\tan x}\times\dfrac{\tan 3x}{3x}\times\dfrac{3}{x^2-1}\right]$

$$=1\times1\times\frac{3}{1}$$

$$=3$$

នាំឱ្យ $\lim\limits_{x\to0}\dfrac{e^{-2\sin x}-1}{x\left(x^2+1\right)}-\lim\limits_{x\to0}\dfrac{e^{\tan 3x}-1}{x\left(x^2+1\right)}=-2-3$

$$=-5$$

ដូចនេះ $\boxed{\lim\limits_{x\to0}\frac{e^{-2\sin x}-e^{\tan 3x}}{x^3+x}=-5}$

យ. $\lim\limits_{x\to0}\frac{xe^{-2x^2}+\sin x-\tan x-x}{x^3}$, រាងមិនកំណត់ $\frac{0}{0}$ យើងបាន

$\lim\limits_{x\to0}\dfrac{xe^{-2x^2}+\sin x-\tan x-x}{x^3}=\lim\limits_{x\to0}\dfrac{xe^{-2x^2}-x+\cos x\tan x-\tan x}{x^3}$

$$=\lim_{x\to0}\frac{x\left(e^{-2x^2}-1\right)+\tan x\left(\cos x-1\right)}{x^3}$$

$$=\lim_{x\to0}\left[\frac{x\left(e^{-2x^2}-1\right)}{x^3}+\frac{\tan x\left(\cos x-1\right)}{x^3}\right]$$

$$=\lim_{x\to0}\frac{x\left(e^{-2x^2}-1\right)}{x^3}+\lim_{x\to0}\frac{\tan x\left(\cos x-1\right)}{x^3}$$

$$=\lim_{x\to0}\frac{e^{-2x^2}-1}{x^2}+\lim_{x\to0}\frac{-\tan x\left(1-\cos x\right)}{x^3}$$

$$=\lim_{x\to0}\frac{e^{-2x^2}-1}{-2x^2}\times(-2)-\lim_{x\to0}\frac{\tan x}{x}\times\frac{2\sin^2\frac{x}{2}}{x^2}$$

$$=-2\lim_{x\to0}\frac{e^{-2x^2}-1}{-2x^2}-\frac{1}{2}\lim_{x\to0}\frac{\tan x}{x}\times\frac{\sin^2\frac{x}{2}}{\left(\frac{x}{2}\right)^2}$$

$$=-2-\frac{1}{2}$$

$$=-\frac{5}{2}$$

ដូចនេះ $\boxed{\lim\limits_{x\to0}\frac{xe^{-2x^2}+\sin x-\tan x-x}{x^3}=-\frac{5}{2}}$

រ. $\lim\limits_{x\to0}\frac{e^{3\sin^2 x}-\cos x\cos 3x}{-e^{2x^2}+\cos 2x}$, រាងមិនកំណត់ $\frac{0}{0}$

ដូចនេះ $\boxed{\lim\limits_{x\to0}\frac{e^{3\sin^2x}-\cos x\cos 3x}{-e^{2x^2}+\cos 2x}=-2}$

ល. $\lim\limits_{x\to0}\frac{4^x+2^x-2}{x}$, រាងមិនកំណត់ $\frac{0}{0}$

យើងបាន $$\begin{aligned}\lim_{x\to0}\frac{4^x+2^x-2}{x}&=\lim_{x\to0}\frac{4^x-1+2^x-1}{x}\\&=\lim_{x\to0}\frac{(4^x-1)+(2^x-1)}{x}\\&=\lim_{x\to0}\left(\frac{4^x-1}{x}+\frac{2^x-1}{x}\right)\\&=\lim_{x\to0}\frac{4^x-1}{x}+\lim_{x\to0}\frac{2^x-1}{x}\\&=\lim_{x\to0}\frac{4^x-1}{x}+\lim_{x\to0}\frac{2^x-1}{x}\\&=\ln4+\ln2\end{aligned}$$

ដូចនេះ $\boxed{\lim\limits_{x\to0}\frac{4^x+2^x-2}{x}=\ln4+\ln2}$

វ. $\lim\limits_{x\to0}\frac{7^{x^2}-\cos 2x}{x^2}$, រាងមិនកំណត់ $\frac{0}{0}$

យើងបាន $$\begin{aligned}\lim_{x\to0}\frac{7^{x^2}-\cos 2x}{x^2}&=\lim_{x\to0}\frac{7^{x^2}-1+1-\cos 2x}{x^2}\\&=\lim_{x\to0}\frac{\left(7^{x^2}-1\right)+2\sin^2x}{x^2}\\&=\lim_{x\to0}\left(\frac{7^{x^2}-1}{x^2}+\frac{2\sin^2x}{x^2}\right)\\&=\lim_{x\to0}\frac{7^{x^2}-1}{x^2}+\lim_{x\to0}\frac{2\sin^2x}{x^2}\\&=\lim_{x\to0}\frac{7^{x^2}-1}{x^2}+2\lim_{x\to0}\frac{\sin^2x}{x^2}\\&=\ln7+2\end{aligned}$$

ដូចនេះ $\boxed{\lim\limits_{x\to0}\frac{7^{x^2}-\cos 2x}{x^2}=\ln7+2}$

ស. $\lim\limits_{x\to0}\frac{2^{\sin x}-3^{\tan x}}{x}$, រាងមិនកំណត់ $\frac{0}{0}$

យើងបាន $$\begin{aligned}\lim_{x\to0}\frac{2^{\sin x}-3^{\tan x}}{x}&=\lim_{x\to0}\frac{2^{\sin x}-1+1-3^{\tan x}}{x}\\&=\lim_{x\to0}\frac{\left(2^{\sin x}-1\right)-\left(3^{\tan x}-1\right)}{x}\\&=\lim_{x\to0}\left(\frac{2^{\sin x}-1}{x}-\frac{3^{\tan x}-1}{x}\right)\\&=\lim_{x\to0}\frac{2^{\sin x}-1}{\sin x}\times\frac{\sin x}{x}-\lim_{x\to0}\frac{3^{\tan x}-1}{\tan x}\times\frac{\tan x}{x}\\&=\ln2\times1-\ln3\times1\\&=\ln2-\ln3\\&=\ln\frac{2}{3}\end{aligned}$$

ដូចនេះ $\boxed{\lim\limits_{x\to0}\frac{2^{\sin x}-3^{\tan x}}{x}=\ln\frac{2}{3}}$

លំហាត់គំរូទី២១

ដោះស្រាយលីមីតខាងក្រោម៖

ក. $\lim\limits_{x\to+\infty}\left(\frac{x+1}{x}\right)^{x+5}$ ខ. $\lim\limits_{x\to0^+}(1+2x)^{\cot x}$ គ. $\lim\limits_{x\to0}(1+\sin x)^{\frac{1}{x}}$

ឃ. $\lim\limits_{x\to\infty}\left(1+\frac{3}{x}\right)^{x}$ ង. $\lim\limits_{x\to\infty}\left(\frac{x-2}{x+3}\right)^{x}$ ច. $\lim\limits_{x\to\infty}\left(1+\frac{1}{x}\right)^{\frac{x+1}{x}}$

ឆ. $\lim\limits_{x\to\infty}\left(\frac{3x-4}{3x+2}\right)^{\frac{x+1}{3}}$ ជ. $\lim\limits_{x\to\infty}\left(1+\frac{2}{x}\right)^{x}$ ឈ. $\lim\limits_{x\to0}(\cos x)^{\frac{1}{\sin x}}$

ញ. $\lim\limits_{x\to0}\sqrt[\sin x]{1+x}$ ដ. $\lim\limits_{x\to0}(x+2^x)^{\frac{1}{x}}$ ឋ. $\lim\limits_{x\to0}\left(1+\tan^2(\sqrt{x})\right)^{\frac{1}{2x}}$

ឌ. $\lim\limits_{x\to0}\left(\frac{x^2-x+1}{x^2+x+1}\right)^{\frac{1}{\sin x}}$ ឍ. $\lim\limits_{x\to0}\left(\frac{\sin x}{x}\right)^{\frac{\tan x}{x-\sin x}}$ ណ. $\lim\limits_{x\to0}(2-\cos x)^{\frac{1}{x^2}}$

ដំណោះស្រាយ

ដោះស្រាយលីមីតខាងក្រោម៖

ក. $\lim\limits_{x\to+\infty}\left(\frac{x+1}{x}\right)^{x+5}$, រាងមិនកំណត់ 1^∞

យើងបាន $\displaystyle\lim_{x\to+\infty}\left(\frac{x+1}{x}\right)^{x+5} = \lim_{x\to+\infty}\left(\frac{x}{x}+\frac{1}{x}\right)^{x+5}$

$$= \lim_{x\to+\infty}\left(1+\frac{1}{x}\right)^{x+5}$$

$$= \lim_{x\to+\infty}\left[\left(1+\frac{1}{x}\right)^{x}\right]^{\frac{1}{x}(x+5)}$$

$$= \lim_{x\to+\infty}\left[\left(1+\frac{1}{x}\right)^{x}\right]^{1+\frac{5}{x}}$$

$$= e^{1+0}$$

$$= e$$

ដូចនេះ $\boxed{\lim\limits_{x\to+\infty}\left(\frac{x+1}{x}\right)^{x+5} = e}$

ខ. $\lim\limits_{x\to0^+}(1+2x)^{\cot x}$, រាងមិនកំណត់ 1^∞

យើងបាន $\displaystyle\lim_{x\to0^+}(1+2x)^{\cot x} = \lim_{x\to0^+}\left[(1+2x)^{\frac{1}{2x}}\right]^{2x\times\cot x}$

$$= \lim_{x\to0^+}\left[(1+2x)^{\frac{1}{2x}}\right]^{2x\times\frac{\cos x}{\sin x}}$$

$$= \lim_{x\to0^+}\left[(1+2x)^{\frac{1}{2x}}\right]^{2\cos x\times\frac{x}{\sin x}}$$

$$= e^{2\cos x\lim\limits_{x\to0^+}\frac{x}{\sin x}}$$

$$= e^{2\times1}$$

$$= e$$

ដូចនេះ $\boxed{\lim\limits_{x\to0^+}(1+2x)^{\cot x} = e^2}$

គ. $\lim\limits_{x\to0}(1+\sin x)^{\frac{1}{x}}$, រាងមិនកំណត់ 1^∞

យើងបាន $\displaystyle\lim_{x\to0}(1+\sin x)^{\frac{1}{x}} = \lim_{x\to0^+}\left[(1+\sin x)^{\frac{1}{\sin x}}\right]^{\frac{\sin x}{x}}$

$$= e^{\lim\limits_{x\to0^+}\frac{\sin x}{x}}$$

$$= e$$

ដូចនេះ $\boxed{\lim\limits_{x\to0}(1+\sin x)^{\frac{1}{x}} = e}$

ឃ. $\lim\limits_{x\to\infty}\left(1+\frac{3}{x}\right)^{x}$, រាងមិនកំណត់ 1^∞

យើងបាន $\displaystyle\lim_{x\to\infty}\left(1+\frac{3}{x}\right)^{x} = \lim_{x\to\infty}\left[\left(1+\frac{3}{x}\right)^{\frac{x}{3}}\right]^{\frac{3}{x}\times x}$

$$= \lim_{x\to\infty}\left[\left(1+\frac{3}{x}\right)^{\frac{x}{3}}\right]^{3}$$

$$= e^3$$

ដូចនេះ $\boxed{\lim\limits_{x\to\infty}\left(1+\frac{3}{x}\right)^{x} = e^3}$

ង. $\lim\limits_{x\to\infty}\left(\frac{x-2}{x+3}\right)^{x}$, រាងមិនកំណត់ 1^∞

យើងបាន $\displaystyle\lim_{x\to\infty}\left(\frac{x-2}{x+3}\right)^{x} = \lim_{x\to\infty}\left(1-\frac{5}{x+3}\right)^{x}$

$$= \lim_{x\to\infty}\left[\left(1-\frac{5}{x+3}\right)^{-\frac{x+3}{5}}\right]^{\left(\frac{\frac{-5x}{x}}{\frac{x+3}{x}}\right)}$$

$$= e^{\lim\limits_{x\to\infty}\left(\frac{\frac{-5x}{x}}{\frac{x+3}{x}}\right)}$$

$$= e^{\lim\limits_{x\to\infty}\left(\frac{-5}{1+\frac{3}{x}}\right)} = e^{-5}$$

ដូចនេះ $\boxed{\lim\limits_{x\to\infty}\left(\frac{x-2}{x+3}\right)^{x} = e^{-5}}$

ច. $\lim\limits_{x\to\infty}\left(1+\frac{1}{x}\right)^{\frac{x+1}{x}}$, រាងមិនកំណត់ 1^∞

យើងបាន $\displaystyle\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^{\frac{x+1}{x}} = \lim_{x\to\infty}\left[\left(1+\frac{1}{x}\right)^{x}\right]^{\frac{1}{x}\times\frac{x+1}{x}}$

$$= \lim_{x\to\infty}\left[\left(1+\frac{1}{x}\right)^{x}\right]^{\frac{x+1}{x^2}}$$

$$= e^{\lim\limits_{x\to\infty}\frac{x+1}{x^2}}$$

$$= e^0$$

$$= 1$$

ដូចនេះ $\boxed{\lim\limits_{x\to\infty}\left(1+\frac{1}{x}\right)^{\frac{x+1}{x}} = 1}$

ឆ. $\lim\limits_{x\to\infty}\left(\frac{3x-4}{3x+2}\right)^{\frac{x+1}{3}}$, រាងមិនកំណត់ 1^∞

យើងបាន $\displaystyle\lim_{x\to\infty}\left(\frac{3x-4}{3x+2}\right)^{\frac{x+1}{3}} = \lim_{x\to\infty}\left(\frac{3x+2-6}{3x+2}\right)^{\frac{x+1}{3}}$

$$= \lim_{x\to\infty}\left(1-\frac{6}{3x+2}\right)^{\frac{x+1}{3}}$$

$$= \lim_{x\to\infty}\left[\left(1-\frac{6}{3x+2}\right)^{\frac{3x-2}{-6}}\right]^{\left(\frac{-6}{3x+2}\right)\left(\frac{x+1}{3}\right)}$$

$$= e^{\lim\limits_{x\to\infty}\left(\frac{-6}{3x+2}\right)\left(\frac{x+1}{3}\right)}$$

$$= e^{\lim\limits_{x\to\infty}\frac{-2x-2}{3x+2}}$$

$$= e^{-\frac{2}{3}}$$

ដូចនេះ $\boxed{\lim\limits_{x\to\infty}\left(\frac{3x-4}{3x+2}\right)^{\frac{x+1}{3}} = e^{-\frac{2}{3}}}$

ជ. $\lim\limits_{x\to\infty}\left(1+\frac{2}{x}\right)^{x}$, រាងមិនកំណត់ 1^∞

យើងបាន $\displaystyle\lim_{x\to\infty}\left(1+\frac{2}{x}\right)^{x} = \lim_{x\to\infty}\left[\left(1+\frac{2}{x}\right)^{\frac{x}{2}}\right]^{\frac{2}{x}\cdot x}$

$$= \lim_{x\to\infty}\left[\left(1+\frac{2}{x}\right)^{\frac{x}{2}}\right]^{2}$$

$$= e^2$$

ដូចនេះ $\boxed{\lim\limits_{x\to\infty}\left(1+\frac{2}{x}\right)^{x} = e^2}$

ឈ. $\lim\limits_{x\to0}(\cos x)^{\frac{1}{\sin x}}$, រាងមិនកំណត់ 1^∞

យើងបាន $\lim\limits_{x\to0}(\cos x)^{\frac{1}{\sin x}} = \lim\limits_{x\to0}\left\{[1-(1-\cos x)]^{\frac{1}{-(1-\cos x)}}\right\}^{-(1-\cos x)\left(\frac{1}{\sin x}\right)}$

$$= \lim_{x\to0}\left\{[1-(1-\cos x)]^{\frac{1}{-(1-\cos x)}}\right\}^{-2\sin^2\frac{x}{2}\left(\frac{1}{\sin x}\right)}$$
$$= e^{\lim\limits_{x\to0} -2\sin^2\frac{x}{2}\left(\frac{1}{\sin x}\right)}$$
$$= e^{\lim\limits_{x\to0} \frac{-2\sin^2\frac{x}{2}}{2\sin\frac{x}{2}\cos\frac{x}{2}}}$$
$$= e^{\lim\limits_{x\to0} \frac{-\sin\frac{x}{2}}{\cos\frac{x}{2}}}$$
$$= e^{-\lim\limits_{x\to0}\tan\frac{x}{2}}$$
$$= e^0$$
$$= 1$$

ដូចនេះ $\boxed{\lim\limits_{x\to0}(\cos x)^{\frac{1}{\sin x}} = 1}$

ញ. $\lim\limits_{x\to0}\sqrt[\sin x]{1+x} = \lim\limits_{x\to0}(1+x)^{\frac{1}{\sin x}}$, រាងមិនកំណត់ 1^∞

យើងបាន $\lim\limits_{x\to0}(1+x)^{\frac{1}{\sin x}} = \lim\limits_{x\to0}\left[(1+x)^{\frac{1}{x}}\right]^{x\cdot\frac{1}{\sin x}}$

$$= \lim_{x\to0}\left[(1+x)^{\frac{1}{x}}\right]^{\frac{x}{\sin x}}$$
$$= e^{\lim\limits_{x\to0}\frac{x}{\sin x}}$$
$$= e^1$$
$$= e$$

ដូចនេះ $\boxed{\lim\limits_{x\to0}\sqrt[\sin x]{1+x} = e}$

ដ. $\lim\limits_{x\to0}(x+2^x)^{\frac{1}{x}}$, រាងមិនកំណត់ 1^∞

យើងបាន $\lim\limits_{x\to0}(x+2^x)^{\frac{1}{x}} = \lim\limits_{x\to0}\left\{[1+(x+2^x-1)]^{\frac{1}{x+2^x-1}}\right\}^{(x+2^x-1)\cdot\frac{1}{x}}$

$$= e^{\lim\limits_{x\to0}(x+2^x-1)\cdot\frac{1}{x}}$$
$$= e^{\lim\limits_{x\to0}\left(1+\frac{2^x-1}{x}\right)}$$
$$= e^{\left(\lim\limits_{x\to0}1+\lim\limits_{x\to0}\frac{2^x-1}{x}\right)}$$
$$= e^{1+\ln2}$$
$$= e\cdot e^{\ln2}$$
$$= e\cdot2$$
$$= 2e$$

ដូចនេះ $\boxed{\lim\limits_{x\to0}(x+2^x)^{\frac{1}{x}} = 2e}$

ឋ. $\lim\limits_{x\to0}\left(1+\tan^2(\sqrt{x})\right)^{\frac{1}{2x}}$, រាងមិនកំណត់ 1^∞

យើងបាន $\lim\limits_{x\to0}\left(1+\tan^2(\sqrt{x})\right)^{\frac{1}{2x}} = \lim\limits_{x\to0}\left[\left(1+\tan^2\sqrt{x}\right)^{\frac{1}{\tan^2\sqrt{x}}}\right]^{\tan^2\sqrt{x}\cdot\frac{1}{2x}}$

$$= e^{\lim\limits_{x\to0}\tan^2\sqrt{x}\cdot\frac{1}{2x}}$$
$$= e^{\lim\limits_{x\to0}\frac{\tan^2\sqrt{x}}{2x}}$$
$$= e^{\lim\limits_{x\to0}\frac{\tan^2\sqrt{x}}{(\sqrt{x})^2}\times\frac{1}{2}}$$
$$= e^{1\times\frac{1}{2}}$$
$$= e^{\frac{1}{2}}$$
$$= \sqrt{e}$$

ដូចនេះ $\boxed{\lim\limits_{x\to0}\left(1+\tan^2\sqrt{x}\right)^{\frac{1}{2x}} = \sqrt{e}}$

ឌ. $\lim\limits_{x\to0}\left(\frac{x^2-x+1}{x^2+x+1}\right)^{\frac{1}{\sin x}}$, រាងមិនកំណត់ 1^∞

យើងបាន $\lim\limits_{x\to0}\left(\frac{x^2-x+1}{x^2+x+1}\right)^{\frac{1}{\sin x}} = \lim\limits_{x\to0}\left(\frac{x^2+x+1-2x}{x^2+x+1}\right)^{\frac{1}{\sin x}}$

$$= \lim_{x\to0}\left[\left(1-\frac{2x}{x^2+x+1}\right)^{\frac{x^2+x+1}{-2x}}\right]^{\left(\frac{-2x}{x^2+x+1}\right)\cdot\frac{1}{\sin x}}$$
$$= e^{\lim\limits_{x\to0}\left(\frac{-2x}{x^2+x+1}\right)\cdot\frac{1}{\sin x}}$$
$$= e^{\lim\limits_{x\to0}\frac{1}{\sin x}\cdot\left(\frac{-2x}{x^2+x+1}\right)}$$
$$= e^{1\times(-2)}$$
$$= e^{-2}$$

ដូចនេះ $\boxed{\lim\limits_{x\to0}\left(\frac{x^2-x+1}{x^2+x+1}\right)^{\frac{1}{\sin x}} = e^{-2}}$

ឍ. $\lim\limits_{x\to0}\left(\frac{\sin x}{x}\right)^{\frac{\tan x}{x-\sin x}}$, រាងមិនកំណត់ 1^∞

យើងបាន $\lim\limits_{x\to0}\left(\frac{\sin x}{x}\right)^{\frac{\tan x}{x-\sin x}} = \lim\limits_{x\to0}\left(1-1+\frac{\sin x}{x}\right)^{\frac{\tan x}{x-\sin x}}$

$$= \lim_{x\to0}\left[1-\left(1-\frac{\sin x}{x}\right)\right]^{\frac{\tan x}{x-\sin x}}$$
$$= \lim_{x\to0}\left[1-\left(\frac{x-\sin x}{x}\right)\right]^{\frac{\tan x}{x-\sin x}}$$
$$= \lim_{x\to0}\left\{\left[1-\left(\frac{x-\sin x}{x}\right)\right]^{\frac{-x}{x-\sin x}}\right\}^{\frac{x-\sin x}{-x}\cdot\frac{\tan x}{x-\sin x}}$$
$$= e^{\lim\limits_{x\to0}\frac{x-\sin x}{-x}\cdot\frac{\tan x}{x-\sin x}}$$
$$= e^{-\lim\limits_{x\to0}\frac{\tan x}{x}}$$
$$= e^{-1}$$

ដូចនេះ $\boxed{\lim\limits_{x\to0}\left(\frac{\sin x}{x}\right)^{\frac{\tan x}{x-\sin x}} = e^{-1}}$

ណ. $\lim\limits_{x\to0}(2-\cos x)^{\frac{1}{x^2}}$, រាងមិនកំណត់ 1^∞

យើងបាន $\lim\limits_{x\to0}(2-\cos x)^{\frac{1}{x^2}} = \lim\limits_{x\to0}[1+(1-\cos 2x)]^{\frac{1}{x^2}}$

$$= \lim_{x\to0}\left(1+2\sin^2 x\right)^{\frac{1}{x^2}}$$
$$= \lim_{x\to0}\left[\left(1+2\sin^2 x\right)^{\frac{1}{2\sin^2 x}}\right]^{2\sin^2 x\cdot\frac{1}{x^2}}$$
$$= e^{2\lim\limits_{x\to0}\frac{\sin^2 x}{x^2}}$$
$$= e^{2\times1}$$
$$= e^2$$

ដូចនេះ $\boxed{\lim\limits_{x\to0}(2-\cos x)^{\frac{1}{x^2}} = e^2}$

មេរៀនទី ៣ លំហាត់ និង ដំណោះស្រាយ

លំហាត់គំរូទី២២

ដោះស្រាយលីមីតខាងក្រោម៖

ក. $\lim\limits_{x\to+\infty}\frac{\ln x}{x^2-x}$ ខ. $\lim\limits_{x\to-\infty}\ln(e^x+1)$ គ. $\lim\limits_{x\to0^+}\ln\left(\frac{x}{x+1}\right)$

ឃ. $\lim\limits_{x\to0^+}\frac{\ln^2x+2\ln x+3}{\ln x+2}$ ង. $\lim\limits_{x\to+\infty}\ln\left(\frac{x^2+x-2}{x^2+1}\right)$ ច. $\lim\limits_{x\to+\infty}\frac{3x\ln x}{x+2}$

ឆ. $\lim\limits_{x\to+\infty}\frac{\ln x^{2020}-2020}{x^{2020}}$ ជ. $\lim\limits_{x\to0}\frac{\sin 3x-\sin x}{\ln(x+1)}$ ឈ. $\lim\limits_{x\to0}\frac{x-\ln(1+3x)}{x+\sin 3x}$

ញ. $\lim\limits_{x\to1}\frac{x^2-1+\ln x}{e^x-e}$ ដ. $\lim\limits_{x\to0}\frac{\ln(1+2x^2)}{1-\cos 2x}$ ឋ. $\lim\limits_{x\to0}\frac{\ln(\cos 2x)}{x^2}$

ឌ. $\lim\limits_{x\to0}\frac{\ln(x^2-x+1)}{x}$ ឍ. $\lim\limits_{x\to0}\frac{\ln(1+x)+\ln(1+2x)}{x}$ ណ. $\lim\limits_{x\to0}\frac{\ln(1+2\sin^2x)}{3\tan^2x}$

ដំណោះស្រាយ

ដោះស្រាយលីមីតខាងក្រោម៖

ក. $\lim\limits_{x\to+\infty}\frac{\ln x}{x^2-x}$, រាងមិនកំណត់ $\frac{\infty}{\infty}$

យើងបាន
$$\begin{aligned}\lim_{x\to+\infty}\frac{\ln x}{x^2-x}&=\lim_{x\to+\infty}\frac{\ln x}{x(x-1)}\\&=\lim_{x\to+\infty}\left[\frac{\ln x}{x}\cdot\frac{1}{(x-1)}\right]\\&=\lim_{x\to+\infty}\frac{\ln x}{x}\times\lim_{x\to+\infty}\frac{1}{x-1}\\&=0\times0\\&=0\end{aligned}$$

ដូចនេះ $\boxed{\lim\limits_{x\to+\infty}\frac{\ln x}{x^2-x}=0}$

ខ. $\lim\limits_{x\to-\infty}\ln(e^x+1)$

យើងបាន
$$\begin{aligned}\lim_{x\to-\infty}\ln(e^x+1)&=\ln(e^{-\infty}+1)\\&=\ln(0+1)\\&=\ln 1\\&=0\end{aligned}$$

ដូចនេះ $\boxed{\lim\limits_{x\to-\infty}\ln(e^x+1)=0}$

គ. $\lim\limits_{x\to0^+}\ln\left(\frac{x}{x+1}\right)$

យើងបាន
$$\begin{aligned}\lim_{x\to0^+}\ln\left(\frac{x}{x+1}\right)&=\lim_{x\to0^+}\ln\left[\frac{x}{x\left(1+\frac{1}{x}\right)}\right]\\&=\lim_{x\to0^+}\ln\frac{1}{1+\frac{1}{x}}\\&=\ln 0^+\\&=-\infty\end{aligned}$$

ដូចនេះ $\boxed{\lim\limits_{x\to0^+}\ln\left(\frac{x}{x+1}\right)=-\infty}$

ឃ. $\lim\limits_{x\to0^+}\frac{\ln^2x+2\ln x+3}{\ln x+2}$, រាងមិនកំណត់ $\frac{\infty}{\infty}$

យើងបាន
$$\begin{aligned}\lim_{x\to0^+}\frac{\ln^2x+2\ln x+3}{\ln x+2}&=\lim_{x\to0^+}\frac{\ln x\left(\ln x+2+\frac{3}{\ln x}\right)}{\ln x\left(1+\frac{2}{\ln x}\right)}\\&=\lim_{x\to0^+}\frac{\ln x+2+\frac{3}{\ln x}}{1+\frac{2}{\ln x}}\\&=\frac{-\infty+2+0}{1+0}\\&=-\infty\end{aligned}$$

ដូចនេះ $\boxed{\lim\limits_{x\to0^+}\frac{\ln^2x+2\ln x+3}{\ln x+2}=-\infty}$

ង. $\lim\limits_{x\to+\infty}\ln\left(\frac{x^2+x-2}{x^2+1}\right)$

យើងមាន

- $x^2+x-2=x^2\left(1+\frac{1}{x}-\frac{2}{x^2}\right)$
- $x^2+1=x^2\left(1+\frac{1}{x^2}\right)$

យើងបាន
$$\begin{aligned}\lim_{x\to+\infty}\ln\left(\frac{x^2+x-2}{x^2+1}\right)&=\lim_{x\to+\infty}\ln\left[\frac{x^2\left(1+\frac{1}{x}-\frac{2}{x^2}\right)}{x^2\left(1+\frac{1}{x^2}\right)}\right]\\&=\lim_{x\to+\infty}\ln\frac{1+\frac{1}{x}-\frac{2}{x^2}}{1+\frac{1}{x^2}}\\&=\ln\frac{1+0-0}{1+0}\\&=\ln 1\\&=0\end{aligned}$$

ដូចនេះ $\boxed{\lim\limits_{x\to+\infty}\ln\left(\frac{x^2+x-2}{x^2+1}\right)=0}$

ច. $\lim\limits_{x\to+\infty}\frac{3x\ln x}{x+2}$, រាងមិនកំណត់ $\frac{\infty}{\infty}$

យើងបាន $\lim\limits_{x\to+\infty}\dfrac{3x\ln x}{x+2}=\lim\limits_{x\to+\infty}\dfrac{3x\ln x}{x\left(1+\frac{2}{x}\right)}$

$=\lim\limits_{x\to+\infty}\dfrac{3\ln x}{1+\frac{2}{x}}$

$=\lim\limits_{x\to+\infty}\dfrac{3(+\infty)}{1+0}$

$=+\infty$

ដូចនេះ $\boxed{\lim\limits_{x\to+\infty}\frac{3x\ln x}{x+2}=+\infty}$

ឆ. $\lim\limits_{x\to+\infty}\frac{\ln x^{2020}-2020}{x^{2020}}$, រាងមិនកំណត់ $\frac{\infty}{\infty}$

យើងបាន $\lim\limits_{x\to+\infty}\dfrac{\ln x^{2020}-2020}{x^{2020}}=\lim\limits_{x\to+\infty}\dfrac{2020(\ln x-1)}{x^{2020}}$

$=2020\lim\limits_{x\to+\infty}\left(\dfrac{\ln x}{x^{2020}}-\dfrac{1}{x^{2020}}\right)$

$=2020(0-0)$

$=0$

ដូចនេះ $\boxed{\lim\limits_{x\to+\infty}\frac{\ln x^{2020}-2020}{x^{2020}}=0}$

ជ. $\lim\limits_{x\to0}\frac{\sin 3x-\sin x}{\ln(x+1)}$, រាងមិនកំណត់ $\frac{0}{0}$

យើងបាន $\lim\limits_{x\to0}\dfrac{\sin 3x-\sin x}{\ln(x+1)}=\lim\limits_{x\to0}\dfrac{\frac{\sin 3x-\sin x}{x}}{\frac{\ln(x+1)}{x}}$

$=\lim\limits_{x\to0}\dfrac{\frac{\sin 3x}{x}-\frac{\sin x}{x}}{\frac{1}{x}\cdot\ln(x+1)}$

$=\lim\limits_{x\to0}\dfrac{\frac{\sin 3x}{x}-\frac{\sin x}{x}}{\ln(x+1)^{\frac{1}{x}}}$

$=\lim\limits_{x\to0}\dfrac{\frac{\sin 3x}{3x}\times3-\frac{\sin x}{x}}{\ln(x+1)^{\frac{1}{x}}}$

$=\dfrac{1\times3-1}{\ln e}$

$=\dfrac{1\times3-1}{1}$

$=2$

ដូចនេះ $\boxed{\lim\limits_{x\to0}\frac{\sin 3x-\sin x}{\ln(x+1)}=2}$

ឈ. $\lim\limits_{x\to0}\frac{x-\ln(1+3x)}{x+\sin 3x}$, រាងមិនកំណត់ $\frac{\infty}{\infty}$

របៀបទី១

យើងបាន $\lim\limits_{x\to0}\dfrac{x-\ln(1+3x)}{x+\sin 3x}=\lim\limits_{x\to0}\dfrac{\frac{x-\ln(1+3x)}{x}}{\frac{x+\sin 3x}{x}}$

$=\lim\limits_{x\to0}\dfrac{\frac{x}{x}-\frac{\ln(1+3x)}{x}}{\frac{x}{x}+\frac{\sin 3x}{x}}$

$=\lim\limits_{x\to0}\dfrac{1-\frac{\ln(1+3x)}{x}}{1+\frac{\sin 3x}{x}}$

$=\lim\limits_{x\to0}\dfrac{1-\frac{1}{x}\cdot\ln(1+3x)}{1+\frac{\sin 3x}{x}}$

$=\lim\limits_{x\to0}\dfrac{1-\ln(1+3x)^{\frac{1}{x}}}{1+\frac{\sin 3x}{x}}$

$=\lim\limits_{x\to0}\dfrac{1-\ln\left[(1+3x)^{\frac{1}{3x}}\right]^{3x\cdot\frac{1}{x}}}{1+\frac{\sin 3x}{x}}$

$=\lim\limits_{x\to0}\dfrac{1-\ln\left[(1+3x)^{\frac{1}{3x}}\right]^{3x\cdot\frac{1}{x}}}{1+\frac{\sin 3x}{3x}\times3}$

$=\lim\limits_{x\to0}\dfrac{1-\ln\left[(1+3x)^{\frac{1}{3x}}\right]^{3}}{1+\frac{\sin 3x}{3x}\times3}$

$=\dfrac{1-\ln e^3}{1+1\times3}$

$=\dfrac{1-3}{4}$

$=-\dfrac{1}{2}$

ដូចនេះ $\boxed{\lim\limits_{x\to0}\frac{x-\ln(1+3x)}{x+\sin 3x}=-\frac{1}{2}}$

របៀបទី២

តាង $u=x$ បើ $x\to0\Rightarrow u\to0$

យើងបាន $\lim\limits_{x\to0}\dfrac{x-\ln(1+3x)}{x+\sin 3x}=\lim\limits_{u\to0}\dfrac{u-\ln(1+3u)}{u+\sin 3u}$

$=\lim\limits_{u\to0}\dfrac{\frac{u-\ln(1+3u)}{3u}}{\frac{u+\sin 3u}{3u}}$

$=\dfrac{\frac{1}{3}-1}{\frac{1}{3}+1}=-\dfrac{1}{2}$

ញ. $\lim\limits_{x\to1}\frac{x^2-1+\ln x}{e^x-e}$, រាងមិនកំណត់ $\frac{0}{0}$

តាង $u=x-1\Rightarrow x=u+1$

បើ $x\to1\Rightarrow u\to0$

យើងបាន $\lim\limits_{x\to 1}\dfrac{x^2-1+\ln x}{e^x-e}=\lim\limits_{u\to 0}\dfrac{(1+u)^2-1+\ln(1+u)}{e^{u+1}-e}$

$$=\lim_{u\to 0}\frac{1+2u+u^2-1+\ln(1+u)}{e\,(e^u-1)}$$
$$=\frac{1}{e}\lim_{u\to 0}\frac{2u+u^2+\ln(1+u)}{e^u-1}$$
$$=\frac{1}{e}\lim_{u\to 0}\frac{\frac{2u+u^2+\ln(1+u)}{u}}{\frac{e^u-1}{u}}$$
$$=\frac{1}{e}\lim_{u\to 0}\frac{2+u+\frac{1}{u}\cdot\ln(1+u)}{\frac{e^u-1}{u}}$$
$$=\frac{1}{e}\lim_{u\to 0}\frac{2+u+\ln(1+u)^{\frac{1}{u}}}{\frac{e^u-1}{u}}$$
$$=\frac{1}{e}\left(\frac{2+0+\ln e}{1}\right)$$
$$=\frac{1}{e}(2+1)$$
$$=\frac{3}{e}$$

ដូចនេះ $\boxed{\lim\limits_{x\to 1}\frac{x^2-1+\ln x}{\ln(e^x-e)}=\frac{3}{e}}$

ដ. $\lim\limits_{x\to 0}\frac{\ln(1+2x^2)}{1-\cos 2x}$, រាងមិនកំណត់ $\frac{\infty}{\infty}$

យើងបាន $\lim\limits_{x\to 0}\dfrac{\ln(1+2x^2)}{1-\cos 2x}=\lim\limits_{x\to 0}\dfrac{\ln\left(1+2x^2\right)}{2\sin^2 x}$

$$=\lim_{x\to 0}\frac{\ln\left(1+2x^2\right)}{2x^2}\times\frac{x^2}{\sin^2 x}$$
$$=\ln e\times 1$$
$$=1$$

ដូចនេះ $\boxed{\lim\limits_{x\to 0}\frac{\ln(1+2x^2)}{1-\cos 2x}=1}$

ឋ. $\lim\limits_{x\to 0}\frac{\ln(\cos 2x)}{x^2}$, រាងមិនកំណត់ $\frac{0}{0}$

យើងបាន $\lim\limits_{x\to 0}\dfrac{\ln(\cos 2x)}{x^2}=\lim\limits_{x\to 0}\dfrac{\ln\left(1-2\sin^2 x\right)}{x^2}$

$$=\lim_{x\to 0}\frac{\ln\left(1-2\sin^2 x\right)}{-2\sin^2 x}\times\frac{-2\sin^2 x}{x^2}$$
$$=\ln e\times(-2\times 1)$$
$$=-2$$

ដូចនេះ $\boxed{\lim\limits_{x\to 0}\frac{\ln(\cos 2x)}{x^2}=-2}$

ឌ. $\lim\limits_{x\to 0}\frac{\ln(x^2-x+1)}{x}$, រាងមិនកំណត់ $\frac{0}{0}$

យើងបាន $\lim\limits_{x\to 0}\dfrac{\ln(x^2-x+1)}{x}=\lim\limits_{x\to 0}\dfrac{\ln[1+x(x-1)]}{x}$

$$=\lim_{x\to 0}\frac{\ln[1+x(x-1)]}{x(x-1)}\times(x-1)$$
$$=\ln e\,(0-1)$$
$$=-1$$

ដូចនេះ $\boxed{\lim\limits_{x\to 0}\frac{\ln(x^2-x+1)}{x}=-1}$

ឍ. $\lim\limits_{x\to 0}\frac{\ln(1+x)+\ln(1+2x)}{x}$, រាងមិនកំណត់ $\frac{0}{0}$

យើងបាន $\lim\limits_{x\to 0}\dfrac{\ln(1+x)+\ln(1+2x)}{x}=\lim\limits_{x\to 0}\left[\dfrac{\ln(1+x)}{x}+\dfrac{\ln(1+2x)}{x}\right]$

$$=\lim_{x\to 0}\frac{\ln(1+x)}{x}+\lim_{x\to 0}\frac{\ln(1+2x)}{x}$$
$$=\lim_{x\to 0}\frac{\ln(1+x)}{x}+2\lim_{x\to 0}\frac{\ln(1+2x)}{2x}$$
$$=\ln e+2\ln e$$
$$=3$$

ដូចនេះ $\boxed{\lim\limits_{x\to 0}\frac{\ln(1+x)+\ln(1+2x)}{x}=3}$

ណ. $\lim\limits_{x\to 0}\frac{\ln(1+2\sin^2 x)}{3\tan^2 x}$, រាងមិនកំណត់ $\frac{0}{0}$

យើងបាន $\lim\limits_{x\to 0}\dfrac{\ln(1+2\sin^2 x)}{3\tan^2 x}=\lim\limits_{x\to 0}\dfrac{\ln\left(1+2\sin^2 x\right)}{2\sin^2 x}\times\dfrac{2\sin^2 x}{x^2}\times\dfrac{x^2}{3\tan^2 x}$

$$=\ln e\times 2\times\frac{1}{3}$$
$$=\frac{2}{3}$$

ដូចនេះ $\boxed{\lim\limits_{x\to 0}\frac{\ln(1+2\sin^2 x)}{3\tan^2 x}=\frac{2}{3}}$

លំហាត់គំរូទី២៣

ដោះស្រាយលីមីតខាងក្រោម៖

ក. $\lim\limits_{x\to 2}\dfrac{\sqrt{2+\sqrt{2+\sqrt{2+\cdots+\sqrt{2+x}}}}-2}{x-2}$, មាន n ឫសការេ

ខ. $\lim\limits_{x\to 0}\dfrac{1-(1+x)(1+2x)\cdots(1+nx)}{x}$

គ. $\lim\limits_{x\to 1}\dfrac{x+x^2+\cdots+x^n-n}{x+x^2+\cdots+x^m-m}$

ឃ. $\lim\limits_{x\to a}\dfrac{x^n-a^n-na^{n-1}x+na^n}{(x-a)^2}$

ង. $\lim\limits_{x\to1}\frac{x^n-nx+n-1}{x-1}$

ច. $\lim\limits_{x\to1}\frac{nx^{n+1}-(n+1)x^n+1}{x^{m+1}-x^m-x+1}$

ឆ. $\lim\limits_{x\to+\infty}\frac{x^{100}+(x+1)^{100}+\cdots+(x+100)^{100}}{x^{100}+100^{100}}$

ជ. $\lim\limits_{x\to+\infty}\left(\frac{1}{2}+\frac{1}{4}+\cdots+\frac{1}{2^n}\right)$

ឈ. $\lim\limits_{x\to+\infty}\left[\frac{1}{\sqrt{n}}\left(\frac{1}{\sqrt{3}+1}+\frac{1}{\sqrt{5}+\sqrt{3}}+\cdots+\frac{1}{\sqrt{2n+1}+\sqrt{2n-1}}\right)\right]$

ញ. $\lim\limits_{n\to+\infty}\left[\frac{1}{1.4.7}+\frac{1}{4.7.10}+\cdots+\frac{1}{(3n-2)(3n+1)(3n+4)}\right]$, $|x|<1$

ដ. $\lim\limits_{x\to+\infty}\left[(1+x)\left(1+x^2\right)\left(1+x^4\right)\cdots\left(1+x^{2^n}\right)\right]$

ឋ. $\lim\limits_{x\to0}\frac{e^x+e^{2x}+\cdots+e^{nx}-n}{x}$

ឌ. $\lim\limits_{x\to0}\frac{(e^x-1)\left(e^{2x}-1\right)\cdots(e^{nx}-1)}{x^n}$

ឍ. $\lim\limits_{x\to1}\left(\frac{n}{1-x^n}-\frac{1}{1-x}\right)$

ណ. $\lim\limits_{x\to+\infty}\frac{n^2-1}{1+2+3+\ldots+n}$

ដំណោះស្រាយ

ដោះស្រាយលីមីតខាងក្រោម៖

ក. $\lim\limits_{x\to2}\frac{\sqrt{2+\sqrt{2+\sqrt{2+\cdots+\sqrt{2+x}}}}-2}{x-2}$ មាន n ឫសការេ , រាងមិនកំណត់ $\frac{0}{0}$

$$=\lim_{x\to2}\frac{\left(\overbrace{\sqrt{2+\sqrt{2+2+\cdots+\sqrt{2+x}}}}^{n}-2\right)\left(\overbrace{\sqrt{2+\sqrt{2+2+\cdots+\sqrt{2+x}}}}^{n}+2\right)}{(x-2)\left(\underbrace{\sqrt{2+\sqrt{2+2+\cdots+\sqrt{2+x}}}}_{n}+2\right)}$$

$$=\lim_{x\to2}\frac{2+\overbrace{\sqrt{2+2+\cdots+\sqrt{2+x}}}^{n-1}-4}{(x-2)\left(\underbrace{\sqrt{2+\sqrt{2+2+\cdots+\sqrt{2+x}}}}_{n}+2\right)}$$

$$=\lim_{x\to2}\frac{\overbrace{\sqrt{2+2+\cdots+\sqrt{2+x}}}^{n-1}-2}{(x-2)\left(\underbrace{\sqrt{2+\sqrt{2+2+\cdots+\sqrt{2+x}}}}_{n}+2\right)}$$

$=\cdots$ (គុណនឹងកន្សោមឆ្លាស់ n ដង យើងបានចុងក្រោយ)

$$=\lim_{x\to2}\frac{x-2}{(x-2)\underbrace{\left(\sqrt{2+\sqrt{2+2+\cdots+\sqrt{2+x}}}+2\right)}_{n}\underbrace{\cdots\cdots}_{(n-1)(n-2)\cdots}\underbrace{\left(\sqrt{x+2}\right)}_{n-(n-1)}}$$

$$=\lim_{x\to2}\frac{1}{\underbrace{\left(\sqrt{2+\sqrt{2+2+\cdots+\sqrt{2+x}}}+2\right)}_{n}\underbrace{\cdots\cdots}_{(n-1)(n-2)\cdots}\underbrace{\left(\sqrt{x+2}\right)}_{n-(n-1)}}$$

$$=\frac{1}{\underbrace{4\cdot4\cdots4}_{n}}$$

$$=\frac{1}{4^n}$$

ដូចនេះ $\boxed{\lim\limits_{x\to2}\frac{\sqrt{2+\sqrt{2+\sqrt{2+\cdots+\sqrt{2+x}}}}-2}{x-2}=\frac{1}{4^n}}$

ខ. $\lim\limits_{x\to0}\frac{1-(1+x)(1+2x)\cdots(1+nx)}{x}$, រាងមិនកំណត់ $\frac{0}{0}$

តាង $A=\lim\limits_{x\to0}\frac{1-(1+x)(1+2x)\cdots(1+nx)}{x}$ យើងបាន

$A=\lim\limits_{x\to0}\frac{1-(1+x)+(1+x)[1-(1-2x)]+\cdots+(1+x)\cdots[1-(1+nx)]}{x}$

$A=\lim\limits_{x\to0}\frac{[-x-2x(x+1)-\cdots-nx]\{(x+1)(1+2x)\cdots[1+(n-1)x]\}}{x}$

$A=\lim\limits_{x\to0}\frac{-x[1+2(x+1)+\cdots+n]\{(x+1)(1+2x)\cdots[1+(n-1)x]\}}{x}$

$$A=-\lim_{x\to 0}[1+2(x+1)+\cdots+n]\{(x+1)(1+2x)\cdots[1+(n-1)x]\}$$

$$A=-(1+2+\cdots+n)\left(\underbrace{1\cdot 1\cdots 1}_{n}\right)$$

$$A=-\left[\frac{n(n+1)}{2}\right]$$

$$A=-\frac{n(n+1)}{2}$$

ដូចនេះ $\boxed{\lim_{x\to 0}\frac{1-(1+x)(1+2x)\cdots(1+nx)}{x}=-\frac{n(n+1)}{2}}$

គ. $\lim_{x\to 1}\frac{x+x^2+\cdots+x^n-n}{x+x^2+\cdots+x^m-m}$, រាងមិនកំណត់ $\frac{0}{0}$

តាង $B=\lim_{x\to 1}\frac{x+x^2+\cdots+x^n-n}{x+x^2+\cdots+x^m-m}$ យើងបាន

$$B=\lim_{x\to 1}\frac{x+x^2+\cdots+x^n-\overbrace{(1+1+\cdots+1)}^{n}}{x+x^2+\cdots+x^m-\underbrace{(1+1+\cdots+1)}_{m}}$$

$$B=\lim_{x\to 1}\frac{(x-1)+\left(x^2-1\right)+\cdots+\left(x^n-1\right)}{(x-1)+\left(x^2-1\right)+\cdots+\left(x^m-1\right)}$$

$$B=\lim_{x\to 1}\frac{(x-1)+(x-1)(x+1)+\cdots+(x-1)\left(x^{n-1}+x^{n-2}+\cdots+1\right)}{(x-1)+(x-1)(x+1)+\cdots+(x-1)\left(x^{m-1}+x^{m-2}+\cdots+1\right)}$$

$$B=\lim_{x\to 1}\frac{(x-1)\left[1+(x+1)+\left(x^2+x+1\right)+\cdots+\left(x^{n-1}+x^{n-2}+\cdots+1\right)\right]}{(x-1)\left[1+(x+1)+\left(x^2+x+1\right)+\cdots+\left(x^{m-1}+x^{m-2}+\cdots+1\right)\right]}$$

$$B=\lim_{x\to 1}\frac{1+(x+1)+\left(x^2+x+1\right)+\cdots+\left(x^{n-1}+x^{n-2}+\cdots+1\right)}{1+(x+1)+\left(x^2+x+1\right)+\cdots+\left(x^{m-1}+x^{m-2}+\cdots+1\right)}$$

$$B=\frac{1+(1+1)+(1+1+1)+\cdots+\overbrace{(1+1+\cdots+1)}^{n}}{1+(1+1)+(1+1+1)+\cdots+\underbrace{(1+1+\cdots+1)}_{m}}$$

$$B=\frac{1+2+3+\cdots+n}{1+2+3+\cdots+m}$$

$$B=\frac{\frac{n(n+1)}{2}}{\frac{m(m+1)}{2}}$$

$$B=\frac{n(n+1)}{m(m+1)}$$

ដូចនេះ $\boxed{\lim_{x\to 1}\frac{x+x^2+\cdots+x^n-n}{x+x^2+\cdots+x^m-m}=\frac{n(n+1)}{m(m+1)}}$

ឃ. $\lim_{x\to a}\frac{x^n-a^n-na^{n-1}x+na^n}{(x-a)^2}$, រាងមិនកំណត់ $\frac{0}{0}$

តាង $C=\lim_{x\to a}\frac{x^n-a^n-na^{n-1}x+na^n}{(x-a)^2}$ យើងបាន

$$C=\lim_{x\to a}\frac{(x-a)\left(x^{n-1}+ax^{n-2}+\cdots+a^{n-2}+a^{n-1}\right)-na^{n-1}(x-a)}{(x-a)^2}$$

$$C=\lim_{x\to a}\frac{(x-a)\left[\left(x^{n-1}+ax^{n-2}+\cdots+a^{n-2}+a^{n-1}\right)-na^{n-1}\right]}{(x-a)^2}$$

$$C=\lim_{x\to a}\frac{\left(x^{n-1}+ax^{n-2}+\cdots+a^{n-2}+a^{n-1}\right)-na^{n-1}}{x-1}$$

$$C=\lim_{x\to a}\frac{\left(x^{n-1}+ax^{n-2}+\cdots+a^{n-2}+a^{n-1}\right)-\overbrace{\left(a^{n-1}+\cdots+a^{n-1}\right)}^{n-1}}{x-1}$$

$$C=\lim_{x\to a}\frac{\left(x^{n-1}-a^{n-1}\right)+\left(ax^{n-2}-a^{n-1}\right)+\cdots+\left(a^{n-2}x-a^{n-1}\right)}{x-1}$$

$$C=\lim_{x\to a}\frac{(x-1)\left(x^{n-2}+\cdots+a^{n-2}\right)+a(x-1)\left(x^{n-3}+\cdots+a^{n-3}\right)+\cdots+a^{n-2}(x-a)}{x-1}$$

$$C=\lim_{x\to a}\frac{(x-1)\left[\left(x^{n-2}+\cdots+a^{n-2}\right)+a\left(x^{n-3}+\cdots+a^{n-3}\right)+\cdots+a^{n-2}\right]}{x-1}$$

$$C=\lim_{x\to a}\frac{(x-1)\left[\left(x^{n-2}+\cdots+a^{n-2}\right)+a\left(x^{n-3}+\cdots+a^{n-3}\right)+\cdots+a^{n-2}\right]}{x-1}$$

$$C=\lim_{x\to a}\left[\left(x^{n-2}+\cdots+a^{n-2}\right)+a\left(x^{n-3}+\cdots+a^{n-3}\right)+\cdots+a^{n-2}\right]$$

$$C=\overbrace{a^{n-2}+\cdots+a^{n-2}}^{n-1}+\overbrace{a^{n-2}+\cdots+a^{n-2}}^{n-2}+\cdots+a^{n-2}$$

$$C=a^{n-2}[1+2+\cdots+(n-2)+(n-1)]$$

$$C=\frac{n(n-1)a^{n-2}}{2}$$

ដូចនេះ $\boxed{\lim_{x\to a}\frac{x^n-a^n-na^{n-1}x+na^n}{(x-a)^2}=\frac{n(n-1)a^{n-2}}{2}}$

ង. $\lim_{x\to 1}\frac{x^n-nx+n-1}{x-1}$, រាងមិនកំណត់ $\frac{0}{0}$

យើងបាន $\lim_{x\to 1}\frac{x^n-nx+n-1}{x-1}=\lim_{x\to 1}\frac{\left(x^n-1\right)-n(x-1)}{x-1}$

$$=\lim_{x\to 1}\frac{(x-1)\left(x^{n-1}+x^{n-2}+\cdots+1\right)-n(x-1)}{x-1}$$

$$=\lim_{x\to 1}\frac{(x-1)\left[\left(x^{n-1}+x^{n-2}+\cdots+1\right)-n\right]}{x-1}$$

$$=\lim_{x\to 1}\left[\overbrace{\left(x^{n-1}+x^{n-2}+\cdots+1\right)}^{n}-n\right]$$

$$=\left[\overbrace{(1+1+\cdots+1)}^{n}-n\right]$$

$$=n-n$$

$$=0$$

ដូចនេះ $\boxed{\lim_{x\to 1}\frac{x^n-nx+n-1}{(x-1)^2}=0}$

ច. $\lim_{x\to 1}\frac{nx^{n+1}-(n+1)x^n+1}{x^{m+1}-x^m-x+1}$, រាងមិនកំណត់ $\frac{0}{0}$

តាង $D=\lim_{x\to 1}\frac{nx^{n+1}-nx^n-x^n+1}{x^{m+1}-x^m-x+1}$ យើងបាន

$$D=\lim_{x\to 1}\frac{\left(nx^{n+1}-nx^n\right)-\left(x^n-1\right)}{x^{m+1}-x^m-x+1}$$

$$D=\lim_{x\to 1}\frac{nx^n(x-1)-\left(x^n-1\right)}{x^m(x-1)-(x-1)}$$

$$D=\lim_{x\to 1}\frac{nx^n(x-1)-(x-1)\left(x^{n-1}+x^{n-2}+\cdots+1\right)}{x^m(x-1)-(x-1)}$$

$$D=\lim_{x\to 1}\frac{(x-1)\left[nx^n-\left(x^{n-1}+x^{n-2}+\cdots+1\right)\right]}{(x-1)\left(x^m-1\right)}$$

$$D=\lim_{x\to 1}\frac{\left[nx^n-\left(x^{n-1}+x^{n-2}+\cdots+1\right)\right]}{\left(x^m-1\right)}$$

$$D=\lim_{x\to 1}\frac{\overbrace{\left(x^n+x^n+\cdots x^n\right)}^{n}-\left(x^{n-1}+x^{n-2}+\cdots+1\right)}{\left(x^m-1\right)}$$

$$D=\lim_{x\to 1}\frac{\left(x^n-x^{n-1}\right)+\left(x^n-x^{n-2}\right)+\cdots+\left(x^n-1\right)}{\left(x^m-1\right)}$$

$$D=\lim_{x\to 1}\frac{x^n(x-1)+x^{x-2}\left(x^2-1\right)+\cdots+x\left(x^{n-1}-1\right)+\left(x^n-1\right)}{\left(x^m-1\right)}$$

$$D=\lim_{x\to 1}\frac{x^n(x-1)+x^{x-2}(x-1)(x+1)+\cdots+x\left(x^{n-1}-1\right)+\left(x^n-1\right)}{\left(x^m-1\right)}$$

$$D=\lim_{x\to 1}\frac{x^n(x-1)+x^{x-2}(x-1)(x+1)+\cdots+x\left(x^{n-1}-1\right)+\left(x^n-1\right)}{\left(x^m-1\right)}$$

$$D=\lim_{x\to 1}\frac{(x-1)\left[x^n+x^{x-2}(x+1)+\cdots+x\left(x^{n-2}+\cdots+1\right)+\left(x^{n-1}+\cdots+1\right)\right]}{(x-1)\left(x^{m-1}+\cdots+1\right)}$$

$$D=\lim_{x\to 1}\frac{\left[x^n+x^{x-2}(x+1)+\cdots+x\left(x^{n-2}+\cdots+1\right)+\left(x^{n-1}+\cdots+1\right)\right]}{x^{m-1}+\cdots+1}$$

$$D=\frac{1+1+\cdots+\overbrace{(1+1+\cdots+1)}^{n-1}+\overbrace{(1+1+\cdots+1)}^{n}}{\underbrace{1+1+\cdots+1}_{m}}$$

$$D=\frac{\frac{n(n+1)}{2}}{m}=\frac{n(n+1)}{2m}$$

ដូចនេះ $\boxed{\lim_{x\to 1}\frac{nx^{n+1}-(n+1)x^n+1}{x^{m+1}-x^m-x+1}=\frac{n(n+1)}{2m}}$

ឆ. $\lim_{x\to+\infty}\frac{x^{100}+(x+1)^{100}+\cdots+(x+100)^{100}}{x^{100}+100^{100}}$, រាងមិនកំណត់ $\frac{\infty}{\infty}$

តាង $E=\lim\limits_{x\to+\infty}\dfrac{x^{100}+\overbrace{(x+1)^{100}+\cdots+(x+100)^{100}}^{100}}{x^{100}+100^{100}}$ យើងបាន

$$E=\lim_{x\to+\infty}\frac{x^{100}+\overbrace{\left[x\left(1+\frac{1}{x}\right)\right]^{100}+\cdots+\left[x\left(x+\frac{100}{x^{100}}\right)\right]^{100}}^{100}}{x^{100}+100^{100}}$$

$$E=\lim_{x\to+\infty}\frac{x^{100}+\overbrace{x^{100}\left(1+\frac{1}{x}\right)^{100}+\cdots+x^{100}\left(1+\frac{100}{x^{100}}\right)^{100}}^{100}}{x^{100}\left(1+\frac{100}{x^{100}}\right)^{100}}$$

$$E=\lim_{x\to+\infty}\frac{x^{100}\left[1+\overbrace{\left(1+\frac{1}{x}\right)^{100}+\cdots+\left(1+\frac{100}{x^{100}}\right)^{100}}^{100}\right]}{x^{100}\left(1+\frac{100}{x^{100}}\right)^{100}}$$

$$E=\lim_{x\to+\infty}\frac{1+\overbrace{\left(1+\frac{1}{x}\right)^{100}+\cdots+\left(1+\frac{100}{x^{100}}\right)^{100}}^{100}}{\left(1+\frac{100}{x^{100}}\right)^{100}}$$

$$E=\frac{1+\overbrace{1+\cdots+1}^{100}}{1}$$

$$E=101$$

ដូចនេះ $\boxed{\lim\limits_{x\to+\infty}\dfrac{x^{100}+(x+1)^{100}+\cdots+(x+100)^{100}}{x^{100}+100^{100}}=101}$

ជ. $\lim\limits_{n\to+\infty}\left(\frac{1}{2}+\frac{1}{4}+\cdots+\frac{1}{2^n}\right)$

តាង $F=\lim\limits_{n\to+\infty}\left(\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\cdots+\frac{1}{2^n}\right)$ យើងបាន

$$F=\lim_{n\to+\infty}\left[\left(1-\frac{1}{2}\right)+\left(\frac{1}{2}-\frac{1}{2^2}\right)+\left(\frac{1}{2^2}-\frac{1}{2^3}\right)+\cdots+\left(\frac{1}{2^{n-1}}-\frac{1}{2^n}\right)\right]$$

$$F=\lim_{n\to+\infty}\left(1-\frac{1}{2^n}\right)$$

$$F=1-0=1$$

ដូចនេះ $\boxed{\lim\limits_{x\to+\infty}\left(\frac{1}{2}+\frac{1}{4}+\cdots+\frac{1}{2^n}\right)=1}$

ឈ. $\lim\limits_{n\to+\infty}\left[\frac{1}{\sqrt{n}}\left(\frac{1}{\sqrt{3}+1}+\frac{1}{\sqrt{5}+\sqrt{3}}+\cdots+\frac{1}{\sqrt{2n+1}+\sqrt{2n-1}}\right)\right]$

តាង $G=\lim\limits_{n\to+\infty}\left[\frac{1}{\sqrt{n}}\left(\frac{1}{\sqrt{3}+1}+\frac{1}{\sqrt{5}+\sqrt{3}}+\cdots+\frac{1}{\sqrt{2n+1}+\sqrt{2n-1}}\right)\right]$

យើងបាន

$$G=\lim_{n\to+\infty}\frac{1}{\sqrt{n}}\sum_{k=1}^{n}\left(\frac{1}{\sqrt{2k+1}+\sqrt{2k-1}}\right)$$

$$G=\lim_{n\to+\infty}\frac{1}{\sqrt{n}}\sum_{k=1}^{n}\left[\frac{\sqrt{2k+1}-\sqrt{2k-1}}{\left(\sqrt{2k+1}+\sqrt{2k-1}\right)\left(\sqrt{2k+1}-\sqrt{2k-1}\right)}\right]$$

$$G=\lim_{n\to+\infty}\frac{1}{\sqrt{n}}\sum_{k=1}^{n}\left[\frac{\sqrt{2k+1}-\sqrt{2k-1}}{(2k+1)-(2k-1)}\right]$$

$$G=\lim_{n\to+\infty}\frac{1}{\sqrt{n}}\sum_{k=1}^{n}\frac{\sqrt{2k+1}-\sqrt{2k-1}}{2}$$

$$G=\lim_{n\to+\infty}\frac{1}{\sqrt{n}}\frac{(\sqrt{3}-1)+(\sqrt{5}-\sqrt{3})+\cdots+\left[\sqrt{2n+1}-(\sqrt{2n-1})\right]}{2}$$

$$G=\lim_{n\to+\infty}\frac{\sqrt{2n+1}-1}{2\sqrt{n}}$$

$$G=\frac{1}{2}\lim_{n\to+\infty}\left(\sqrt{2+\frac{1}{n}}-\frac{1}{\sqrt{n}}\right)$$

$$G=\frac{1}{2}\left(\sqrt{2}-0\right)=\frac{\sqrt{2}}{2}$$

ដូចនេះ $\boxed{\lim\limits_{x\to+\infty}\frac{1}{\sqrt{n}}\left(\frac{1}{\sqrt{3}+1}+\frac{1}{\sqrt{5}+\sqrt{3}}+\cdots+\frac{1}{\sqrt{2n+1}+\sqrt{2n-1}}\right)=\frac{\sqrt{2}}{2}}$

ញ. $\lim\limits_{n\to+\infty}\left[\frac{1}{1\cdot4\cdot7}+\frac{1}{4\cdot7\cdot10}+\cdots+\frac{1}{(3n-2)(3n+1)(3n+4)}\right]$

តាង $H=\lim\limits_{n\to+\infty}\left[\frac{1}{1\cdot4\cdot7}+\frac{1}{4\cdot7\cdot10}+\cdots+\frac{1}{(3n-2)(3n+1)(3n+4)}\right]$

យើងបាន

$$H=\lim_{n\to+\infty}\sum_{k=1}^{n}\frac{1}{(3k-2)(3k+1)(3k+4)}$$

$$H=\lim_{n\to+\infty}\sum_{k=1}^{n}\frac{1}{6}\left[\frac{1}{(3k-2)(3k+1)}-\frac{1}{(3k+1)(3k+4)}\right]$$

$$H=\frac{1}{6}\lim_{n\to+\infty}\left\{\left(\frac{1}{1\cdot4}-\frac{1}{4\cdot7}\right)+\cdots+\left[\frac{1}{(3n-2)(3n+1)}-\frac{1}{(3n+1)(3n+4)}\right]\right\}$$

$$H=\frac{1}{6}\lim_{n\to+\infty}\left[\frac{1}{4}-\frac{1}{(3n+1)(3n+4)}\right]$$

$$H=\frac{1}{6}\left(\frac{1}{4}-0\right)$$

$$H=\frac{1}{24}$$

ដូចនេះ $\boxed{\lim\limits_{x\to+\infty}\left[\frac{1}{1\cdot4\cdot7}+\frac{1}{4\cdot7\cdot10}+\cdots+\frac{1}{(3n-2)(3n+1)(3n+4)}\right]=\frac{1}{24}}$

ដ. $\lim\limits_{n\to+\infty}\left[(1+x)\left(1+x^2\right)\left(1+x^4\right)\cdots\left(1+x^{2^n}\right)\right]$, $|x|<1$

តាង $I=\lim\limits_{n\to+\infty}\left[(1+x)\left(1+x^2\right)\left(1+x^4\right)\cdots\left(1+x^{2^n}\right)\right]$ យើងបាន

$$I=\lim_{n\to+\infty}\left(\frac{1-x^2}{1-x}\cdot\frac{1-x^4}{1+x^2}\cdots\frac{1-x^{2^{n+1}}}{1-x^{2^n}}\right)$$

$$I=\lim_{n\to+\infty}\frac{1-x^{2^{n+1}}}{1-x}$$

$$I=\lim_{n\to+\infty}\frac{1}{1-x}$$

$I=\frac{1}{1-x}$, ព្រោះ $\left(\lim\limits_{n\to+\infty}x^{2^{n+1}}=0\ ,\ |x|<1\right)$

ដូចនេះ $\boxed{\lim\limits_{n\to+\infty}(1+x)\left(1+x^2\right)\left(1+x^4\right)\cdots\left(1+x^{2^n}\right)=\frac{1}{1-x}}$

ឋ. $\lim\limits_{x\to0}\frac{e^x+e^{2x}+\cdots+e^{nx}-n}{x}$, រាងមិនកំណត់ $\frac{0}{0}$

តាង $J=\lim\limits_{x\to0}\frac{e^x+e^{2x}+\cdots+e^{nx}-n}{x}$ យើងបាន

$$J=\lim_{x\to0}\frac{e^x+e^{2x}+\cdots+e^{nx}-\overbrace{(1+1+\cdots+1)}^{n}}{x}$$

$$J=\lim_{x\to0}\frac{(e^x-1)+(e^{2x}-1)+\cdots+(e^{nx}-1)}{x}$$

$$J=\lim_{x\to0}\left(\frac{e^x-1}{x}+\frac{e^{2x}-1}{x}+\cdots+\frac{e^{nx}-1}{x}\right)$$

$$J=\lim_{x\to0}\left(\frac{e^x-1}{x}+2\frac{e^{2x}-1}{2x}+\cdots+n\frac{e^{nx}-1}{nx}\right)$$

$$J=1+2+\cdots+n$$

$$J=\frac{n(n+1)}{2}$$

ដូចនេះ $\boxed{\lim\limits_{x\to0}\frac{e^x+e^{2x}+\cdots+e^{nx}-n}{x}=\frac{n(n+1)}{2}}$

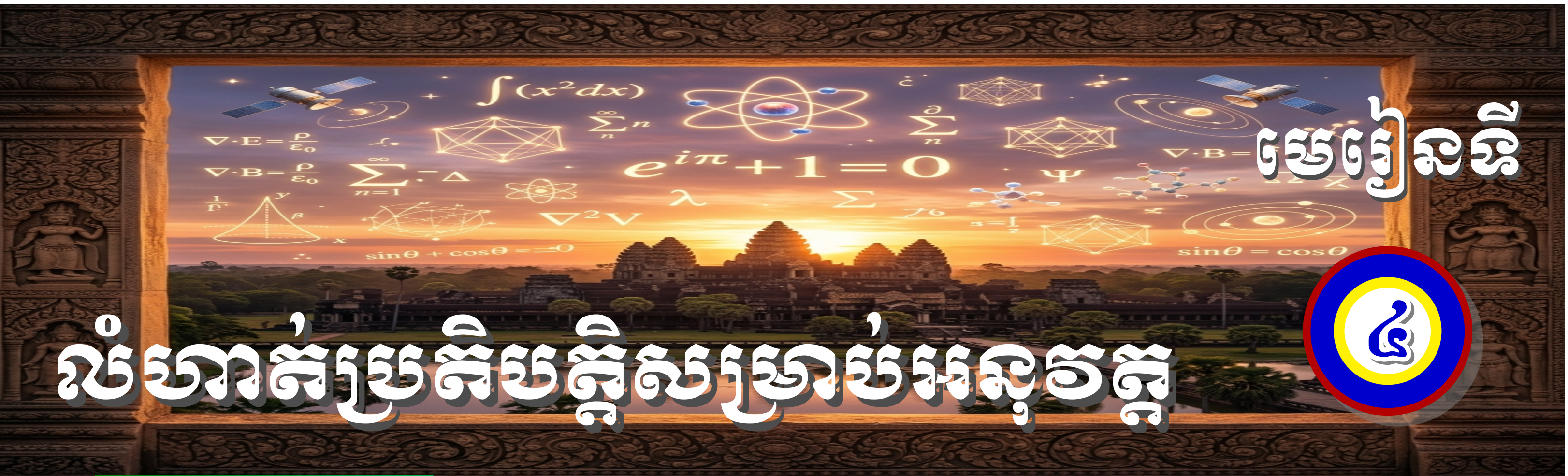
ឌ. $\lim\limits_{x\to0}\frac{(e^x-1)(e^{2x}-1)\cdots(e^{nx}-1)}{x^n}$, រាងមិនកំណត់ $\frac{0}{0}$

តាង $K=\lim\limits_{x\to0}\frac{(e^x-1)(e^{2x}-1)\cdots(e^{nx}-1)}{x^n}$ យើងបាន

$$K=\lim_{x\to0}\frac{(e^x-1)}{x}\cdot\frac{(e^{2x}-1)}{x}\cdots\frac{(e^{nx}-1)}{x}$$

$$K=\lim_{x\to0}\frac{(e^x-1)}{x}\cdot2\frac{(e^{2x}-1)}{2x}\cdots n\frac{(e^{nx}-1)}{nx}$$

$$K=1\cdot2\cdots n$$



មេរៀនទី ៤

លំហាត់ប្រតិបត្តិសម្រាប់អនុវត្ត

លំហាត់អនុវត្តទី១

ចូរបង្ហាញលីមីតខាងក្រោមដោយប្រើនិយមន័យ៖

ក. $\lim\limits_{x\to1}(x^2+1)=2$

ខ. $\lim\limits_{x\to2}x^2=4$

គ. $\lim\limits_{x\to-1}(x^2-1)=0$

ឃ. $\lim\limits_{x\to7}\frac{8}{x-3}=2$

ង. $\lim\limits_{x\to-1^+}\frac{x+5}{x+1}=+\infty$

ច. $\lim\limits_{x\to-2^+}\frac{x-9}{x+2}=-\infty$

ឆ. $\lim\limits_{x\to+\infty}\frac{x^2-2}{x^2-1}=1$

ជ. $\lim\limits_{x\to-\infty}\frac{4x+1}{2x+1}=2$

ឈ. $\lim\limits_{x\to+\infty}(1+6x-x^2)=-\infty$

ញ. $\lim\limits_{x\to+\infty}\frac{5x^2+4x-1}{x+2}=+\infty$

ដ. $\lim\limits_{x\to-\infty}(x^3)=-\infty$

ឋ. $\lim\limits_{x\to-\infty}\sqrt{x-1}=-\infty$

ឌ. $\lim\limits_{x\to4}(x^2+x-11)=9$

ឍ. $\lim\limits_{x\to0}\frac{1}{x}=\infty$

ណ. $\lim\limits_{x\to1}\frac{1}{(x-1)^2}=+\infty$

ត. $\lim\limits_{x\to0}\frac{x+2}{x^2}=\infty$

ថ. $\lim\limits_{x\to3}\frac{2x^2+3x-27}{x^2-4x+3}=\frac{15}{2}$

ទ. $\lim\limits_{x\to+\infty}\frac{x-1}{x+1}=1$

ធ. $\lim\limits_{x\to+\infty}\frac{x+1}{x-1}=1$

ន. $\lim\limits_{x\to+\infty}\frac{2x}{x+1}=2$

ដំណោះស្រាយ

មេរៀនទី 4 លំហាត់ប្រតិបត្តិសម្រាប់អនុវត្ត

លំហាត់អនុវត្តទី២

គណនាតម្លៃ $\alpha > 0$ ទៅតាមតម្លៃ ε នីមួយៗដែលគេឲ្យដូចខាងក្រោម៖

ក. $\lim\limits_{x\to 1}(5x-3)=2$, $\varepsilon = 0.00005$

ខ. $\lim\limits_{x\to 2}(3x-2)=1$, $\varepsilon = 0.000000099$

គ. $\lim\limits_{x\to 1}(2x-1)=1$, $\varepsilon = 44\times 10^{-6}$

ឃ. $\lim\limits_{x\to 2}(3x+1)=7$, $\varepsilon = 0.0030$

ង. $\lim\limits_{x\to 4}(2x-1)=7$, $\varepsilon = 0.000006$

ច. $\lim\limits_{x\to -3}(2x+5)=-1$, $\varepsilon = 0.10$

ឆ. $\lim\limits_{x\to 3}(2x+4)=10$, $\varepsilon = 0.00001$

ជ. $\lim\limits_{x\to 2}(x-1)^2=1$, $\varepsilon = 0.00021$

ឈ. $\lim\limits_{x\to 1}\sqrt{x}=1$, $\varepsilon = (\sqrt{2}-1)0.0001$

ញ. $\lim\limits_{x\to 2}(4x-5)=3$, $\varepsilon = 8\times 10^{-3}$

ដំណោះស្រាយ

មេរៀនទី ៤ លំហាត់ប្រតិបត្តិសម្រាប់អនុវត្ត

លំហាត់អនុវត្តទី៣

ចូរដោះស្រាយលីមីតខាងក្រោមឲ្យបានត្រឹមត្រូវ៖

ក. $\lim\limits_{x\to 2}(x^2-4)$

ខ. $\lim\limits_{x\to 0}\frac{x^3-4x}{2x^2+3x}$

គ. $\lim\limits_{x\to -1}\frac{x^3}{(x+1)^2}$

ឃ. $\lim\limits_{x\to -1}\frac{(x+1)^2(x-1)}{x^3+1}$

ង. $\lim\limits_{x\to 0}\frac{x^3-2x^2+x}{2x^3+x^2-2x}$

ច. $\lim\limits_{x\to 1}\frac{x^2+2x+3}{(x-1)^2}$

ឆ. $\lim\limits_{x\to 0}\frac{x^4-4x^3+x^2}{x^3+x^2+x}$

ជ. $\lim\limits_{x\to -1}\frac{x^3+x^2+x+1}{x^4+x^2-2}$

ឈ. $\lim\limits_{x\to 2}\frac{(x+1)^2}{2-x}$

ញ. $\lim\limits_{x\to 2}\frac{x-2}{x^2-3x+2}$

ដ. $\lim\limits_{x\to 3}\frac{x^2-6x+9}{2x^2+x-21}$

ឋ. $\lim\limits_{x\to -3}\frac{x^3+27}{2x^2-3x-27}$

ឌ. $\lim\limits_{x\to 2}\frac{x^3-6x^2+11x-6}{x^2-6x+8}$

ឍ. $\lim\limits_{x\to 2}\frac{x^3-5x^2+8x-4}{x^3-3x^2+4}$

ណ. $\lim\limits_{x\to 3}\left(\frac{1}{x-3}-\frac{6}{x^2-9}\right)$

ត. $\lim\limits_{x\to 1}\frac{x^3+x^2-x-1}{x^3-x^2-x+1}$

ថ. $\lim\limits_{x\to 4}\left(\frac{2x^2-4x-24}{x^2-16}-\frac{1}{4-x}\right)$

ទ. $\lim\limits_{x\to 2}\frac{x^3+3x^2-9x-2}{x^3-x-6}$

ធ. $\lim\limits_{x\to 1}\frac{x^3-3x+2}{x^4-4x+3}$

ន. $\lim\limits_{x\to 2}\left(\frac{1}{1-x}-\frac{3}{1-x^2}\right)$

ដំណោះស្រាយ

មេរៀនទី 4 លំហាត់ប្រតិបត្តិសម្រាប់អនុវត្ត

លំហាត់អនុវត្តទី៤

ចូរដោះស្រាយលីមីតខាងក្រោមឲ្យបានត្រឹមត្រូវ៖

ក. $\lim\limits_{x\to 2}\frac{x^2-3x+2}{x^2-2x}$

ខ. $\lim\limits_{x\to 2}\frac{x^2-x-2}{x^2-2x}$

គ. $\lim\limits_{x\to 2}\frac{x^2+5}{x^2-3}$

ឃ. $\lim\limits_{x\to 1}\frac{3x^4-4x^3+1}{(x-1)^2}$

ង. $\lim\limits_{x\to -2}\frac{3x+6}{x^3+8}$

ច. $\lim\limits_{x\to 2}\frac{x+1}{x-1}$

ឆ. $\lim\limits_{x\to -2}\frac{x^3+3x^2+2x}{x^2-x-6}$

ជ. $\lim\limits_{x\to 1}\frac{x^2-2x+1}{x^3-x}$

ឈ. $\lim\limits_{x\to 4}\frac{x^2+7x-44}{x^2-6x+8}$

ញ. $\lim\limits_{x\to 1}\frac{x^2-4}{x^2-3x+2}$

ដ. $\lim\limits_{x\to 1}\frac{x^3-5x+4}{x^3-1}$

ឋ. $\lim\limits_{x\to 2}\frac{x^2-4}{x-2}$

ឌ. $\lim\limits_{x\to 2}\frac{x^2-4}{x^2-3x+2}$

ឍ. $\lim\limits_{x\to 1}\left(\frac{1}{x^2-1}-\frac{2}{x^4-1}\right)$

ណ. $\lim\limits_{x\to 3}\frac{x-3}{x^2-5x+6}$

ត. $\lim\limits_{x\to 1}\frac{x^2-4x+3}{x^3-x^2+2x-2}$

ថ. $\lim\limits_{x\to 1}\frac{x^9-3x+2}{x^6+5x-6}$

ទ. $\lim\limits_{x\to 2}\frac{x-2}{x^3-2x^2-x-2}$

ធ. $\lim\limits_{x\to 1}\frac{x^m-1}{x^n-1}$

ន. $\lim\limits_{x\to 1}\frac{x^n-1}{x-1}$

ដំណោះស្រាយ

មេរៀនទី 4 លំហាត់ប្រតិបត្តិសម្រាប់អនុវត្ត

លំហាត់អនុវត្តទី៥

ចូរដោះស្រាយលីមីតខាងក្រោមឲ្យបានត្រឹមត្រូវ៖

ក. $\lim\limits_{x\to+\infty}\dfrac{\sqrt{x+\sqrt{x+\sqrt{x}}}}{\sqrt{x}}$

ខ. $\lim\limits_{x\to+\infty}\dfrac{\sqrt{x}+\sqrt[3]{x}+\sqrt[4]{x}}{\sqrt{2x+3}}$

គ. $\lim\limits_{x\to+\infty}\dfrac{\sqrt{x^2+x-3}+2x}{\sqrt{x^2+4}+x}$

ឃ. $\lim\limits_{x\to\infty}\dfrac{x^2-1}{2x^2+1}$

ង. $\lim\limits_{x\to-\infty}\dfrac{x^3+x^2-4}{2x^3+x+11}$

ច. $\lim\limits_{x\to\infty}\dfrac{3x^2+2x-1}{x^3-x+2}$

ឆ. $\lim\limits_{x\to\infty}\left(\dfrac{x^3}{x^2+2}-x\right)$

ជ. $\lim\limits_{x\to\infty}\dfrac{x^2+3x-4}{3x^2-2x+5}$

ឈ. $\lim\limits_{x\to\infty}\dfrac{x(x-1)(x-2)}{x^2+6x-9}$

ញ. $\lim\limits_{x\to\infty}\dfrac{\sqrt{x^2+9}}{x+3}$

ដ. $\lim\limits_{x\to\infty}\left(\dfrac{x^2+x-1}{2x^2-x+1}\right)^3$

ឋ. $\lim\limits_{x\to\infty}\dfrac{x^2+2x+1}{5x}$

ឌ. $\lim\limits_{x\to\infty}\dfrac{x^3+x^4-1}{2x^5-x^2+x}$

ឍ. $\lim\limits_{x\to\infty}\dfrac{\left(\sqrt{x^2+1}+x\right)^2}{\sqrt[3]{x^6+1}}$

ណ. $\lim\limits_{x\to-\infty}\dfrac{x^6+7x^4-40}{1-x-5x^7}$

ត. $\lim\limits_{x\to\infty}\dfrac{(x+1)(x-2)}{3x^2+6x-5}$

ថ. $\lim\limits_{x\to\infty}\dfrac{\sqrt{1+x^2}}{x}$

ទ. $\lim\limits_{x\to\infty}\left(\dfrac{3x^2+2x+1}{x^2-3x+1}\right)^4$

ធ. $\lim\limits_{x\to-\infty}\dfrac{5x^3-x^2+x}{1+x^2+3x^3}$

ន. $\lim\limits_{x\to\infty}\dfrac{1+x-3x^3}{1+x^2+3x^3}$

ប. $\lim\limits_{x\to-\infty}\left(\dfrac{x^3-8}{x^4+16}\right)^{10}$

ផ. $\lim\limits_{x\to\infty}\dfrac{(x+3)(x+4)(x+5)}{x^4+x-11}$

ព. $\lim\limits_{x\to-\infty}\dfrac{8x-2x^5+x^6}{11x+5x^3+3x^5}$

ភ. $\lim\limits_{x\to\infty}\left(\dfrac{x^3}{2x^2-1}-\dfrac{x^2}{2x+1}\right)$

ម. $\lim\limits_{x\to\infty}\left(x^2-\dfrac{x^4-1}{x^2-2}\right)$

យ. $\lim\limits_{x\to\infty}\dfrac{(x-1)^{100}(6x+1)^{200}}{(3x+5)^{300}}$

រ. $\lim\limits_{x\to\infty}\dfrac{\sqrt[4]{x^5}+\sqrt[5]{x^3}+\sqrt[6]{x^8}}{\sqrt[3]{x^4+2}}$

ល. $\lim\limits_{x\to-\infty}\dfrac{x^2(2x+1)(3x-2)}{2x^2(5x-8)(x+6)}$

វ. $\lim\limits_{x\to\infty}\left(\dfrac{2x^8+8x^6+6x^4}{4x^8-x^6+12x^4}\right)^5$

ស. $\lim\limits_{x\to-\infty}\dfrac{(2x-3)^{20}(3x+2)^{30}}{(2x+1)^{50}}$

ដំណោះស្រាយ

មេរៀនទី 4 លំហាត់ប្រតិបត្តិសម្រាប់អនុវត្ត

លំហាត់អនុវត្តទី៦

ចូរដោះស្រាយលីមីតខាងក្រោមឲ្យបានត្រឹមត្រូវ៖

ក. $\lim_{x\to1}\frac{\sqrt[3]{x}-1}{x-1}$

ខ. $\lim_{x\to0}\frac{\sqrt[n]{a+x}-\sqrt[n]{a}}{x}$

គ. $\lim_{x\to1}\frac{1-\sqrt[2021]{x}}{1-x}$

ឃ. $\lim_{x\to0}\frac{\sqrt[5]{(x+1)^3}-1}{x}$

ង. $\lim_{x\to1}\frac{1-\sqrt[3]{2-x}}{1-x}$

ច. $\lim_{x\to0}\frac{\sqrt[3]{x+1}-1}{x(x+2)}$

ឆ. $\lim_{x\to2}\frac{x^2-x-2}{\sqrt[3]{x+6}-2}$

ជ. $\lim_{x\to1}\frac{\sqrt[5]{x^2+x-1}-1}{1+\sqrt[3]{1-2x}}$

ឈ. $\lim_{x\to0}\frac{\sqrt{1+2x}-1}{3x}$

ញ. $\lim_{x\to0}\frac{\sqrt{1+x}-\sqrt{1-x}}{x}$

ដ. $\lim_{x\to0}\frac{x-\sqrt{x}}{\sqrt{x}}$

ឋ. $\lim_{x\to5}\frac{\sqrt{x-1}-2}{x^2-25}$

ឌ. $\lim_{x\to3}\frac{\sqrt{x+6}-3}{x^3-5x^2+3x+9}$

ឍ. $\lim_{x\to9}\frac{3-\sqrt{x}}{27-\sqrt{x^3}}$

ណ. $\lim_{x\to0}\frac{\sqrt[3]{1+x}-\sqrt[3]{1-x}}{x}$

ត. $\lim_{x\to1}\frac{x^{\frac{2}{3}}-1}{x^{\frac{3}{5}}-1}$

ថ. $\lim_{x\to1}\frac{1-\sqrt[n]{x}}{1-\sqrt[m]{x}}$

ទ. $\lim_{x\to1}\frac{1-\sqrt{x}}{1-x^2}$

ធ. $\lim_{x\to4}\frac{3-\sqrt{5+x}}{1-\sqrt{5-x}}$

ន. $\lim_{x\to1}\frac{\sqrt{x+8}-\sqrt{8x+1}}{\sqrt{5-x}-\sqrt{7x-3}}$

ប. $\lim_{x\to0}\frac{\sqrt{4+x}-\sqrt{4-x}}{4x}$

ផ. $\lim_{x\to7}\frac{2-\sqrt{x-3}}{x^2-49}$

ព. $\lim_{x\to0}\frac{\sqrt{1+x}-\sqrt{1-x}}{x}$

ភ. $\lim_{x\to3}\frac{\sqrt{x^2-2x+6}-\sqrt{x^2+2x-6}}{x^2-4x+3}$

ម. $\lim_{x\to-1}\frac{x+1}{2-\sqrt{4+x+x^2}}$

យ. $\lim_{x\to0}\frac{\sqrt{x^2+1}-1}{\sqrt{x^2+16}-4}$

រ. $\lim_{x\to0}\frac{\sqrt{x^2+1}-1}{\sqrt{x^2+9}-3}$

ល. $\lim_{x\to2}\frac{\sqrt[3]{10-x}-2}{x-2}$

វ. $\lim_{x\to1}\frac{x-1}{\sqrt[3]{26+x}-3}$

ស. $\lim_{x\to0}\frac{\sqrt[3]{1+x^2+1}-1}{x^2}$

ដំណោះស្រាយ

មេរៀនទី 4 លំហាត់ប្រតិបត្តិសម្រាប់អនុវត្ត

លំហាត់អនុវត្តទី៧

ចូរដោះស្រាយលីមីតខាងក្រោមឲ្យបានត្រឹមត្រូវ៖

ក. $\lim\limits_{x\to\infty}(-3x^2+5x+2)$

ខ. $\lim\limits_{x\to\infty}(-2+x-x^4)$

គ. $\lim\limits_{x\to\infty}(-x^3+x^2-4x+5)$

ឃ. $\lim\limits_{x\to+\infty}(1+x-x^5)$

ង. $\lim\limits_{x\to-\infty}(1-3x-x^3)$

ច. $\lim\limits_{x\to\infty}(\sqrt{x-2}-\sqrt{x})$

ឆ. $\lim\limits_{x\to\infty}(\sqrt{x^2+x}-x)$

ជ. $\lim\limits_{x\to-\infty}(\sqrt{x^2+x}-x)$

ឈ. $\lim\limits_{x\to+\infty}(\sqrt{x-3}-\sqrt{x})$

ញ. $\lim\limits_{x\to+\infty}\sqrt{x}(\sqrt{x-3}-\sqrt{x})$

ដ. $\lim\limits_{x\to+\infty}x(\sqrt{x^2+1}-x)$

ឋ. $\lim\limits_{x\to-\infty}x(\sqrt{x^2+1}-x)$

ឌ. $\lim\limits_{x\to+\infty}(\sqrt{x^2+1}-x)$

ឍ. $\lim\limits_{x\to-\infty}(\sqrt{x^2+1}-x)$

ណ. $\lim\limits_{x\to+\infty}(\sqrt{x+1}-\sqrt{x^2+3})$

ត. $\lim\limits_{x\to\pm\infty}(x\sqrt{x^2+1}-x)$

ថ. $\lim\limits_{x\to-\infty}(\sqrt{2x^2+x}+3x-1)$

ទ. $\lim\limits_{x\to\infty}(\sqrt{x^2-1}-\sqrt{x^2+3x})$

ធ. $\lim\limits_{x\to+\infty}(\sqrt[3]{x^3+3x^2+1}-x)$

ន. $\lim\limits_{x\to-\infty}(\sqrt[4]{x^4+4}-x)$

ដំណោះស្រាយ

លំហាត់អនុវត្តទី៨

ចូរដោះស្រាយលីមីតខាងក្រោមឲ្យបានត្រឹមត្រូវ៖

ក. $\lim\limits_{x\to\infty}\frac{2x-7}{3x}$

ខ. $\lim\limits_{x\to\infty}\frac{2x+1}{x}$

គ. $\lim\limits_{x\to\infty}\frac{7+x^2}{-3+x^2}$

ឃ. $\lim\limits_{x\to\infty}\frac{2x}{3+(x-1)^2}$

ង. $\lim\limits_{x\to\infty}\frac{3x^2-4x+4}{(x-1)^2}$

ច. $\lim\limits_{x\to\infty}\frac{\sqrt{x+2}-\sqrt{2}}{x}$

ឆ. $\lim\limits_{x\to\infty}\frac{\sqrt{x+5}-\sqrt{5}}{\sqrt{x}-5}$

ជ. $\lim\limits_{x\to\infty}\frac{\sqrt{x^2+9}-\sqrt{x^2-9}}{6x}$

ឈ. $\lim\limits_{x\to\infty}\frac{\sqrt{x-1}-2x}{x-7}$

ញ. $\lim\limits_{x\to\infty}\frac{\sqrt{x}-6x}{3x+1}$

ដ. $\lim\limits_{x\to\infty}\frac{\sqrt{x^2+1}+\sqrt{x}}{\sqrt[4]{x^3+x}-x}$

ឋ. $\lim\limits_{x\to\infty}\frac{\sqrt{x^2+1}+\sqrt{x}}{\sqrt[4]{x^2+1}-x}$

ឌ. $\lim\limits_{x\to\infty}\frac{\sqrt[3]{x}-2\sqrt{x^3}}{\sqrt[4]{x^5}+x\sqrt{x}}$

ឍ. $\lim\limits_{x\to-\infty}\frac{\sqrt{x^2+1}-\sqrt[3]{x^2+1}}{2\sqrt[4]{x^4+1}-\sqrt[5]{x^4+1}}$

ណ. $\lim\limits_{x\to+\infty}\frac{\sqrt{x}}{\sqrt{x+\sqrt{x}}}$

ត. $\lim\limits_{x\to+\infty}\frac{\sqrt{x+\sqrt{x+\sqrt{x}}}}{\sqrt{x}}$

ថ. $\lim\limits_{x\to\infty}\frac{\sqrt{x^2+2x-3}+2x}{\sqrt{x^2+4}+x}$

ទ. $\lim\limits_{x\to\infty}\frac{\sqrt[4]{x^4+2}}{\sqrt[3]{5+27x^3}}$

ធ. $\lim\limits_{x\to+\infty}\frac{\sqrt{x+\sqrt{x+\sqrt{x}}}}{\sqrt{x+1}}$

ន. $\lim\limits_{x\to+\infty}\frac{\sqrt{x}-\sqrt{x+1}}{x+1}$

ដំណោះស្រាយ

មេរៀនទី 4 លំហាត់ប្រតិបត្តិសម្រាប់អនុវត្ត

លំហាត់អនុវត្តទី៩

ចូរដោះស្រាយលីមីតខាងក្រោមឲ្យបានត្រឹមត្រូវ៖

ក. $\lim\limits_{x\to\frac{\pi}{6}}(\sin^2 x+\cos^2 x)$

ខ. $\lim\limits_{x\to\frac{\pi}{6}}(\sin x-1)^2$

គ. $\lim\limits_{x\to\pi}\tan x$

ឃ. $\lim\limits_{x\to 7\pi}\cos\frac{x}{3}$

ង. $\lim\limits_{x\to\frac{13\pi}{4}}(\cot x-\tan x)$

ច. $\lim\limits_{x\to 0}\frac{\sin x+\cos^2 x}{x^2+1}$

ឆ. $\lim\limits_{x\to\frac{\pi}{4}}\frac{\cos x-\tan x}{\sin^2 x}$

ជ. $\lim\limits_{x\to\frac{\pi}{3}}\frac{\tan x-\sin x}{\cos x-1}$

ឈ. $\lim\limits_{x\to\frac{\pi}{6}}\frac{\sin^2 x+\tan^2 x}{2\sin x}$

ញ. $\lim\limits_{x\to 0}\frac{\cos x+\tan x}{\tan^2 x+3}$

ដ. $\lim\limits_{x\to\frac{\pi}{4}}\frac{\cos x+\sin x}{\cos^2 2x}$

ឋ. $\lim\limits_{x\to 0}\frac{\sin 10x}{10x}$

ឌ. $\lim\limits_{x\to 0}\frac{\sin 3x}{2x}$

ឍ. $\lim\limits_{x\to 0}\frac{\tan 8x}{x}$

ណ. $\lim\limits_{x\to 0}\frac{\sin 3x}{5x}$

ត. $\lim\limits_{x\to 0}\frac{\tan 5x}{\sin 4x}$

ថ. $\lim\limits_{x\to 0}\frac{\tan 5x}{\tan 6x}$

ទ. $\lim\limits_{x\to 0}\frac{\tan x}{3x}$

ធ. $\lim\limits_{x\to 0}\frac{1-\cos x}{x}$

ន. $\lim\limits_{x\to 0}\frac{x}{\sqrt{1-\cos x}}$

ប. $\lim\limits_{x\to 0}\frac{\sin x}{x^3}$

ផ. $\lim\limits_{x\to 0}\frac{1-\cos 2x}{x\sin x}$

ព. $\lim\limits_{x\to 0}\frac{\sin^3\frac{x}{2}}{x^3}$

ភ. $\lim\limits_{x\to 0}\frac{\sin 4x+\sin 7x}{\sin 3x}$

ម. $\lim\limits_{x\to 0}\frac{\sqrt{1-\cos x^2}}{1-\cos x}$

យ. $\lim\limits_{x\to 0}\frac{\tan x-\sin x}{x^3}$

រ. $\lim\limits_{x\to\infty}x\sin\frac{\pi}{x}$

ល. $\lim\limits_{x\to -1}\frac{x^3+1}{\sin(x+1)}$

វ. $\lim\limits_{x\to 0^+}\frac{|\sin x|}{x}$

ស. $\lim\limits_{x\to 1}\frac{\tan(x-1)}{\sqrt{x}-1}$

ហ. $\lim\limits_{x\to 0}\frac{\sqrt{\cos x}-1}{\sin^2 x}$

ឡ. $\lim\limits_{x\to 0}\frac{\sin 3x+\sin 5x}{\sin 2x}$

អ. $\lim\limits_{x\to 0}\frac{\cos x-\cos^3 x}{x^2}$

ដំណោះស្រាយ

មេរៀនទី 4 លំហាត់ប្រតិបត្តិសម្រាប់អនុវត្ត

លំហាត់អនុវត្តទី១០

ចូរដោះស្រាយលីមីតខាងក្រោមឲ្យបានត្រឹមត្រូវ៖

ក. $\lim\limits_{x\to 0}\frac{2\sin x}{3x}$ ខ. $\lim\limits_{x\to 0}\frac{1-\cos x}{2x}$ គ. $\lim\limits_{x\to 0}\frac{2\tan x}{x}$

ឃ. $\lim\limits_{x\to 0}\frac{\sin ax}{\sin bx}$ ង. $\lim\limits_{x\to 0}\frac{\sin 4x}{3x}$ ច. $\lim\limits_{x\to 0}\frac{\tan mx}{\sin nx}$

ឆ. $\lim\limits_{x\to 0}\frac{1-\cos x}{x\sin x}$ ជ. $\lim\limits_{x\to \frac{\pi}{4}}\frac{\cos x-\sin x}{\cos 2x}$ ឈ. $\lim\limits_{x\to 0}\frac{\sin 3x}{x}$

ញ. $\lim\limits_{x\to 0}\frac{\tan 3x}{2x}$ ដ. $\lim\limits_{x\to 0}\frac{\sin 2x}{\sin 5x}$ ឋ. $\lim\limits_{x\to 0}\frac{1-\cos 2x}{x^2}$

ឌ. $\lim\limits_{x\to 0}\frac{\cos x-\cos^2 x}{x}$ ឍ. $\lim\limits_{x\to 0}\frac{\cos 3x-\cos^2 3x}{6x}$ ណ. $\lim\limits_{x\to 0}\frac{1-\cos 4x}{x^2}$

ត. $\lim\limits_{x\to 0}\frac{\sin x-\sin 3x}{x}$ ថ. $\lim\limits_{x\to 0}\frac{\sin 2x-3x}{\sin 3x-2x}$ ទ. $\lim\limits_{x\to 0}\frac{\sin x-2x}{x-2\sin x}$

ធ. $\lim\limits_{x\to 0}\frac{x+\sin x}{2x-3\sin 2x}$ ន. $\lim\limits_{x\to 0}\frac{1-\cos^2 x}{3x^2}$ ប. $\lim\limits_{x\to 0}\frac{1-\cos 5x}{x^2}$

ផ. $\lim\limits_{x\to 0}\frac{\sin(3x+1989\pi)}{\sin(4x+1987\pi)}$ ព. $\lim\limits_{x\to 0}\frac{1-\cos x}{\sin^2 x}$ ភ. $\lim\limits_{x\to 0}\left(\frac{2}{\sin^2 x}-\frac{1}{1-\cos x}\right)$

ម. $\lim\limits_{x\to 0}\frac{\sin x+1-\cos x}{x}$ យ. $\lim\limits_{x\to 0}\frac{\sin x-2\sin 4x}{\tan x}$ រ. $\lim\limits_{x\to \frac{\pi}{6}}\frac{2\sin^2 x-3\sin x+1}{4\sin x-1}$

ល. $\lim\limits_{x\to 0}\frac{1-\cos nx}{1-\cos mx}$ វ. $\lim\limits_{x\to 0}\frac{3x+\sin 2x}{3x-\sin 2x}$ ស. $\lim\limits_{x\to 0}\frac{(1+x^2)-\cos x}{\tan^2 x}$

ហ. $\lim\limits_{x\to 0}\frac{\sin(a+x)-\sin(a-x)}{x}$ ឡ. $\lim\limits_{x\to 0}\frac{\tan x-\sin x}{x}$ អ. $\lim\limits_{x\to 0}\frac{1+\sin x-\cos x}{1-\sin x-\cos x}$

ដំណោះស្រាយ

មេរៀនទី 4 លំហាត់ប្រតិបត្តិសម្រាប់អនុវត្ត

មេរៀនទី 4 លំហាត់ប្រតិបត្តិសម្រាប់អនុវត្ត

លំហាត់អនុវត្តទី១១

ចូរដោះស្រាយលីមីតខាងក្រោមឲ្យបានត្រឹមត្រូវ៖

ក. $\lim\limits_{x\to 0} x\cot 2x$

ខ. $\lim\limits_{x\to 0}\sqrt{\dfrac{1-\cos x}{x^2}}$

គ. $\lim\limits_{x\to \pi}\dfrac{\sqrt{1-\tan x}-\sqrt{1+\tan x}}{\sin 2x}$

ឃ. $\lim\limits_{x\to 0}\dfrac{\ln x}{\ln(\sin x)}$

ង. $\lim\limits_{x\to 0}\dfrac{\ln(\sin 2x)}{\ln(\sin x)}$

ច. $\lim\limits_{x\to 0}\dfrac{2\tan x-\sin x}{x}$

ឆ. $\lim\limits_{x\to 0}\dfrac{x(a-b)}{\sin ax-\sin bx}, a\neq b$

ជ. $\lim\limits_{x\to 0}\dfrac{\sin(\sin(\sin x))}{x}$

ឈ. $\lim\limits_{x\to 0}\dfrac{\sin 5x+x\sin 10x}{\sin 5x+x\sin 10x}$

ញ. $\lim\limits_{x\to 0}\dfrac{(1-\cos x)\sin x}{\tan^3 x}$

ដំណោះស្រាយ

មេរៀនទី 4 លំហាត់ប្រតិបត្តិសម្រាប់អនុវត្ត

លំហាត់អនុវត្តទី១២

ចូរដោះស្រាយលីមីតខាងក្រោមឲ្យបានត្រឹមត្រូវ៖

ក. $\lim\limits_{x\to 1}\frac{\sin(\ln x)}{\ln x}$

ខ. $\lim\limits_{x\to -\infty}\frac{\tan(e^x)}{e^x}$

គ. $\lim\limits_{x\to 0}\frac{\sin(x^2)}{x}$

ឃ. $\lim\limits_{x\to 0}\frac{\sin(\sin^2 2x)}{x^2}$

ង. $\lim\limits_{x\to 0}\frac{1-\cos x}{x^2}$

ច. $\lim\limits_{x\to 1}\frac{\cos\left(\frac{\pi x}{2}\right)}{1-x}$

ឆ. $\lim\limits_{x\to \frac{\pi}{2}}\left(\frac{\pi}{2}-x\right)\tan x$

ជ. $\lim\limits_{x\to 1}(1-x)\tan\frac{\pi x}{2}$

ឈ. $\lim\limits_{x\to 0}\frac{\sin^{-1}2x}{3x}$

ញ. $\lim\limits_{x\to 0}\frac{x}{\tan^{-1}2x}$

ដ. $\lim\limits_{x\to 0}\frac{\sin^{-1}(x^2)}{(\tan^{-1}x)^2}$

ឋ. $\lim\limits_{x\to -3}\frac{x^2-9}{\tan^{-1}(x+3)}$

ឌ. $\lim\limits_{x\to 0}\frac{2x-\sin^{-1}x}{2x+\tan^{-1}x}$

ឍ. $\lim\limits_{x\to \frac{\pi}{6}}\frac{\sin\left(x-\frac{\pi}{6}\right)}{\sqrt{3}-2\cos x}$

ណ. $\lim\limits_{x\to \frac{\pi}{4}}\frac{\cos x-\sin x}{\left(\frac{\pi}{4}-x\right)(\cos x+\sin x)}$

ត. $\lim\limits_{x\to a}\frac{\cos\sqrt{x}-\cos\sqrt{a}}{x-a}$

ថ. $\lim\limits_{x\to 0}\frac{x(5^x-1)}{1-\cos x}$

ទ. $\lim\limits_{x\to 0}\frac{(1-\cos 2x)\sin 5x}{x^2\sin 3x}$

ធ. $\lim\limits_{x\to 0}\frac{\sin 4x}{1-\sqrt{1-x}}$

ន. $\lim\limits_{x\to 0}\frac{\sin x}{\sqrt{x^2}}$

ប. $\lim\limits_{x\to 0}\frac{\cos x+4\tan x}{2-x-2x^4}$

ផ. $\lim\limits_{x\to 0}\frac{\sqrt{x+4}-2}{\sin 5x}$

ព. $\lim\limits_{x\to 1}\frac{\sin(1-x)}{\sqrt{x}-1}$

ភ. $\lim\limits_{a\to \pi}\frac{\sin a}{1-\frac{a^2}{\pi^2}}$

ដំណោះស្រាយ

មេរៀនទី ៤ លំហាត់ប្រតិបត្តិសម្រាប់អនុវត្ត

លំហាត់អនុវត្តទី១៣

ចូរដោះស្រាយលីមីតខាងក្រោមឲ្យបានត្រឹមត្រូវ៖

ក. $\lim\limits_{x\to+\infty} e^{-x}$

ខ. $\lim\limits_{x\to-\infty} e^{2-3x}$

គ. $\lim\limits_{x\to+\infty} (e^x-7x+6)$

ឃ. $\lim\limits_{x\to-\infty} e^x+3$

ង. $\lim\limits_{x\to+\infty} (2e^x-x^2+1)$

ច. $\lim\limits_{x\to0}(e^x+2x^2-6)$

ឆ. $\lim\limits_{x\to+\infty} (x^3+x)e^{-x}$

ជ. $\lim\limits_{x\to-\infty} \frac{e^{2x}+1}{e^x+3}$

ឈ. $\lim\limits_{x\to+\infty} \frac{xe^x+2}{x^2-1}$

ញ. $\lim\limits_{x\to-\infty} xe^x$

ដ. $\lim\limits_{x\to+\infty} \frac{2e^x-1}{x^2+x-3}$

ឋ. $\lim\limits_{x\to-\infty} (xe^x+e^x+2)$

ឌ. $\lim\limits_{x\to+\infty} \frac{x^3-e^x}{2e^x+3x+1}$

ឍ. $\lim\limits_{x\to+\infty} \frac{x-e^x}{2e^x+3}$

ណ. $\lim\limits_{x\to-\infty} \frac{e^x-e^{-x}}{e^x+e^{-x}}$

ត. $\lim\limits_{x\to+\infty} \frac{3e^{x-2}}{x^3}$

ថ. $\lim\limits_{x\to+\infty} (x+1)e^{-x}$

ទ. $\lim\limits_{x\to0} \frac{e^{2x}-e^{-2x}}{x}$

ធ. $\lim\limits_{x\to0} \frac{e^x-e^{-x}-2}{x^2}$

ន. $\lim\limits_{x\to0} \frac{2e^{2x}-3e^x+1}{x}$

ដំណោះស្រាយ

លំហាត់អនុវត្តទី១៤

ចូរដោះស្រាយលីមីតខាងក្រោមឲ្យបានត្រឹមត្រូវ៖

ក. $\lim\limits_{x\to 0}\frac{e^x+\sin x-1}{x}$

ខ. $\lim\limits_{x\to 0}\frac{e^{2x}-1}{e^{3x}-1}$

គ. $\lim\limits_{x\to 0}\frac{e^{\sin x}+\sin 2x-1}{\sin x}$

ឃ. $\lim\limits_{x\to 0}\frac{(e^{-3x+2}-e^2)\sin\pi x}{4x^2}$

ង. $\lim\limits_{x\to 0}\frac{\sin x+e^x-1}{x^2+x}$

ច. $\lim\limits_{x\to 1}\frac{1-x}{e^x-e}$

ឆ. $\lim\limits_{x\to 0}\frac{5^x-4^x}{x^2-x}$

ជ. $\lim\limits_{x\to 0}\frac{2-(e^x+e^{-x})}{x^2}$

ឈ. $\lim\limits_{x\to 0}\frac{e^{\sin x}-e^{\sin 3x}}{\sin 2x}$

ញ. $\lim\limits_{x\to 0}\frac{e^{ax}-e^{bx}}{x}$

ដ. $\lim\limits_{x\to 0}\frac{e^{\sin^2 2x}-\cos(\sin 3x)}{x^2}$

ឋ. $\lim\limits_{x\to 0}\frac{e^{x^2}-\cos x}{x^2}$

ឌ. $\lim\limits_{x\to 0}\frac{e^{x+1}-e}{\sin\pi x}$

ឍ. $\lim\limits_{x\to \pi}\frac{\sin 2x}{e^{x+\pi}-e^{2\pi}}$

ណ. $\lim\limits_{x\to 0}\frac{e^x+\sin x-1}{1-\sqrt{x+1}}$

ត. $\lim\limits_{x\to 0}\frac{(e^x-1)(1-\cos 2x)\sin 7x}{x^4}$

ថ. $\lim\limits_{x\to 0}\frac{x^2}{e^{x^2}-4^{x^2}}$

ទ. $\lim\limits_{x\to 0}\frac{e^x-e^{-x}}{\sin 2x}$

ធ. $\lim\limits_{x\to 0}\frac{e^{2x}+3e^{2x}-5}{e^{3x}-1}$

ន. $\lim\limits_{x\to 0}\frac{e^{-x^2}-\cos 2x}{x^2}$

ដំណោះស្រាយ

លំហាត់អនុវត្តទី១៥

ចូរដោះស្រាយលីមីតខាងក្រោមឲ្យបានត្រឹមត្រូវ៖

ក. $\lim\limits_{x\to+\infty}\frac{\ln x}{x^2-x}$

ខ. $\lim\limits_{x\to-\infty}\ln(e^x+1)$

គ. $\lim\limits_{x\to0^+}\ln\left(\frac{x}{x+1}\right)$

ឃ. $\lim\limits_{x\to\infty}x\ln(x^2+1)$

ង. $\lim\limits_{x\to+\infty}(x\ln x-x^2+5)$

ច. $\lim\limits_{x\to1^-}x\ln(4-3x-x^2)$

ឆ. $\lim\limits_{x\to+\infty}x[\ln(x+1)-\ln x]$

ជ. $\lim\limits_{x\to0^+}(x\ln x-x^2+2)$

ឈ. $\lim\limits_{x\to0^+}\frac{x^2\ln x+2}{\ln x+1}$

ញ. $\lim\limits_{x\to0^+}\left(\frac{1}{x}-\frac{\ln x}{x}\right)$

ដ. $\lim\limits_{x\to+\infty}\frac{e^x\ln x+1}{x^2}$

ឋ. $\lim\limits_{x\to0}\frac{\ln(1+x)}{1-e^x}$

ឌ. $\lim\limits_{x\to1}\frac{x^2-1+\ln x}{e^x-e}$

ឍ. $\lim\limits_{x\to+\infty}[x+3-3\ln(e^x+1)]$

ណ. $\lim\limits_{x\to0}\frac{\ln(1+x)}{e^{2x}-1}$

ត. $\lim\limits_{x\to+\infty}\left(x+\frac{\ln x}{2x+1}\right)$

ថ. $\lim\limits_{x\to0}\frac{\ln(1+2x^2)}{1-\cos 2x}$

ទ. $\lim\limits_{x\to0}\frac{\ln(\cos 2x)}{x^2}$

ធ. $\lim\limits_{x\to0}\frac{\ln(x^2-x+1)}{x}$

ន. $\lim\limits_{x\to0}\frac{\ln(1+x)+\ln(1+2x)}{x}$

ដំណោះស្រាយ

លំហាត់អនុវត្តទី១៦

ចូរដោះស្រាយលីមីតខាងក្រោមឲ្យបានត្រឹមត្រូវ៖

ក. $\lim\limits_{n\to+\infty}\left(\frac{1+n}{n}\right)^{n+2020}$

ខ. $\lim\limits_{x\to+\infty}\left(\frac{2x+3}{2x+1}\right)^{x+1}$

គ. $\lim\limits_{x\to+\infty}\left(1+\frac{1}{x}\right)^{\frac{x+1}{x}}$

ឃ. $\lim\limits_{x\to\infty}\left(1+\frac{1}{x}\right)^{3x}$

ង. $\lim\limits_{x\to\infty}\left(1+\frac{1}{x^2}\right)^{3x-4}$

ច. $\lim\limits_{x\to\infty}\left(1+\frac{1}{5x}\right)^{2x+6}$

ឆ. $\lim\limits_{x\to\infty}\left(1+\frac{7}{3x}\right)^{x-1}$

ជ. $\lim\limits_{x\to\infty}\left(1-\frac{1}{3x}\right)^{x}$

ឈ. $\lim\limits_{x\to\infty}\left(1-\frac{5}{x}\right)^{x}$

ញ. $\lim\limits_{x\to\infty}\left(\frac{x-1}{x+1}\right)^{x}$

ដ. $\lim\limits_{x\to\infty}\left(\frac{x+6}{x+5}\right)^{x}$

ឋ. $\lim\limits_{x\to\infty}\left(\frac{x-3}{x}\right)^{\frac{x}{2}}$

ឌ. $\lim\limits_{x\to\infty}\left(\frac{7x+10}{1+7x}\right)^{\frac{x}{3}}$

ឍ. $\lim\limits_{x\to\infty}\left(\frac{2x-1}{2x+1}\right)^{x}$

ណ. $\lim\limits_{x\to\infty}\left(\frac{6+4x}{2+4x}\right)^{3-2x}$

ត. $\lim\limits_{x\to\infty}\left(\frac{x+5}{x+4}\right)^{2x-1}$

ថ. $\lim\limits_{x\to\infty}\left(\frac{2x+5}{2x}\right)^{3x-7}$

ទ. $\lim\limits_{x\to\infty}\left(\frac{2x+1}{2x-3}\right)^{3x}$

ធ. $\lim\limits_{x\to\infty}\left(\frac{x-1}{x+3}\right)^{5-4x}$

ន. $\lim\limits_{x\to\infty}\left(\frac{x}{x+1}\right)^{x}$

ប. $\lim\limits_{x\to\infty}\left(\frac{3x-2}{3x+1}\right)^{2x}$

ផ. $\lim\limits_{x\to\infty}\left(\frac{2x-5}{2x-2}\right)^{4x^2}$

ព. $\lim\limits_{x\to\infty}\left(\frac{3x+6}{3x-1}\right)^{x^2}$

ភ. $\lim\limits_{x\to\infty}\left(\frac{x^2+2x+1}{x^2+3}\right)^{x}$

ម. $\lim\limits_{x\to0}(1+2x)^{\frac{1}{x}}$

យ. $\lim\limits_{x\to0}(1+5x)^{\frac{1}{8x}}$

រ. $\lim\limits_{x\to\infty}\left[\ln\left(\frac{3x+1}{3x-5}\right)^{2-x}\right]$

ល. $\lim\limits_{x\to\infty}x\left[\ln(x+3)-\ln x\right]$

វ. $\lim\limits_{x\to\infty}x\left[\ln x-\ln(x+2)\right]$

ស. $\lim\limits_{x\to\infty}(x+1)\left[\ln(x+1)-\ln x\right]$

ដំណោះស្រាយ

មេរៀនទី ៤ លំហាត់ប្រតិបត្តិសម្រាប់អនុវត្ត

លំហាត់អនុវត្តទី១៧

ចូរដោះស្រាយលីមីតខាងក្រោមឲ្យបានត្រឹមត្រូវ៖

ក. $\lim\limits_{x\to0^+}(1+2x)^{\cot x}$

ខ. $\lim\limits_{x\to0}(1+\sin x)^{\frac{1}{x}}$

គ. $\lim\limits_{x\to0}(2-e^x)^{\frac{1}{x}}$

ឃ. $\lim\limits_{x\to0}(1+\sin x)^{\cot x}$

ង. $\lim\limits_{x\to0}(\cos x)^{\frac{1}{\sin x}}$

ច. $\lim\limits_{x\to0}(1+3\tan^2 x)^{\cot^2 x}$

ឆ. $\lim\limits_{x\to0}\sqrt[\sin x]{1+x}$

ជ. $\lim\limits_{x\to0}\left(\frac{2x+3}{2x+1}\right)^{\frac{1}{\sin 3x}}$

ឈ. $\lim\limits_{x\to0}(x+2^x)^{\frac{1}{x}}$

ញ. $\lim\limits_{x\to0}(\cos x+\sin x)^{\frac{1}{x}}$

ដ. $\lim\limits_{x\to0}(1+\tan^2\sqrt{x})^{\frac{1}{2x}}$

ឋ. $\lim\limits_{x\to0}\left(\frac{x^2-x+1}{x^2+x+1}\right)^{\frac{1}{\sin x}}$

ឌ. $\lim\limits_{x\to0}\left(\frac{\sin x}{x}\right)^{\frac{\tan x}{x-\sin x}}$

ឍ. $\lim\limits_{x\to0}(2-2\cos x)^{\frac{1}{x^2}}$

ណ. $\lim\limits_{x\to1}(1+\sin\pi x)^{\cot\pi x}$

ត. $\lim\limits_{x\to+\infty}(1+2x^2)^{\cot^2 x}$

ថ. $\lim\limits_{n\to0}\left(1+\sin\frac{n}{2}\right)^{\frac{1}{n}}$

ទ. $\lim\limits_{x\to0}(\cos x+\sin^2 x)^{\frac{1}{x^2}}$

ធ. $\lim\limits_{x\to+\infty}(x)^{\frac{1}{x}}$

ន. $\lim\limits_{x\to\infty}\left(\frac{x+a}{x+b}\right)^x$

ដំណោះស្រាយ

លំហាត់អនុវត្តទី១៨

ដោះស្រាយលីមីតខាងក្រោមឲ្យបានត្រឹមត្រូវ៖

ក. $\lim\limits_{n\to\infty}\left(\frac{1}{n^2}+\frac{2}{n^2}+\cdots+\frac{n-1}{n^2}\right)$

ខ. $\lim\limits_{n\to\infty}\left[\frac{1+3+5+\cdots+(2n-1)}{n+1}-\frac{1}{2n+3}\right]$

គ. $\lim\limits_{n\to\infty}\frac{1+2+3+\cdots+n}{\sqrt{9n^4+1}}$

ឃ. $\lim\limits_{n\to\infty}\frac{1+3+5+\cdots+(2n-1)}{1+2+3+\cdots+n}$

ង. $\lim\limits_{n\to\infty}\left[\frac{1+3+5+\cdots+(2n-1)}{n+3}-n\right]$

ច. $\lim\limits_{n\to\infty}\frac{1+4+7+\cdots+(3n-2)}{\sqrt{5n^2+n+1}}$

ឆ. $\lim\limits_{n\to\infty}\frac{1+\frac{1}{3}+\frac{1}{3^2}+\cdots+\frac{1}{3^n}}{1+\frac{1}{5}+\frac{1}{5^2}+\cdots+\frac{1}{5^n}}$

ជ. $\lim\limits_{n\to\infty}\frac{1-3+5-7+\cdots+(4n-)+(4n-1)}{\sqrt{n^2+1}+\sqrt{n^2+n+1}}$

ឈ. $\lim\limits_{n\to\infty}\frac{1-2+3-4+\cdots+(2n-1)-2n}{\sqrt{9n^2+1}}$

ញ. $\lim\limits_{n\to\infty}\frac{\sqrt[3]{n^3+5}-\sqrt{3n^4+2}}{1+2+3+\cdots+(2n-1)}$

ដ. $\lim\limits_{n\to\infty}\left(\frac{n+2}{1+2+3+\cdots+n}-\frac{2}{3}\right)$

ឋ. $\lim\limits_{n\to\infty}\left(\frac{5}{6}+\frac{13}{56}+\cdots+\frac{3^n+2^n}{6^n}\right)$

ឌ. $\lim\limits_{n\to\infty}\frac{2-5+4-7+\cdots+2n-(2n+3)}{n+3}$

ឍ. $\lim\limits_{n\to\infty}\frac{1+2+3+\cdots+n}{n-n^2+3}$

ណ. $\lim\limits_{n\to\infty}\frac{n^2+\sqrt{n}-1}{2+7+12+\cdots+(5n-3)}$

ត. $\lim\limits_{n\to\infty}\left(\frac{3}{4}+\frac{5}{16}+\frac{9}{64}+\cdots+\frac{1+2^n}{4^2}\right)$

ថ. $\lim\limits_{n\to\infty}\frac{2+4+6+\cdots+2n}{1+3+5+\cdots+(2n-1)}$

មេរៀនទី 4 លំហាត់ប្រតិបត្តិសម្រាប់អនុវត្ត

ដំណោះស្រាយ

លំហាត់អនុវត្តទី១៩

ដោះស្រាយលីមីតខាងក្រោមឲ្យបានត្រឹមត្រូវៈ

ក. $\lim\limits_{n\to\infty} \frac{1-2+3-4+\cdots+(2n-1)-2n}{\sqrt[3]{n^3+2n+2}}$

ខ. $\lim\limits_{n\to\infty} \frac{3+6+9+\cdots+3n}{n^2+4}$

គ. $\lim\limits_{n\to\infty} \left(\frac{2+4+\cdots+2n}{n+3} - n\right)$

ឃ. $\lim\limits_{n\to\infty} \left(\frac{7}{10} + \frac{29}{100} + \cdots + \frac{2^n+5^n}{10^n}\right)$

ង. $\lim\limits_{n\to\infty} \left(\frac{27}{100} + \frac{27}{100^2} + \cdots + \frac{27}{100^n}\right)$

ច. $\lim\limits_{n\to\infty} \left(\sqrt{2}\cdot\sqrt[4]{2}\cdot\sqrt[8]{2}\cdots\sqrt[2^n]{2}\right)$

ឆ. $\lim\limits_{n\to\infty} \sqrt{a\sqrt{a\sqrt{a\cdots\sqrt{a}}}}$

ជ. $\lim\limits_{n\to\infty} \sqrt{2\sqrt{2\sqrt{2\cdots\sqrt{2}}}}$

ឈ. $\lim\limits_{n\to\infty} \sqrt{a\sqrt{b\sqrt{a\sqrt{b\cdots\sqrt{a\sqrt{b}}}}}}$

ញ. $\lim\limits_{n\to\infty} \sqrt{3\sqrt{5\sqrt{3\sqrt{5\cdots\sqrt{3\sqrt{5}}}}}}$

ដ. $\lim\limits_{n\to\infty} \left[1 - \frac{1}{3} + \frac{1}{9} - \frac{1}{27} + \cdots + \frac{(-1)^{2n-1}}{3^n}\right]$

ឋ. $\lim\limits_{n\to\infty} \left(\frac{1+a+a^2+\cdots+a^n}{1+b+b^2+\cdots+b^n}\right)$

ដំណោះស្រាយ

លំហាត់អនុវត្តទី២០

ដោយប្រើទ្រឹស្តីបទឡូពីតាល់ (De L'hospital) ចូរដោះស្រាយលីមីតខាងក្រោម៖

ក. $\lim\limits_{x\to\infty}\dfrac{\sqrt{x^2+1}+\sqrt{x}}{\sqrt[4]{x^3+x}-x}$

ខ. $\lim\limits_{x\to\infty}\dfrac{\sqrt{x^2+1}+\sqrt{x}}{\sqrt[4]{x^2+1}-x}$

គ. $\lim\limits_{x\to\infty}\dfrac{\sqrt[3]{x}-2\sqrt{x^3}}{\sqrt[4]{x^5}+x\sqrt{x}}$

ឃ. $\lim\limits_{x\to-\infty}\dfrac{\sqrt{x^2+1}-\sqrt[3]{x^2+1}}{2\sqrt[4]{x^4+1}-\sqrt[5]{x^4+1}}$

ង. $\lim\limits_{x\to+\infty}\dfrac{\sqrt{x}}{\sqrt{x+\sqrt{x}}}$

ច. $\lim\limits_{x\to+\infty}\dfrac{\sqrt{x+\sqrt{x+\sqrt{x}}}}{\sqrt{x}}$

ឆ. $\lim\limits_{x\to\infty}\dfrac{\sqrt{x^2+2x-3}+2x}{\sqrt{x^2+4}+x}$

ជ. $\lim\limits_{x\to\infty}\dfrac{\sqrt[4]{x^4+2}}{\sqrt[3]{5+27x^3}}$

ឈ. $\lim\limits_{x\to+\infty}\dfrac{\sqrt{x+\sqrt{x+\sqrt{x}}}}{\sqrt{x+1}}$

ញ. $\lim\limits_{x\to+\infty}\dfrac{\sqrt{x}-\sqrt{x+1}}{x+1}$

ដ. $\lim\limits_{x\to1}\dfrac{x^3-2x^2+x}{x^2-1}$

ឋ. $\lim\limits_{x\to3}\dfrac{x^3-9x}{x^4-3x^3-x+3}$

ឌ. $\lim\limits_{x\to1}\dfrac{\sqrt[3]{8x}-2x}{\sqrt[4]{x}-x}$

ឍ. $\lim\limits_{x\to1}\dfrac{\sqrt{x+1}-\sqrt{2}}{x^2-1}$

ណ. $\lim\limits_{x\to0}\dfrac{5^x-1}{x}$

ត. $\lim\limits_{x\to1}\dfrac{3x^3-3}{3^x-3}$

ថ. $\lim\limits_{x\to0}\dfrac{e^x-e^{-x}}{x}$

ទ. $\lim\limits_{x\to0}\dfrac{e^x-1}{x^2}$

ធ. $\lim\limits_{x\to\frac{\pi}{3}}\dfrac{1-\cos x}{\pi-3x}$

ន. $\lim\limits_{x\to0}\dfrac{\sin x}{e^x-1}$

ដំណោះស្រាយ

លំហាត់អនុវត្តទី២១

ដោយប្រើទ្រឹស្តីបទឡូពីតាល់ (De L'hospital) ចូរដោះស្រាយលីមីតខាងក្រោម៖

ក. $\lim\limits_{x\to 0}\dfrac{\arctan 3x}{\arcsin 2x}$ ខ. $\lim\limits_{x\to 0}\dfrac{e^x-e^{-x}}{2x-\sin x}$ គ. $\lim\limits_{x\to 0}\dfrac{x^3+\pi x}{\sin 3x}$

ឃ. $\lim\limits_{x\to 0}\dfrac{\ln(1+4x)}{3^x-1}$ ង. $\lim\limits_{x\to 0}\dfrac{\sin 4x}{\ln(1+\sin x)}$ ច. $\lim\limits_{x\to 0}\dfrac{3\ln(1-2x)}{2\arctan 3x}$

ឆ. $\lim\limits_{x\to 0}\dfrac{\arctan x+x^2}{2^{3x}-3^{2x}}$ ជ. $\lim\limits_{x\to \frac{\pi}{2}}\dfrac{\arctan(x-\frac{\pi}{2})}{\pi-2x}$ ឈ. $\lim\limits_{x\to 0}\dfrac{e^{3x}-e^{-2x}}{2\arcsin x-\sin x}$

ញ. $\lim\limits_{x\to 0}\dfrac{\ln(\cos 3x)}{\arctan 4x}$ ដ. $\lim\limits_{x\to 0}\dfrac{(1+x)^2-(1+2x)}{x^2+4x^3}$ ឋ. $\lim\limits_{x\to -1}\dfrac{(x^2+3x+2)^2}{x^3-3x-2}$

ឌ. $\lim\limits_{x\to 0}\dfrac{1-\cos^4 x}{4x^2}$ ឍ. $\lim\limits_{x\to 0}\dfrac{1-\cos x}{2x\sin x}$ ណ. $\lim\limits_{x\to 0}\dfrac{x-\sin x}{e^x-e^{-x}-2x}$

ត. $\lim\limits_{x\to 0}\dfrac{e^{3x}-3x-1}{\sin^2 x}$ ថ. $\lim\limits_{x\to 0}\dfrac{x^3}{x-\arctan x}$ ទ. $\lim\limits_{x\to 1}\dfrac{\sin(\ln x)}{\ln x}$

ធ. $\lim\limits_{x\to -\infty}\dfrac{\tan(e^x)}{e^x}$ ន. $\lim\limits_{x\to 0}\dfrac{\sin(x^2)}{x}$ ប. $\lim\limits_{x\to 0}\dfrac{\sin(\sin^2 2x)}{x^2}$

ផ. $\lim\limits_{x\to 0}\dfrac{1-\cos x}{x^2}$ ព. $\lim\limits_{x\to 1}\dfrac{\cos(\frac{\pi x}{2})}{1-x}$ ភ. $\lim\limits_{x\to 0}\dfrac{(1-\cos 2x)\sin 5x}{x^2\sin 3x}$

ម. $\lim\limits_{x\to 0}\dfrac{\sin 4x}{1-\sqrt{1-x}}$ យ. $\lim\limits_{x\to 0}\dfrac{\sin x}{\sqrt{x^2}}$ រ. $\lim\limits_{x\to 0}\dfrac{\cos x+4\tan x}{2-x-2x^4}$

ល. $\lim\limits_{x\to 0}\dfrac{\sqrt{x+4}-2}{\sin 5x}$ វ. $\lim\limits_{x\to 1}\dfrac{\sin(1-x)}{\sqrt{x}-1}$ ស. $\lim\limits_{a\to \pi}\dfrac{\sin a}{1-\frac{a^2}{\pi^2}}$

ដំណោះស្រាយ

មេរៀនទី 4 លំហាត់ប្រតិបត្តិសម្រាប់អនុវត្ត

លំហាត់ប្រឡងថ្នាក់ជាតិ

លំហាត់អនុវត្តទី២២

គណនាលីមីត $\lim\limits_{x\to 0}\frac{2-\sqrt{x+4}}{x}$។

ដំណោះស្រាយ

លំហាត់អនុវត្តទី២៣

ាអនុគមន៍កំណត់ដោយ $f(x)=\begin{cases} x-2+\dfrac{e}{\ln x} & \text{បើ}x>0 \\ m; & \text{បើ}x\neq 1 \end{cases}$ ។ គណនា $\lim\limits_{x\to 0^+} f(x)$។

កំណត់តម្លៃើម្បីអោយអនុគមន៍ាប់ខាងស្ដាំត្រង់=0

ដំណោះស្រាយ

លំហាត់អនុវត្តទី២៤

គណនាលីមីត៖

ក. $\lim\limits_{x\to 0}\dfrac{\sin x+e^x-1}{x^2+x}$

ខ. $\lim\limits_{x\to +\infty}\dfrac{\ln x+2}{x+1}$

ដំណោះស្រាយ

លំហាត់អនុវត្តទី២៥

ក. គណនាលីមីត៖ $\lim\limits_{x\to+\infty}\dfrac{e^x\ln x+1}{x^2}$។

ខ. គណនាដេរីវេនៃ $=e^x\ln(x+1), x>-1$។ ទាញរកព្រីមីទីវនៃ(x)=e$^x\ln(x+1)+\dfrac{e^x+1}{x+1}$ ដោយដឹងថាព្រីមីទីវមានតម្លៃស្មើ ពោះ=0

ដំណោះស្រាយ

លំហាត់អនុវត្តទី២៦

គណនាលីមីត៖

ក. $\lim\limits_{x\to\frac{\pi}{3}} \dfrac{\sqrt{2\cos x}-1}{2\cos 2x+1}$

ខ. $\lim\limits_{x\to 0} \dfrac{\dfrac{1}{(x+a)^2}-\dfrac{1}{a^2}}{x}$

ដំណោះស្រាយ

លំហាត់អនុវត្តទី២៧

គណនាលីមីត៖

ក. $\lim\limits_{x\to\frac{\pi}{4}} \dfrac{2\sin\left(x-\frac{\pi}{4}\right)}{\frac{\pi}{4}-x}$

ខ. $\lim\limits_{x\to 0} \dfrac{2-\sin 5x}{\sqrt{5}-\sqrt{x+5}}$

គ. $\lim\limits_{x\to 0} \dfrac{1-\cos^2 3x}{-2x^2}$

ឃ. $\lim\limits_{x\to 0} \dfrac{x^2+x}{|x|}$

ដំណោះស្រាយ

លំហាត់អនុវត្តទី២៨

គណនាលីមីត៖

ក. $\lim\limits_{x \to 1} \dfrac{x^3 - x^2 + x - 1}{x - 1}$

ខ. $\lim\limits_{x \to \frac{\pi}{2}} \dfrac{\sin^2 x - 1}{1 + \sin x}$

គ. $\lim\limits_{x \to +\infty} \left(\sqrt{x^2 + x} - x\right)$

ឃ. $\lim\limits_{x \to 0} \dfrac{(e^{-x} + e^{x}) \sin^2 x}{2x^2}$

ង. $\lim\limits_{x \to +\infty} \left[\ln(x+2) - \ln x - \dfrac{2}{x+2} + \dfrac{1}{4}\right]$

ដំណោះស្រាយ

លំហាត់អនុវត្តទី២៩

គណនាលីមីត៖

ក. $\lim_{x\to 2}\dfrac{x^3-8}{\sqrt{x+2}-2}$

ខ. $\lim_{x\to 0}\dfrac{\cos x-1}{\sin^2 x}$

គ. $\lim_{x\to 0}\dfrac{2\sin 3x}{x}$

ដំណោះស្រាយ

លំហាត់អនុវត្តទី៣០

គណនាលីមីត៖

ក. $\lim\limits_{x\to 1}\dfrac{1-x^2}{x^2+2-3x}$

ខ. $\lim\limits_{x\to 3}\dfrac{\sqrt{x+6}-3}{x^3-27}$

គ. $\lim\limits_{x\to 0}\dfrac{5\sin 5x}{x}$

ដំណោះស្រាយ

លំហាត់អនុវត្តទី៣១

គណនាលីមីត៖

ក. $\lim\limits_{x\to 1}\dfrac{1-x^2}{x^3-x^2+x-1}$

ខ. $\lim\limits_{x\to 0}\dfrac{\sin 3x}{-x}$

គ. $\lim\limits_{x\to 0}\dfrac{\sqrt{x+2}-\sqrt{2-x}}{\sin x}$

ដំណោះស្រាយ

លំហាត់អនុវត្តទី៣២

គណនាលីមីត៖

ក. $\lim\limits_{x\to 1}\dfrac{x^2(x-2)+x^2+x-1}{1-x}$

ខ. $\lim\limits_{x\to 0}\dfrac{-2x}{\sin 3x}$

គ. $\lim\limits_{x\to \frac{\pi}{3}}\dfrac{\sin x-\sqrt{3}\cos x}{2(\pi-3x)}$

ដំណោះស្រាយ

លំហាត់អនុវត្តទី៣៣

(២០១៩) គណនាលីមីត

ក. $\lim\limits_{x\to+\infty}\left[\sqrt{x^2+2x+3}-(x+1)\right]$

ខ. $\lim\limits_{x\to 0}\frac{-2x\sin x}{1-\cos^2 x}$

គ. $\lim\limits_{x\to-\frac{\pi}{2}}\frac{1+\sin x}{\sin^4 x-1}$

ដំណោះស្រាយ

លំហាត់អនុវត្តទី៣៤

គណនាលីមីត

ក. $\lim\limits_{x\to 3}\sqrt{3x^2-11}$

ខ. $\lim\limits_{x\to \frac{\pi}{3}}\dfrac{\sqrt{3}\sin\left(x-\frac{\pi}{3}\right)}{\left(\frac{\pi}{3}-x\right)}$

គ. $\lim\limits_{x\to +\infty}(2x-7-11\ln x)$

ដំណោះស្រាយ

លំហាត់អនុវត្តទី៣៥

គណនាលីមីត៖

ក. $\lim\limits_{x \to 0} \dfrac{e^x + 1}{2e^x}$

ខ. $\lim\limits_{x \to 1} \dfrac{\sqrt{x+3} - 2}{\sqrt{x} - 1}$

គ. $\lim\limits_{x \to \frac{\pi}{3}} \dfrac{\sqrt{3}\left(\frac{\pi}{3} - x\right)}{\sin\left(x - \frac{\pi}{3}\right)}$

ដំណោះស្រាយ

លំហាត់អនុវត្តទី៣៦

គណនាលីមីត

ក. $\lim\limits_{x\to+\infty}\left[x\ln\left(1+\frac{1}{x}\right)\right]$ គេដឹងថា $\lim\limits_{x\to 0}\frac{\ln(1+x)}{x}=1$

ខ. $\lim\limits_{x\to-\frac{\pi}{2}}\frac{1-\sin^2 x}{2+2\sin x}$

គ. $\lim\limits_{x\to 3}\frac{x^3-27}{27(\sqrt{x+6}-3)}$

ដំណោះស្រាយ

លំហាត់អនុវត្តទី៣៧

គណនាលីមីត៖

ក. $\lim\limits_{x\to 0}(2e^{3x}-xe^{x}+2e^{x}+4)$

ខ. រកតម្លៃ $\lim\limits_{x\to+\infty}\frac{8x^2-x+1}{ax^2+8}=1$

គ. $\lim\limits_{x\to-\infty}\left(x+\sqrt{x^2+9}\right)$

ឃ. $\lim\limits_{x\to 0}\frac{\sin x-\sin x\cos x}{x^3}$

ដំណោះស្រាយ

លំហាត់អនុវត្តទី៣៨

[២០២៥] គណនាលីមីត៖

ក. $\lim\limits_{x\to 1}(\sqrt{x^2-1}+3x)$

ខ. $\lim\limits_{x\to 0}\dfrac{e^{4x}-1}{e^x-1}$

គ. $\lim\limits_{x\to 2}\dfrac{2-\sqrt{x+2}}{x^3-8}$

ឃ. $\lim\limits_{x\to \frac{\pi}{3}}\dfrac{x-\dfrac{\pi}{3}}{\sin x-\sqrt{3}\cos x}$

ដំណោះស្រាយ

លំហាត់អនុវត្តទី៣៩

[២០២៦] គណនា៖

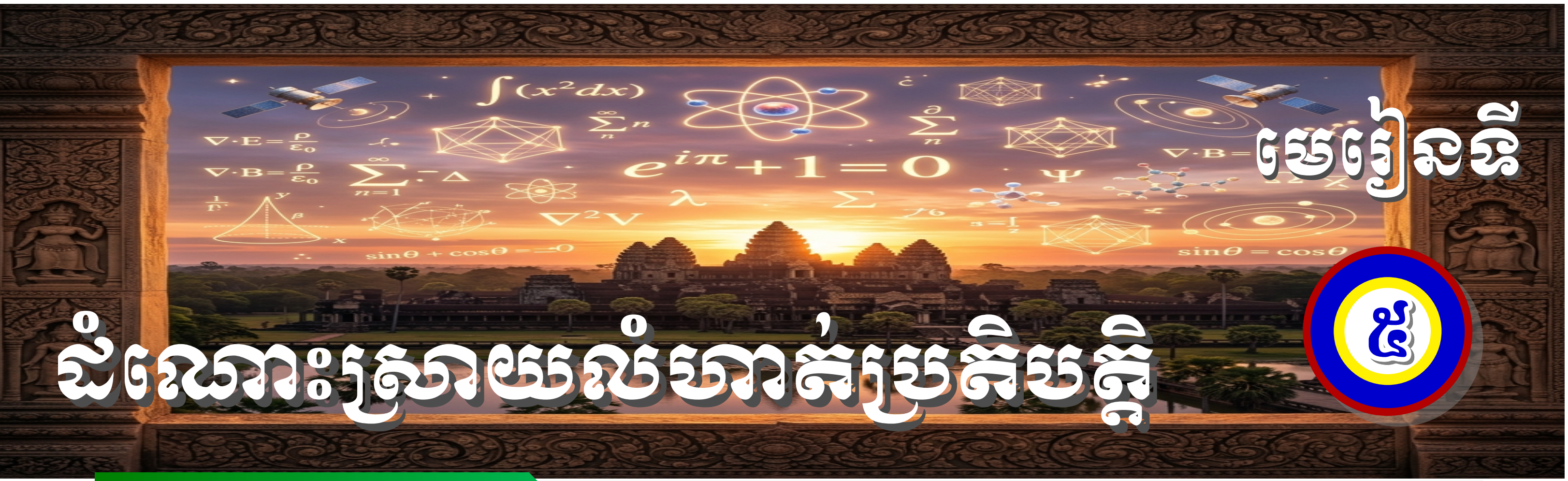
ក. $\lim\limits_{x\to 3}\sqrt{\dfrac{3x^2+9}{x^2-5}}$

ខ. $\lim\limits_{x\to +\infty}\dfrac{1+3x^2}{\sqrt{x^4+1}}$

គ. $\lim\limits_{x\to 1}\dfrac{\sin(2x-2)}{1-x}$

ឃ. $\lim\limits_{x\to e}\dfrac{2(\ln^2 x+1)-4\ln x}{4(1-\ln x)}$

ដំណោះស្រាយ



មេរៀនទី ៥

ដំណោះស្រាយលំហាត់ប្រតិបត្តិ

ដំណោះស្រាយលំហាត់ទី១

ប្រធាន៖ ចូរបង្ហាញលីមីតខាងក្រោមដោយប្រើនិយមន័យ៖

ក. បង្ហាញថា $\lim_{x\to 1}(x^2+1)=2$

តាមនិយមន័យ គ្រប់ $\varepsilon>0$ មាន $\delta>0$ ដែល $0<|x-1|<\delta \implies |(x^2+1)-2|<\varepsilon$។

យើងមាន៖ $|(x^2+1)-2|=|x^2-1|=|x-1||x+1|$។

ឧបមាថា $\delta\le 1$ នោះ៖ $0<|x-1|<1 \implies -1<x-1<1 \implies 0<x<2 \implies 1<x+1<3$។

នាំឲ្យ $|x+1|<3$។

ដូចនេះ៖ $|(x^2+1)-2|=|x-1||x+1|<3|x-1|$។

ដើម្បីឲ្យ $|(x^2+1)-2|<\varepsilon$ យើងគ្រាន់តែឲ្យ $3|x-1|<\varepsilon \implies |x-1|<\frac{\varepsilon}{3}$។

យក $\delta=\min\left(1,\frac{\varepsilon}{3}\right)$។ បើ $0<|x-1|<\delta$ នោះ៖ $|(x^2+1)-2|<\varepsilon$ ពិតមែន។

ដូចនេះ៖ $\lim_{x\to 1}(x^2+1)=2$។

ខ. បង្ហាញថា $\lim_{x\to 2}x^2=4$

តាមនិយមន័យ គ្រប់ $\varepsilon>0$ មាន $\delta>0$ ដែល $0<|x-2|<\delta \implies |x^2-4|<\varepsilon$។

យើងមាន៖ $|x^2-4|=|x-2||x+2|$។

ឧបមាថា $\delta\le 1$ នោះ៖ $0<|x-2|<1 \implies -1<x-2<1 \implies 1<x<3 \implies 3<x+2<5$។

នាំឲ្យ $|x+2|<5$។

ដូចនេះ៖ $|x^2-4|=|x-2||x+2|<5|x-2|$។

ដើម្បីឲ្យ $|x^2-4|<\varepsilon$ យើងគ្រាន់តែឲ្យ $5|x-2|<\varepsilon \implies |x-2|<\frac{\varepsilon}{5}$។

យក $\delta=\min\left(1,\frac{\varepsilon}{5}\right)$។ បើ $0<|x-2|<\delta$ នោះ៖ $|x^2-4|<\varepsilon$ ពិតមែន។

ដូចនេះ៖ $\lim_{x\to 2}x^2=4$។

គ. បង្ហាញថា $\lim_{x\to -1}(x^2-1)=0$

គ្រប់ $\varepsilon>0$ មាន $\delta>0$ ដែល $0<|x+1|<\delta \implies |x^2-1|<\varepsilon$។

យើងមាន៖ $|x^2-1|=|x+1||x-1|$។

ឧបមាថា $\delta\le 1$ នោះ៖ $0<|x+1|<1 \implies -1<x+1<1 \implies -2<x<0 \implies -3<x-1<-1$។

នាំឲ្យ $|x-1|<3$។

ដូចនេះ៖ $|x^2-1|=|x+1||x-1|<3|x+1|$។

យក $\delta=\min\left(1,\frac{\varepsilon}{3}\right)$ នោះបើ $0<|x+1|<\delta$ គេបាន $|x^2-1|<3\left(\frac{\varepsilon}{3}\right)=\varepsilon$។

ដូចនេះ៖ $\lim_{x\to -1}(x^2-1)=0$។

ឃ. បង្ហាញថា $\lim_{x\to 7}\frac{8}{x-3}=2$

គ្រប់ $\varepsilon>0$ មាន $\delta>0$ ដែល $0<|x-7|<\delta \implies \left|\frac{8}{x-3}-2\right|<\varepsilon$។

យើងមាន៖ $\left|\frac{8}{x-3}-2\right|=\left|\frac{8-2x+6}{x-3}\right|=\left|\frac{14-2x}{x-3}\right|=\frac{2|x-7|}{|x-3|}$។

ឧបមាថា $\delta\le 1$ នោះ៖ $0<|x-7|<1 \implies -1<x-7<1 \implies 6<x<8 \implies 3<x-3<5$។

នាំឲ្យ $|x-3|>3 \implies \frac{1}{|x-3|}<\frac{1}{3}$។

ដូចនេះ៖ $\left|\frac{8}{x-3}-2\right|<\frac{2}{3}|x-7|$។

យក $\delta=\min\left(1,\frac{3\varepsilon}{2}\right)$ នោះបើ $0<|x-7|<\delta$ គេបាន $\left|\frac{8}{x-3}-2\right|<\frac{2}{3}\left(\frac{3\varepsilon}{2}\right)=\varepsilon$។

ដូចនេះ៖ $\lim_{x\to 7}\frac{8}{x-3}=2$។

ង. បង្ហាញថា $\lim\limits_{x\to -1^+}\frac{x+5}{x+1}=+\infty$

តាមនិយមន័យ គ្រប់ $M>0$ មាន $\delta>0$ ដែល $0<x-(-1)<\delta \implies \frac{x+5}{x+1}>M$។
ដោយ $x\to -1^+$ នោះ $x>-1 \implies x+1>0$។
ឧបមាថា $\delta\le 1$ នោះ $0<x+1<1 \implies -1<x<0 \implies 4<x+5<5$។

នាំឲ្យ $x+5>4$។
ដូចនេះ $\frac{x+5}{x+1}>\frac{4}{x+1}$។
ដើម្បីឲ្យ $\frac{x+5}{x+1}>M$ យើងឲ្យ $\frac{4}{x+1}>M \implies x+1<\frac{4}{M}$។
យក $\delta=\min\left(1,\frac{4}{M}\right)$។ បើ $0<x+1<\delta$ នោះ $\frac{x+5}{x+1}>M$ ពិតមែន។

ដូចនេះ $\lim\limits_{x\to -1^+}\frac{x+5}{x+1}=+\infty$។

ច. បង្ហាញថា $\lim\limits_{x\to -2^+}\frac{x-9}{x+2}=-\infty$

គ្រប់ $M>0$ មាន $\delta>0$ ដែល $0<x+2<\delta \implies \frac{x-9}{x+2}<-M$។
ដោយ $x\to -2^+$ នោះ $x>-2 \implies x+2>0$។
ឧបមាថា $\delta\le 1$ នោះ $0<x+2<1 \implies -2<x<-1 \implies -11<x-9<-10$។

នាំឲ្យ $x-9<-10$។
ដោយ $x+2>0$ ដូចនេះ $\frac{x-9}{x+2}<\frac{-10}{x+2}$។
ដើម្បីឲ្យ $\frac{x-9}{x+2}<-M$ យើងឲ្យ $\frac{-10}{x+2}<-M \implies \frac{10}{x+2}>M \implies x+2<\frac{10}{M}$។
យក $\delta=\min\left(1,\frac{10}{M}\right)$។

ឆ. បង្ហាញថា $\lim\limits_{x\to +\infty}\frac{x^2-2}{x^2-1}=1$

គ្រប់ $\varepsilon>0$ មាន $N>0$ ដែល $x>N \implies \left|\frac{x^2-2}{x^2-1}-1\right|<\varepsilon$។
យើង មាន៖ $\left|\frac{x^2-2}{x^2-1}-1\right|=\left|\frac{x^2-2-x^2+1}{x^2-1}\right|=\left|\frac{-1}{x^2-1}\right|=\frac{1}{|x^2-1|}$។
ឧបមា ថា $N\ge 2$ នោះ $x>2 \implies x^2>4 \implies x^2-1>3>0$។

ដូចនេះ $\frac{1}{|x^2-1|}=\frac{1}{x^2-1}$។
ដើម្បីឲ្យ $\frac{1}{x^2-1}<\varepsilon \implies x^2-1>\frac{1}{\varepsilon} \implies x^2>1+\frac{1}{\varepsilon} \implies x>\sqrt{1+\frac{1}{\varepsilon}}$។
យក $N=\max\left(2,\sqrt{1+\frac{1}{\varepsilon}}\right)$។

ជ. បង្ហាញថា $\lim\limits_{x\to -\infty}\frac{4x+1}{2x+1}=2$

គ្រប់ $\varepsilon>0$ មាន $N>0$ ដែល $x<-N \implies \left|\frac{4x+1}{2x+1}-2\right|<\varepsilon$។
យើងមាន៖ $\left|\frac{4x+1-4x-2}{2x+1}\right|=\left|\frac{-1}{2x+1}\right|=\frac{1}{|2x+1|}$។
ដោយ $x\to-\infty$ ឧបមាថា $x<-1 \implies 2x<-2 \implies 2x+1<-1<0$។

ដូចនេះ $|2x+1|=-(2x+1)=-2x-1$។
យើងឲ្យ $\frac{1}{-2x-1}<\varepsilon \implies -2x-1>\frac{1}{\varepsilon} \implies -2x>1+\frac{1}{\varepsilon} \implies x<-\frac{1}{2}\left(1+\frac{1}{\varepsilon}\right)$។
យក $N=\frac{1}{2}\left(1+\frac{1}{\varepsilon}\right)$។

ឈ. បង្ហាញថា $\lim\limits_{x\to +\infty}(1+6x-x^2)=-\infty$

គ្រប់ $M>0$ មាន $N>0$ ដែល $x>N \implies 1+6x-x^2<-M$។
យើងមាន៖ $1+6x-x^2=x^2\left(\frac{1}{x^2}+\frac{6}{x}-1\right)$។
ឧបមា ថា $N\ge 12$ នោះ $x>12 \implies \frac{6}{x}<\frac{1}{2}$ និង $\frac{1}{x^2}<\frac{1}{144}<\frac{1}{4}$។

ដូចនេះ $\frac{1}{x^2}+\frac{6}{x}-1<\frac{1}{4}+\frac{1}{2}-1=-\frac{1}{4}$។
នាំឲ្យ $1+6x-x^2<-\frac{1}{4}x^2$។
យើងឲ្យ $-\frac{1}{4}x^2<-M \implies x^2>4M \implies x>2\sqrt{M}$។
យក $N=\max(12,2\sqrt{M})$។

ញ. បង្ហាញថា $\lim\limits_{x\to +\infty}\frac{5x^2+4x-1}{x+2}=+\infty$

គ្រប់ $M>0$ មាន $N>0$ ដែល $x>N \implies \frac{5x^2+4x-1}{x+2}>M$។
ឧបមាថា $x>1$ នោះ $4x>0 \implies 5x^2+4x-1>5x^2-1$។ ដោយ $x>1 \implies x^2>1 \implies 5x^2-1>4x^2$។

ម្យ៉ាងទៀត $x+2<x+2x=3x$ (ព្រោះ $x>1 \implies 2<2x$)។
ដូចនេះ $\frac{5x^2+4x-1}{x+2}>\frac{4x^2}{3x}=\frac{4}{3}x$។
យើងឲ្យ $\frac{4}{3}x>M \implies x>\frac{3M}{4}$។
យក $N=\max\left(1,\frac{3M}{4}\right)$។

មេរៀនទី 5 ដំណោះស្រាយលំហាត់ប្រតិបត្តិ

ដ. បង្ហាញថា $\lim_{x\to-\infty}(x^3)=-\infty$

គ្រប់ $M>0$ មាន $N>0$ ដែល $x<-N \implies x^3<-M$។
យើងឲ្យ $x^3<-M \implies x<-\sqrt[3]{M}$។
យក $N=\sqrt[3]{M}$។ នោះបើ $x<-N$ គេបាន $x^3<-N^3=-M$ ពិតមែន។

ឋ. $\lim_{x\to-\infty}\sqrt{x-1}=-\infty$

ចំណាំ៖ ប្រធានលំហាត់នេះមានកំហុសបច្ចេកទេស ព្រោះដែនកំណត់នៃអនុគមន៍ $f(x)=\sqrt{x-1}$ គឺ $x-1\ge 0 \implies x\ge 1$។ ដូចនេះវាមិនអាចមានលីមីតត្រង់ $-\infty$ បានទេ ព្រោះអនុគមន៍មិនមានន័យ។ (បើកែប្រធានទៅជា $\lim_{x\to+\infty}\sqrt{x-1}=+\infty$ វិញទើបត្រឹមត្រូវ)។

ឌ. បង្ហាញថា $\lim_{x\to4}(x^2+x-11)=9$

គ្រប់ $\varepsilon>0$ មាន $\delta>0$ ដែល $0<|x-4|<\delta \implies |(x^2+x-11)-9|<\varepsilon$។
យើងមាន៖ $|x^2+x-20|=|x-4||x+5|$។
ឧបមាថា $\delta\le 1 \implies |x-4|<1 \implies -1<x-4<1 \implies 8<x+5<10 \implies |x+5|<10$។
ដូចនេះ៖ $|x^2+x-20|<10|x-4|$។
យក $\delta=\min\left(1,\frac{\varepsilon}{10}\right)$។

ឍ. បង្ហាញថា $\lim_{x\to0}\frac{1}{x}=\infty$

គ្រប់ $M>0$ មាន $\delta>0$ ដែល $0<|x|<\delta \implies \left|\frac{1}{x}\right|>M$។
យើងមាន៖ $\frac{1}{|x|}>M \implies |x|<\frac{1}{M}$។
យក $\delta=\frac{1}{M}$។

ណ. បង្ហាញថា $\lim_{x\to1}\frac{1}{(x-1)^2}=+\infty$

គ្រប់ $M>0$ មាន $\delta>0$ ដែល $0<|x-1|<\delta \implies \frac{1}{(x-1)^2}>M$។
យើងមាន៖ $\frac{1}{(x-1)^2}>M \implies (x-1)^2<\frac{1}{M} \implies |x-1|<\frac{1}{\sqrt{M}}$។
យក $\delta=\frac{1}{\sqrt{M}}$។

ត. បង្ហាញថា $\lim_{x\to0}\frac{x+2}{x^2}=\infty$

គ្រប់ $M>0$ មាន $\delta>0$ ដែល $0<|x|<\delta \implies \frac{x+2}{x^2}>M$។
ឧបមាថា $\delta\le 1 \implies -1<x<1 \implies 1<x+2<3 \implies x+2>1$។

ដូចនេះ៖ $\frac{x+2}{x^2}>\frac{1}{x^2}$។
យើងឲ្យ $\frac{1}{x^2}>M \implies x^2<\frac{1}{M} \implies |x|<\frac{1}{\sqrt{M}}$។
យក $\delta=\min\left(1,\frac{1}{\sqrt{M}}\right)$។

ថ. បង្ហាញថា $\lim_{x\to3}\frac{2x^2+3x-27}{x^2-4x+3}=\frac{15}{2}$

គ្រប់ $\varepsilon>0$ មាន $\delta>0$ ដែល $0<|x-3|<\delta \implies \left|\frac{2x^2+3x-27}{x^2-4x+3}-\frac{15}{2}\right|<\varepsilon$។
ដោយ $2x^2+3x-27=(x-3)(2x+9)$ និង $x^2-4x+3=(x-3)(x-1)$ នោះសម្រាប់ $x\neq 3$ គេបាន $\frac{2x+9}{x-1}$។
$\left|\frac{2x+9}{x-1}-\frac{15}{2}\right|=\left|\frac{4x+18-15x+15}{2(x-1)}\right|=\left|\frac{-11x+33}{2(x-1)}\right|=\frac{11|x-3|}{2|x-1|}$។
ឧបមា $\delta\le 1 \implies 2<x<4 \implies 1<x-1<3 \implies |x-1|>1 \implies \frac{1}{|x-1|}<1$។
ដូចនេះកន្សោមយើងតូចជាង $\frac{11}{2}|x-3|$។
យក $\delta=\min\left(1,\frac{2\varepsilon}{11}\right)$។

ទ. បង្ហាញថា $\lim_{x\to+\infty}\frac{x-1}{x+1}=1$

គ្រប់ $\varepsilon>0$ មាន $N>0$ ដែល $x>N \implies \left|\frac{x-1}{x+1}-1\right|<\varepsilon$។
$\left|\frac{x-1-x-1}{x+1}\right|=\left|\frac{-2}{x+1}\right|=\frac{2}{x+1}$ (ព្រោះ $x\to+\infty \implies x+1>0$)។
ឧបមា $x>0$ នោះ $x+1>x \implies \frac{2}{x+1}<\frac{2}{x}$។
យើងឲ្យ $\frac{2}{x}<\varepsilon \implies x>\frac{2}{\varepsilon}$។
យក $N=\frac{2}{\varepsilon}$។

ឆ. បង្ហាញថា $\lim\limits_{x\to+\infty}\frac{x+1}{x-1}=1$

គ្រប់ $\varepsilon>0$ មាន $N>0$ ដែល $x>N \implies \left|\frac{x+1}{x-1}-1\right|<\varepsilon$។

$\left|\frac{x+1-x+1}{x-1}\right|=\frac{2}{x-1}$ (ព្រោះ $x\to+\infty \implies x-1>0$)។

ឧបមា $x>2 \implies x-1>\frac{x}{2} \implies \frac{2}{x-1}<\frac{4}{x}$។

យើងឲ្យ $\frac{4}{x}<\varepsilon \implies x>\frac{4}{\varepsilon}$។

យក $N=\max\left(2,\frac{4}{\varepsilon}\right)$។

ឌ. បង្ហាញថា $\lim\limits_{x\to+\infty}\frac{2x}{x+1}=2$

គ្រប់ $\varepsilon>0$ មាន $N>0$ ដែល $x>N \implies \left|\frac{2x}{x+1}-2\right|<\varepsilon$។

$\left|\frac{2x-2x-2}{x+1}\right|=\left|\frac{-2}{x+1}\right|=\frac{2}{x+1}$។

ដោយ $x>0$ នាំឲ្យ $x+1>x \implies \frac{2}{x+1}<\frac{2}{x}$។

យើងឲ្យ $\frac{2}{x}<\varepsilon \implies x>\frac{2}{\varepsilon}$។

យក $N=\frac{2}{\varepsilon}$។

ដំណោះស្រាយលំហាត់ទី២

ប្រធាន៖ គណនាតម្លៃ $\alpha>0$ ទៅតាមតម្លៃ ε នីមួយៗដែលគេឲ្យដូចខាងក្រោម៖

ក. $\lim\limits_{x\to1}(5x-3)=2$, $\varepsilon=0.00005$

យើងត្រូវរក $\alpha>0$ ដែលកាលណា $0<|x-1|<\alpha \implies |(5x-3)-2|<\varepsilon$។

យើងមាន៖ $|(5x-3)-2|=|5x-5|=5|x-1|$។

ដើម្បីឲ្យ $5|x-1|<\varepsilon \implies |x-1|<\frac{\varepsilon}{5}$។

ដូចនេះយើងអាចយក $\alpha=\frac{\varepsilon}{5}=\frac{0.00005}{5}=0.00001$។

ខ. $\lim\limits_{x\to2}(3x-2)=1$, $\varepsilon=0.000000099$

ចំណាំ៖ ប្រធាន លំហាត់ នេះ មាន កំហុស ដោយសារកាលណា $x\to2$ នោះលីមីតគឺ $3(2)-2=4$ មិនមែន 1 ទេ។ ដូចនេះយើងដោះស្រាយដោយសន្មត់ថាប្រធានគឺ $\lim\limits_{x\to2}(3x-2)=4$។

យើងត្រូវរក $\alpha>0$ ដែល $0<|x-2|<\alpha \implies |(3x-2)-4|<\varepsilon$។

យើងមាន៖ $|(3x-2)-4|=|3x-6|=3|x-2|$។

ដើម្បីឲ្យ $3|x-2|<\varepsilon \implies |x-2|<\frac{\varepsilon}{3}$។

ដូចនេះ៖ $\alpha=\frac{\varepsilon}{3}=\frac{0.000000099}{3}=0.000000033$។

គ. $\lim\limits_{x\to1}(2x-1)=1$, $\varepsilon=44\times10^{-6}$

យើងមាន៖ $|(2x-1)-1|=|2x-2|=2|x-1|$។

ដើម្បីឲ្យ $2|x-1|<\varepsilon \implies |x-1|<\frac{\varepsilon}{2}$។

ដូចនេះ៖ $\alpha=\frac{\varepsilon}{2}=\frac{44\times10^{-6}}{2}=22\times10^{-6}$។

ឃ. $\lim\limits_{x\to2}(3x+1)=7$, $\varepsilon=0.0030$

យើងមាន៖ $|(3x+1)-7|=|3x-6|=3|x-2|$។

ដើម្បីឲ្យ $3|x-2|<\varepsilon \implies |x-2|<\frac{\varepsilon}{3}$។

ដូចនេះ៖ $\alpha=\frac{\varepsilon}{3}=\frac{0.0030}{3}=0.0010$។

ង. $\lim\limits_{x\to4}(2x-1)=7$, $\varepsilon=0.000006$

យើងមាន៖ $|(2x-1)-7|=|2x-8|=2|x-4|$។

ដើម្បីឲ្យ $2|x-4|<\varepsilon \implies |x-4|<\frac{\varepsilon}{2}$។

ដូចនេះ៖ $\alpha=\frac{\varepsilon}{2}=\frac{0.000006}{2}=0.000003$។

ច. $\lim\limits_{x\to-3}(2x+5)=-1$, $\varepsilon=0.10$

យើងមាន៖ $|(2x+5)-(-1)|=|2x+6|=2|x+3|$។

ដើម្បីឲ្យ $2|x+3|<\varepsilon \implies |x+3|<\frac{\varepsilon}{2}$។

ដូចនេះ៖ $\alpha=\frac{\varepsilon}{2}=\frac{0.10}{2}=0.05$។

ឆ. $\lim\limits_{x\to3}(2x+4)=10$, $\varepsilon=0.00001$

យើងមាន៖ $|(2x+4)-10|=|2x-6|=2|x-3|$។

ដើម្បីឲ្យ $2|x-3|<\varepsilon \implies |x-3|<\frac{\varepsilon}{2}$។

ដូចនេះ៖ $\alpha=\frac{\varepsilon}{2}=\frac{0.00001}{2}=0.000005$។

ជ. $\lim\limits_{x\to2}(x-1)^2=1$, $\varepsilon=0.00021$

យើងមាន៖ $|(x-1)^2-1|=|x^2-2x|=|x||x-2|$។

ឧបមាថា $\alpha\le1$ នោះ៖ $0<|x-2|<1 \implies -1<x-2<1 \implies 1<x<3$។

នាំឲ្យ $|x|<3$។

ដូចនេះ៖ $|(x-1)^2-1|=|x||x-2|<3|x-2|$។

យើងឲ្យ $3|x-2|<\varepsilon \implies |x-2|<\frac{\varepsilon}{3}$។

ដូចនេះ៖ $\alpha=\min\left(1,\frac{\varepsilon}{3}\right)=\min\left(1,\frac{0.00021}{3}\right)=\min(1,0.00007)=0.00007$។

ឈ. $\lim_{x\to 1}\sqrt{x}=1$, $\varepsilon=(\sqrt{2}-1)0.0001$

យើងមាន៖ $|\sqrt{x}-1|=\left|\frac{(\sqrt{x}-1)(\sqrt{x}+1)}{\sqrt{x}+1}\right|=\frac{|x-1|}{\sqrt{x}+1}$។

ដោយ $\sqrt{x}>0$ នាំឲ្យ $\sqrt{x}+1>1 \implies \frac{1}{\sqrt{x}+1}<1$។

ដូចនេះ $|\sqrt{x}-1|=\frac{|x-1|}{\sqrt{x}+1}<|x-1|$។

យើងគ្រាន់តែឲ្យ $|x-1|<\varepsilon \implies \alpha=\varepsilon=(\sqrt{2}-1)0.0001$។

ញ. $\lim_{x\to 2}(4x-5)=3$, $\varepsilon=8\times 10^{-3}$

យើងមាន៖ $|(4x-5)-3|=|4x-8|=4|x-2|$។

ដើម្បីឲ្យ $4|x-2|<\varepsilon \implies |x-2|<\frac{\varepsilon}{4}$។

ដូចនេះ $\alpha=\frac{\varepsilon}{4}=\frac{8\times 10^{-3}}{4}=2\times 10^{-3}$។

ដំណោះស្រាយលំហាត់ទី៣

ប្រធាន៖ ចូរដោះស្រាយលីមីតខាងក្រោមឲ្យបានត្រឹមត្រូវ៖

ក. $\lim_{x\to 2}(x^2-4)$

យើងជំនួស $x=2$ ដោយផ្ទាល់ទៅក្នុងកន្សោមលីមីត៖

$$\begin{aligned}\lim_{x\to 2}(x^2-4)\\ &=(2)^2-4\\ &=4-4\\ &=0\end{aligned}$$

ដូចនេះ $\lim_{x\to 2}(x^2-4)=0$។

ខ. $\lim_{x\to 0}\frac{x^3-4x}{2x^2+3x}$

ពេលយើងជំនួស $x=0$ ទៅក្នុងកន្សោមភាគយកនិងភាគបែង គេបានរាងមិនកំណត់ $\frac{0}{0}$។

ដើម្បីគណនាលីមីតនេះ យើងត្រូវចាប់ x ជាកត្តារួមដើម្បីសម្រួលកត្តាដែលធ្វើឲ្យកន្សោមស្មើ 0៖

ភាគយក៖ $x^3-4x=x(x^2-4)$

ភាគបែង៖ $2x^2+3x=x(2x+3)$

យើងអាចសរសេរលីមីតឡើងវិញបានថា៖

$$\begin{aligned}\lim_{x\to 0}\frac{x^3-4x}{2x^2+3x}\\ &=\lim_{x\to 0}\frac{x(x^2-4)}{x(2x+3)}\end{aligned}$$

សម្រួលកត្តា x ចោល គេបាន៖

$$\lim_{x\to 0}\frac{x^2-4}{2x+3}$$

ជំនួស $x=0$ ម្តងទៀត៖

$$\begin{aligned}\frac{0^2-4}{2(0)+3}\\ &=\frac{-4}{3}\\ &=-\frac{4}{3}\end{aligned}$$

ដូចនេះ $\lim_{x\to 0}\frac{x^3-4x}{2x^2+3x}=-\frac{4}{3}$។

គ. $\lim\limits_{x\to -1}\frac{x^3}{(x+1)^2}$

ជំនួស $x=-1$ ចូលក្នុងកន្សោម យើងបាន៖

ភាគយក៖ $(-1)^3=-1$

ភាគបែង៖ $(-1+1)^2=0^2=0$

ដោយសារភាគយកជានាមអវិជ្ជមាន (-1) និងភាគបែងជិត 0 និងមានសញ្ញាវិជ្ជមានជានិច្ច (ដោយសារជាការេនៃចំនួន $(x+1)^2>0$) នោះលទ្ធផលនៃប្រភាគនឹងមានតម្លៃខិតទៅរក $-\infty$។

$$\lim_{x\to -1}\frac{x^3}{(x+1)^2}$$
$$=\frac{-1}{0^+}$$
$$=-\infty$$

ដូចនេះ $\lim\limits_{x\to -1}\frac{x^3}{(x+1)^2}=-\infty$។

ឃ. $\lim\limits_{x\to -1}\frac{(x+1)^2(x-1)}{x^3+1}$

នៅពេល ជំនួស $x=-1$ យើង នឹង ទទួល បាន រាងមិនកំណត់ $\frac{0}{0}$។

ដើម្បីសម្រួល យើងបំបែកភាគបែង x^3+1 តាមរូបមន្ត $A^3+B^3=(A+B)(A^2-AB+B^2)$ ដែលក្នុងករណីនេះ $A=x$ និង $B=1$ នោះគេបាន៖

$$x^3+1=(x+1)(x^2-x+1)$$

លីមីតក្លាយជា៖

$$\lim_{x\to -1}\frac{(x+1)^2(x-1)}{x^3+1}$$
$$=\lim_{x\to -1}\frac{(x+1)^2(x-1)}{(x+1)(x^2-x+1)}$$

សម្រួលកត្តា $(x+1)$ ចោល៖

$$=\lim_{x\to -1}\frac{(x+1)(x-1)}{x^2-x+1}$$

ជំនួស $x=-1$ ម្តងទៀត យើងបាន៖

$$=\frac{(-1+1)(-1-1)}{(-1)^2-(-1)+1}$$
$$=\frac{0\cdot(-2)}{1+1+1}$$
$$=\frac{0}{3}$$
$$=0$$

ដូចនេះ $\lim\limits_{x\to -1}\frac{(x+1)^2(x-1)}{x^3+1}=0$។

ង. $\lim_{x\to 0}\frac{x^3-2x^2+x}{2x^3+x^2-2x}$
ជំនួស $x=0$ កន្សោមមានរាងមិនកំណត់ $\frac{0}{0}$។
យើងចាប់កត្តា x ពីភាគយក និងភាគបែង៖
ភាគយក៖ $x^3-2x^2+x=x(x^2-2x+1)$
ភាគបែង៖ $2x^3+x^2-2x=x(2x^2+x-2)$
យើងបាន៖

$$\begin{aligned}&\lim_{x\to 0}\frac{x(x^2-2x+1)}{x(2x^2+x-2)}\\&=\lim_{x\to 0}\frac{x^2-2x+1}{2x^2+x-2}\end{aligned}$$

ជំនួស $x=0$ ជាលើកទីពីរ៖

$$\begin{aligned}&=\frac{0^2-2(0)+1}{2(0)^2+0-2}\\&=\frac{1}{-2}\\&=-\frac{1}{2}\end{aligned}$$

ដូចនេះ $\lim_{x\to 0}\frac{x^3-2x^2+x}{2x^3+x^2-2x}=-\frac{1}{2}$។

ឬ. $\lim_{x\to 1}\frac{x^4-1}{x^3-1}$
ជំនួស $x=1$ កន្សោមមានរាងមិនកំណត់ $\frac{0}{0}$។
តាមរូបមន្ត៖
ភាគយក៖ $x^4-1=(x^2-1)(x^2+1)=(x-1)(x+1)(x^2+1)$
ភាគបែង៖ $x^3-1=(x-1)(x^2+x+1)$
លីមីតក្លាយជា៖

$$\lim_{x\to 1}\frac{(x-1)(x+1)(x^2+1)}{(x-1)(x^2+x+1)}$$

ក្រោយសម្រួលកត្តា $(x-1)$ រួច យើងទទួលបាន៖

$$\lim_{x\to 1}\frac{(x+1)(x^2+1)}{x^2+x+1}$$

ជំនួស $x=1$៖

$$\begin{aligned}&=\frac{(1+1)(1^2+1)}{1^2+1+1}\\&=\frac{2\times 2}{3}\\&=\frac{4}{3}\end{aligned}$$

ដូចនេះ $\lim_{x\to 1}\frac{x^4-1}{x^3-1}=\frac{4}{3}$។

ឆ. $\lim_{x\to 0}\frac{1+x+x^2-1}{x}$

កន្សោមនេះមានរាង $\frac{0}{0}$។

យើងធ្វើការបូកដកលុបចោលលេខ 1 នៅភាគយក៖

$$1+x+x^2-1=x+x^2$$

ចាប់កត្តា x ជាកត្តារួម៖

$$x+x^2=x(1+x)$$

លីមីតក្លាយជា៖

$$\lim_{x\to 0}\frac{x(1+x)}{x}$$

ក្រោយសម្រួល x ជាមួយភាគបែង យើងទទួលបាន៖

$$\lim_{x\to 0}(1+x)$$

ជំនួស $x=0$ ចូល យើងបាន៖

$$=1+0=1$$

ដូចនេះ $\lim_{x\to 0}\frac{1+x+x^2-1}{x}=1$។

ជ. $\lim_{x\to 0}\frac{(1+x)^3-1}{x}$

ពន្លាតភាគយកតាមរូបមន្ត $(A+B)^3=A^3+3A^2B+3AB^2+B^3$ ៖

$$\begin{aligned}(1+x)^3-1&=(1^3+3(1^2)x+3(1)x^2+x^3)-1\\&=1+3x+3x^2+x^3-1\\&=3x+3x^2+x^3\end{aligned}$$

ចាប់ x ជាកត្តាពីភាគយក៖

$$3x+3x^2+x^3=x(3+3x+x^2)$$

លីមីតក្លាយជា៖

$$\lim_{x\to 0}\frac{x(3+3x+x^2)}{x}$$

សម្រួល x រួចជំនួស $x=0$៖

$$\begin{aligned}&\lim_{x\to 0}(3+3x+x^2)\\&=3+3(0)+0^2\\&=3\end{aligned}$$

ដូចនេះ $\lim_{x\to 0}\frac{(1+x)^3-1}{x}=3$។

ឈ. $\lim_{x\to a}\frac{x^2-a^2}{x^3-a^3}$
ជំនួស $x=a$ កន្សោមមានរាងមិនកំណត់ $\frac{0}{0}$។
តាមរូបមន្តកត្តា៖
ភាគយក៖ $x^2-a^2=(x-a)(x+a)$
ភាគបែង៖ $x^3-a^3=(x-a)(x^2+ax+a^2)$
ជំនួសចូលក្នុងលីមីត៖

$$\lim_{x\to a}\frac{(x-a)(x+a)}{(x-a)(x^2+ax+a^2)}$$

សម្រួលកត្តា $(x-a)$ រួចជំនួស $x=a$៖

$$\lim_{x\to a}\frac{x+a}{x^2+ax+a^2}=\frac{a+a}{a^2+a(a)+a^2}=\frac{2a}{3a^2}$$

សម្រួលនឹង a ម្តងទៀត ទទួលបាន $\frac{2}{3a}$ (លុះត្រាតែ $a\neq 0$)។
ដូចនេះ $\lim_{x\to a}\frac{x^2-a^2}{x^3-a^3}=\frac{2}{3a}$។

ញ. $\lim_{x\to -1}\frac{x^2-x-2}{x^2+2x+1}$
ជំនួស $x=-1$ កន្សោមមានរាងមិនកំណត់ $\frac{0}{0}$។
យើងត្រូវបំបែកភាគយក និងភាគបែងជាផលគុណកត្តា៖
ភាគយក៖ x^2-x-2 គឺមានឫស $x=-1$ នាំឲ្យកត្តាគឺ $(x+1)$។ ដូចនេះ៖ $x^2-x-2=(x+1)(x-2)$។
ភាគបែង៖ x^2+2x+1 ជារូបមន្តការ៉េនៃទ្វេធា $(x+1)^2$។
លីមីតក្លាយជា៖

$$\lim_{x\to -1}\frac{(x+1)(x-2)}{(x+1)^2}=\lim_{x\to -1}\frac{x-2}{x+1}$$

នៅពេលនេះ បើយើងជំនួស $x=-1$ យើងបានភាគយក $-1-2=-3$ និងភាគបែង $-1+1=0$។
ដោយភាគយកជាចំនួនអវិជ្ជមាន និងភាគបែង ខិតទៅរក 0 នោះលទ្ធផលគឺ $-\infty$ ឬ $+\infty$ (អាស្រ័យលើសញ្ញានៃ $x+1$)។
សរុបមក លីមីតនេះមានតម្លៃស្មើនឹង ∞។
ដូចនេះ $\lim_{x\to -1}\frac{x^2-x-2}{x^2+2x+1}=\infty$។

ជ. $\lim_{x\to-2}\frac{x^3+3x^2+2x}{x^2-x-6}$

ជួស $x=-2$ ចូល ទទួលបានរាងមិនកំណត់ $\frac{0}{0}$។
យើងធ្វើការបំបែកជាកត្តា៖
ភាគយក៖ $x^3+3x^2+2x=x(x^2+3x+2)$ ។ ដោយ $x^2+3x+2=(x+1)(x+2)$ ដូចនេះភាគយកគឺ $x(x+1)(x+2)$។
ភាគបែង៖ $x^2-x-6=(x-3)(x+2)$ (ផលបូក $-3+2=-1$ ផលគុណ $-3\times2=-6$)។
ជំនួសចូលលីមីត និងសម្រួលកត្តា $(x+2)$៖

$$\lim_{x\to-2}\frac{x(x+1)(x+2)}{(x-3)(x+2)}$$
$$=\lim_{x\to-2}\frac{x(x+1)}{x-3}$$

ជំនួស $x=-2$ ៖

$$=\frac{-2(-2+1)}{-2-3}$$
$$=\frac{-2(-1)}{-5}$$
$$=\frac{2}{-5}$$
$$=-\frac{2}{5}$$

ដូចនេះ $\lim_{x\to-2}\frac{x^3+3x^2+2x}{x^2-x-6}=-\frac{2}{5}$។

ឋ. $\lim_{x\to1}\frac{x^3-3x+2}{x^4-4x+3}$

រាងមិនកំណត់ $\frac{0}{0}$ ពេលជំនួស $x=1$។
បំបែកកត្តា (ដោយសារមានឫស $x=1$ ត្រូវមានកត្តា $x-1$)៖
ភាគយក៖
$x^3-3x+2=x^3-x-2x+2=x(x^2-1)-2(x-1)=x(x-1)(x+1)-2(x-1)=(x-1)(x^2+x-2)=(x-1)(x-1)(x+2)=(x-1)^2(x+2)$។
ភាគបែង៖
$x^4-4x+3=x^4-x-3x+3=x(x^3-1)-3(x-1)=x(x-1)(x^2+x+1)-3(x-1)=(x-1)(x^3+x^2+x-3)$។
ចំពោះ x^3+x^2+x-3 ក៏មានឫស 1 ដែរ៖
$x^3+x^2+x-3=x^3-1+x^2-1+x-1=(x-1)(x^2+x+1)+(x-1)(x+1)+(x-1)=(x-1)(x^2+2x+3)$។
ដូចនេះ ភាគបែងគឺ $(x-1)^2(x^2+2x+3)$។
ជំនួសចូលលីមីត និងសម្រួល $(x-1)^2$៖

$$\lim_{x\to1}\frac{(x-1)^2(x+2)}{(x-1)^2(x^2+2x+3)}$$
$$=\lim_{x\to1}\frac{x+2}{x^2+2x+3}$$

ជំនួស $x=1$ ៖

$$=\frac{1+2}{1+2+3}$$
$$=\frac{3}{6}$$
$$=\frac{1}{2}$$

ដូចនេះ $\lim_{x\to1}\frac{x^3-3x+2}{x^4-4x+3}=\frac{1}{2}$។

ឌ. $\lim\limits_{x\to1}\frac{x^4-1}{x-1}$

រាងមិនកំណត់ $\frac{0}{0}$។
ប្រើរូបមន្តទូទៅ A^n-B^n សម្រាប់ x^4-1 ៖

$$x^4-1=(x-1)(x^3+x^2+x+1)$$

លីមីតក្លាយជា៖

$$\lim_{x\to1}\frac{(x-1)(x^3+x^2+x+1)}{x-1}$$
$$=\lim_{x\to1}(x^3+x^2+x+1)$$

ជំនួស $x=1$៖

$$=1^3+1^2+1+1=4$$

ដូចនេះ $\lim\limits_{x\to1}\frac{x^4-1}{x-1}=4$។

ណ. $\lim\limits_{x\to1}\frac{x^3-3x^2+3x-1}{x^3-x}$

កន្សោមមានរាងមិនកំណត់ $\frac{0}{0}$ ពេល $x\to1$។
យើងកត់សម្គាល់ឃើញថាភាគយកជារូបមន្តគូបនៃទ្វេធា៖
$x^3-3x^2+3x-1=(x-1)^3$
ចំពោះភាគបែង គឺអាចចាប់កត្តាបាន៖
$x^3-x=x(x^2-1)=x(x-1)(x+1)$
លីមីតក្លាយជា៖

$$\lim_{x\to1}\frac{(x-1)^3}{x(x-1)(x+1)}$$
$$=\lim_{x\to1}\frac{(x-1)^2}{x(x+1)}$$

ជំនួស $x=1$ ម្តងទៀត៖

$$=\frac{(1-1)^2}{1(1+1)}$$
$$=\frac{0}{2}$$
$$=0$$

ដូចនេះ $\lim\limits_{x\to1}\frac{x^3-3x^2+3x-1}{x^3-x}=0$។

ឍ. $\lim\limits_{x\to3}\frac{x^2-5x+6}{x^2-8x+15}$

ជួស $x=3$ កន្សោមមានរាងមិនកំណត់ $\frac{0}{0}$។
យើងបំបែកភាគយក និងភាគបែងជាកត្តា៖
ភាគយក៖ $x^2-5x+6=(x-3)(x-2)$
ភាគបែង៖ $x^2-8x+15=(x-3)(x-5)$
លីមីតក្លាយជា៖

$$\lim_{x\to3}\frac{(x-3)(x-2)}{(x-3)(x-5)}$$

ក្រោយពីសម្រួលកត្តា $(x-3)$ រួច យើងជំនួស $x=3$៖

$$\lim_{x\to3}\frac{x-2}{x-5}=\frac{3-2}{3-5}$$
$$=\frac{1}{-2}$$
$$=-\frac{1}{2}$$

ដូចនេះ $\lim\limits_{x\to3}\frac{x^2-5x+6}{x^2-8x+15}=-\frac{1}{2}$។

ត. $\lim\limits_{x\to2}\frac{x^2-4}{x^2-3x+2}$

ជួស $x=2$ គេបានរាងមិនកំណត់ $\frac{0}{0}$។
បំបែកកត្តាភាគយក និងភាគបែង៖
ភាគយក៖ $x^2-4=(x-2)(x+2)$
ភាគបែង៖ $x^2-3x+2=(x-2)(x-1)$
លីមីតក្លាយជា៖

$$\lim_{x\to2}\frac{(x-2)(x+2)}{(x-2)(x-1)}$$
$$=\lim_{x\to2}\frac{x+2}{x-1}$$

ជំនួស $x=2$ ៖

$$=\frac{2+2}{2-1}=\frac{4}{1}=4$$

ដូចនេះ $\lim\limits_{x\to2}\frac{x^2-4}{x^2-3x+2}=4$។

ឃ. $\lim\limits_{x\to 1}\frac{x^2-4x+3}{x^3-x^2+2x-2}$
ជំនួស $x=1$ គេបានរាងមិនកំណត់ $\frac{0}{0}$។
បំបែកកត្តា៖
ភាគយក៖ $x^2-4x+3=(x-1)(x-3)$
ភាគបែង (ចាប់កត្តាជាក្រុម)៖ $x^3-x^2+2x-2=x^2(x-1)+2(x-1)=(x-1)(x^2+2)$
លីមីតក្លាយជា៖

$$\lim_{x\to 1}\frac{(x-1)(x-3)}{(x-1)(x^2+2)} = \lim_{x\to 1}\frac{x-3}{x^2+2}$$

ជំនួស $x=1$ ៖

$$= \frac{1-3}{1^2+2} = \frac{-2}{3} = -\frac{2}{3}$$

ដូចនេះ $\lim\limits_{x\to 1}\frac{x^2-4x+3}{x^3-x^2+2x-2}=-\frac{2}{3}$។

ង. $\lim\limits_{x\to 1}\frac{x^n-1}{x-1}$
រាងមិនកំណត់ $\frac{0}{0}$ ពេល $x\to 1$។
ប្រើរូបមន្តពន្លាតទូទៅនៃកត្តា A^n-B^n ៖

$$x^n-1 = (x-1)(x^{n-1}+x^{n-2}+\ldots+x+1)$$

លីមីតក្លាយជា៖

$$\lim_{x\to 1}\frac{(x-1)(x^{n-1}+x^{n-2}+\ldots+x+1)}{x-1} = \lim_{x\to 1}(x^{n-1}+x^{n-2}+\ldots+x+1)$$

ដោយក្នុងវង់ក្រចកមានទាំងអស់ n តួ ហើយគ្រប់តួសុទ្ធតែស្មើនឹង 1 នៅពេល $x=1$ នោះផលបូកស្មើនឹង n។
ដូចនេះ $\lim\limits_{x\to 1}\frac{x^n-1}{x-1}=n$។

ឈ. $\lim_{x\to1}\frac{x^m-1}{x^n-1}$

ដើម្បីដោះស្រាយលីមីតនេះ យើងត្រូវចែកទាំងភាគយក និងភាគបែងនឹង $(x-1)$ ដើម្បីអាចប្រើលទ្ធផលពីលំហាត់ខាងលើបាន។

$$\lim_{x\to1}\frac{x^m-1}{x^n-1}$$
$$=\lim_{x\to1}\frac{\frac{x^m-1}{x-1}}{\frac{x^n-1}{x-1}}$$

តាមលទ្ធផលដែលយើងបានបង្ហាញខាងលើ ៖

$$\lim_{x\to1}\frac{x^m-1}{x-1}=m$$

$$\lim_{x\to1}\frac{x^n-1}{x-1}=n$$

ជំនួសចូល យើងបានលទ្ធផលចុងក្រោយ៖

$$\frac{m}{n}$$

ដូចនេះ $\lim_{x\to1}\frac{x^m-1}{x^n-1}=\frac{m}{n}$ (ដោយ $n\neq0$)។

ញ. $\lim_{x\to0}\frac{(1+x)^n-1}{x}$

រាងមិនកំណត់ $\frac{0}{0}$។

យើងតាង $t=1+x$ ។ នៅពេល $x\to0$ នោះ $t\to1$ ហើយ $x=t-1$។

ជំនួសចូលក្នុងលីមីត យើងបាន៖

$$\lim_{t\to1}\frac{t^n-1}{t-1}$$

ផ្អែកតាមលទ្ធផលនៃលំហាត់ទី ១៨ ខាងលើ ផលធៀបនេះគឺមានតម្លៃស្មើនឹង n។

ដូចនេះ $\lim_{x\to0}\frac{(1+x)^n-1}{x}=n$។

ដំណោះស្រាយលំហាត់ទី៤

ប្រធាន៖ ចូរដោះស្រាយលីមីតខាងក្រោមឲ្យបានត្រឹមត្រូវ៖

ក. $\lim_{x\to 2}\frac{x^2-3x+2}{x^2-2x}$

ជួស $x=2$ យើងនឹងទទួលបានរាងមិនកំណត់ $\frac{0}{0}$។

យើងត្រូវបំបែកជាកត្តា៖

ភាគយក៖ $x^2-3x+2=(x-1)(x-2)$

ភាគបែង៖ $x^2-2x=x(x-2)$

ជំនួសចូលក្នុងលីមីត៖

$$\lim_{x\to 2}\frac{(x-1)(x-2)}{x(x-2)}$$

សម្រួលកត្តា $(x-2)$ ចោលរួចជំនួស $x=2$ ៖

$$\lim_{x\to 2}\frac{x-1}{x}$$
$$=\frac{2-1}{2}$$
$$=\frac{1}{2}$$

ដូចនេះ $\lim_{x\to 2}\frac{x^2-3x+2}{x^2-2x}=\frac{1}{2}$។

ខ. $\lim_{x\to 2}\frac{x^2-x-2}{x^2-2x}$

ជួស $x=2$ យើងទទួលបានរាងមិនកំណត់ $\frac{0}{0}$។

បំបែកជាកត្តា៖

ភាគយក៖ $x^2-x-2=(x-2)(x+1)$

ភាគបែង៖ $x^2-2x=x(x-2)$

ជំនួសចូល និងសម្រួល $(x-2)$ ៖

$$\lim_{x\to 2}\frac{(x-2)(x+1)}{x(x-2)}$$
$$=\lim_{x\to 2}\frac{x+1}{x}$$

ជំនួស $x=2$ ៖

$$=\frac{2+1}{2}=\frac{3}{2}$$

ដូចនេះ $\lim_{x\to 2}\frac{x^2-x-2}{x^2-2x}=\frac{3}{2}$។

គ. $\lim_{x\to 2}\frac{x^2+5}{x^2-3}$

ដោយមិនមានរាងមិនកំណត់ យើងអាចជំនួស $x=2$ ចូលដោយផ្ទាល់បាន៖

$$\lim_{x\to 2}\frac{x^2+5}{x^2-3}=\frac{(2)^2+5}{(2)^2-3}$$
$$=\frac{4+5}{4-3}$$
$$=\frac{9}{1}$$
$$=9$$

ដូចនេះ $\lim_{x\to 2}\frac{x^2+5}{x^2-3}=9$។

ឃ. $\lim_{x\to 1}\frac{3x^4-4x^3+1}{(x-1)^2}$

ជួស $x=1$ យើងនឹងបានរាងមិនកំណត់ $\frac{0}{0}$។

យើងអាចបំបែកភាគយកជាផលគុណកត្តាដោយប្រើក្បួនហ័រន័រ ឬ បំបែកជាក្រុម៖

$3x^4-4x^3+1=3x^4-3x^3-x^3+1$

$=3x^3(x-1)-(x^3-1)$

$=3x^3(x-1)-(x-1)(x^2+x+1)$

$=(x-1)[3x^3-(x^2+x+1)]$

$=(x-1)(3x^3-x^2-x-1)$

$=(x-1)(3x^3-3x^2+2x^2-2x+x-1)$

$=(x-1)[3x^2(x-1)+2x(x-1)+(x-1)]$

$=(x-1)^2(3x^2+2x+1)$

ជំនួសចូលលីមីត និងសម្រួលកត្តា $(x-1)^2$៖

$$\lim_{x\to 1}\frac{(x-1)^2(3x^2+2x+1)}{(x-1)^2}$$
$$=\lim_{x\to 1}(3x^2+2x+1)$$

ជំនួស $x=1$៖

$$=3(1)^2+2(1)+1=3+2+1=6$$

ដូចនេះ $\lim_{x\to 1}\frac{3x^4-4x^3+1}{(x-1)^2}=6$។

ង. $\lim_{x\to-2}\frac{3x+6}{x^3+8}$

ជំនួស $x=-2$ គេបានរាងមិនកំណត់ $\frac{0}{0}$។
បំបែកកត្តា៖
ភាគយក៖ $3x+6=3(x+2)$
ភាគបែង៖ $x^3+8=x^3+2^3=(x+2)(x^2-2x+4)$
ជំនួសចូល និងសម្រួល $(x+2)$ ៖

$$\begin{aligned}&\lim_{x\to-2}\frac{3(x+2)}{(x+2)(x^2-2x+4)}\\&=\lim_{x\to-2}\frac{3}{x^2-2x+4}\end{aligned}$$

ជំនួស $x=-2$ ៖

$$\begin{aligned}&=\frac{3}{(-2)^2-2(-2)+4}\\&=\frac{3}{4+4+4}\\&=\frac{3}{12}\\&=\frac{1}{4}\end{aligned}$$

ដូចនេះ $\lim_{x\to-2}\frac{3x+6}{x^3+8}=\frac{1}{4}$។

ច. $\lim_{x\to2}\frac{x+1}{x-1}$

កន្សោមនេះមិនមានរាងមិនកំណត់ទេ អាចជួសផ្ទាល់បាន៖

$$\begin{aligned}\lim_{x\to2}\frac{x+1}{x-1}&=\frac{2+1}{2-1}\\&=\frac{3}{1}\\&=3\end{aligned}$$

ដូចនេះ $\lim_{x\to2}\frac{x+1}{x-1}=3$។

ឆ. $\lim_{x\to-2}\frac{x^3+3x^2+2x}{x^2-x-6}$

ជំនួស $x=-2$ គេបានរាងមិនកំណត់ $\frac{0}{0}$។
បំបែកកត្តា៖
ភាគយក៖ $x^3+3x^2+2x=x(x^2+3x+2)=x(x+1)(x+2)$
ភាគបែង៖ $x^2-x-6=(x-3)(x+2)$
ជំនួសចូល និងសម្រួល $(x+2)$ ៖

$$\begin{aligned}&\lim_{x\to-2}\frac{x(x+1)(x+2)}{(x-3)(x+2)}\\&=\lim_{x\to-2}\frac{x(x+1)}{x-3}\end{aligned}$$

ជំនួស $x=-2$ ៖

$$\begin{aligned}&=\frac{-2(-2+1)}{-2-3}\\&=\frac{-2(-1)}{-5}\\&=-\frac{2}{5}\end{aligned}$$

ដូចនេះ $\lim_{x\to-2}\frac{x^3+3x^2+2x}{x^2-x-6}=-\frac{2}{5}$។

ជ. $\lim_{x\to1}\frac{x^2-2x+1}{x^3-x}$

រាងមិនកំណត់ $\frac{0}{0}$ ពេល $x\to1$។
បំបែកកត្តា៖
ភាគយក៖ $x^2-2x+1=(x-1)^2$
ភាគបែង៖ $x^3-x=x(x^2-1)=x(x-1)(x+1)$
ជំនួសចូល និងសម្រួល $(x-1)$ ៖

$$\begin{aligned}&\lim_{x\to1}\frac{(x-1)^2}{x(x-1)(x+1)}\\&=\lim_{x\to1}\frac{x-1}{x(x+1)}\end{aligned}$$

ជំនួស $x=1$ ៖

$$=\frac{1-1}{1(1+1)}=\frac{0}{2}=0$$

ដូចនេះ $\lim_{x\to1}\frac{x^2-2x+1}{x^3-x}=0$។

ឈ. $\lim\limits_{x\to 4}\frac{x^2+7x-44}{x^2-6x+8}$

រាងមិនកំណត់ $\frac{0}{0}$ ពេល $x\to 4$។

បំបែកកត្តា៖

ភាគយក៖ $x^2+7x-44=(x-4)(x+11)$

ភាគបែង៖ $x^2-6x+8=(x-4)(x-2)$

ជំនួសចូល និងសម្រួល $(x-4)$ ៖

$$\lim_{x\to 4}\frac{(x-4)(x+11)}{(x-4)(x-2)}$$
$$=\lim_{x\to 4}\frac{x+11}{x-2}$$

ជំនួស $x=4$ ៖

$$=\frac{4+11}{4-2}=\frac{15}{2}$$

ដូចនេះ $\lim\limits_{x\to 4}\frac{x^2+7x-44}{x^2-6x+8}=\frac{15}{2}$។

ញ. $\lim\limits_{x\to 1}\frac{x^2-4}{x^2-3x+2}$

ជំនួស $x=1$ យើងបាន៖

ភាគយក៖ $1^2-4=-3$

ភាគបែង៖ $1^2-3(1)+2=0$

ដោយសារភាគយកជាចំនួនថេរ -3 ហើយភាគបែងស្មើ 0 នោះលទ្ធផលរបស់វាគឺជាអនន្ត ∞។ (សញ្ញាអាស្រ័យលើតម្លៃ $x\to 1^+$ ឬ $x\to 1^-$)។

ដូចនេះ លីមីតនេះស្មើនឹង ∞។

ដ. $\lim\limits_{x\to 1}\frac{x^3-5x+4}{x^3-1}$

រាងមិនកំណត់ $\frac{0}{0}$ ពេល $x\to 1$។

បំបែកកត្តា៖

ភាគយក៖ $x^3-5x+4=x^3-x-4x+4=x(x^2-1)-4(x-1)=x(x-1)(x+1)-4(x-1)=(x-1)(x^2+x-4)$

ភាគបែង៖ $x^3-1=(x-1)(x^2+x+1)$

ជំនួសចូល និងសម្រួល $(x-1)$ ៖

$$\lim_{x\to 1}\frac{(x-1)(x^2+x-4)}{(x-1)(x^2+x+1)}$$
$$=\lim_{x\to 1}\frac{x^2+x-4}{x^2+x+1}$$

ជំនួស $x=1$ ៖

$$=\frac{1^2+1-4}{1^2+1+1}$$
$$=\frac{-2}{3}$$
$$=-\frac{2}{3}$$

ដូចនេះ $\lim\limits_{x\to 1}\frac{x^3-5x+4}{x^3-1}=-\frac{2}{3}$។

ឋ. $\lim\limits_{x\to 2}\frac{x^2-4}{x-2}$

រាងមិនកំណត់ $\frac{0}{0}$ ពេល $x\to 2$។

ភាគយក៖ $x^2-4=(x-2)(x+2)$

ជំនួស និងសម្រួល $(x-2)$ ៖

$$\lim_{x\to 2}\frac{(x-2)(x+2)}{x-2}$$
$$=\lim_{x\to 2}(x+2)$$

ជំនួស $x=2$ ៖

$$=2+2=4$$

ដូចនេះ $\lim\limits_{x\to 2}\frac{x^2-4}{x-2}=4$។

ខ. $\lim_{x\to2}\frac{x^2-4}{x^2-3x+2}$
រាងមិនកំណត់ $\frac{0}{0}$ ពេល $x\to2$។
ភាគយក៖ $x^2-4=(x-2)(x+2)$
ភាគបែង៖ $x^2-3x+2=(x-2)(x-1)$
ជំនួស និងសម្រួល $(x-2)$ ៖

$$\lim_{x\to2}\frac{(x-2)(x+2)}{(x-2)(x-1)}$$
$$=\lim_{x\to2}\frac{x+2}{x-1}$$

ជំនួស $x=2$ ៖

$$=\frac{2+2}{2-1}=\frac{4}{1}=4$$

ដូចនេះ $\lim_{x\to2}\frac{x^2-4}{x^2-3x+2}=4$។

ឃ. $\lim_{x\to1}\left(\frac{1}{x^2-1}-\frac{2}{x^4-1}\right)$
ជួស $x=1$ វាមានរាង $\infty-\infty$។ យើងត្រូវតែម្រូវភាគបែងរួម៖
ភាគបែងរួមគឺ $x^4-1=(x^2-1)(x^2+1)$ ដូចនេះប្រភាគទី១ ត្រូវគុណនឹង x^2+1។

$$\frac{1(x^2+1)}{(x^2-1)(x^2+1)}-\frac{2}{x^4-1}=\frac{x^2+1-2}{x^4-1}$$
$$=\frac{x^2-1}{x^4-1}$$

លីមីតក្លាយជា៖

$$\lim_{x\to1}\frac{x^2-1}{(x^2-1)(x^2+1)}$$
$$=\lim_{x\to1}\frac{1}{x^2+1}$$

ជំនួស $x=1$ ៖

$$=\frac{1}{1^2+1}=\frac{1}{2}$$

ដូចនេះ $\lim_{x\to1}\left(\frac{1}{x^2-1}-\frac{2}{x^4-1}\right)=\frac{1}{2}$។

ណ. $\lim_{x\to3}\frac{x-3}{x^2-5x+6}$
រាងមិនកំណត់ $\frac{0}{0}$។
ភាគបែង៖ $x^2-5x+6=(x-3)(x-2)$
ជំនួសចូល និងសម្រួល $(x-3)$ ៖

$$\lim_{x\to3}\frac{x-3}{(x-3)(x-2)}$$
$$=\lim_{x\to3}\frac{1}{x-2}$$

ជំនួស $x=3$ ៖

$$=\frac{1}{3-2}=1$$

ដូចនេះ $\lim_{x\to3}\frac{x-3}{x^2-5x+6}=1$។

ត. $\lim_{x\to1}\frac{x^2-4x+3}{x^3-x^2+2x-2}$
រាងមិនកំណត់ $\frac{0}{0}$។
ភាគយក៖ $x^2-4x+3=(x-1)(x-3)$
ភាគបែង៖ $x^3-x^2+2x-2=x^2(x-1)+2(x-1)=(x-1)(x^2+2)$
ជំនួសចូល និងសម្រួល $(x-1)$ ៖

$$\lim_{x\to1}\frac{(x-1)(x-3)}{(x-1)(x^2+2)}$$
$$=\lim_{x\to1}\frac{x-3}{x^2+2}$$

ជំនួស $x=1$ ៖

$$=\frac{1-3}{1^2+2}$$
$$=\frac{-2}{3}$$
$$=-\frac{2}{3}$$

ដូចនេះ $\lim_{x\to1}\frac{x^2-4x+3}{x^3-x^2+2x-2}=-\frac{2}{3}$។

ប. $\lim\limits_{x\to 1}\frac{x^9-3x+2}{x^6+5x-6}$

រាងមិនកំណត់ $\frac{0}{0}$ ពេល $x\to 1$។

យើងអាចបំបែកកត្តា ឬក៏ប្រើវិធានឡូពីតាល់។ ដើម្បីងាយស្រួលយល់តាមរបៀបកត្តា យើងអាចបំបែកវាជាពីរផ្នែក៖

ភាគយក៖

$x^9-3x+2=(x^9-1)-3(x-1)$

$=(x-1)(x^8+x^7+\ldots+x+1)-3(x-1)$

$=(x-1)(x^8+x^7+\ldots+x+1-3)$

ភាគបែង៖

$x^6+5x-6=(x^6-1)+5(x-1)$

$=(x-1)(x^5+x^4+\ldots+x+1)+5(x-1)$

$=(x-1)(x^5+x^4+\ldots+x+1+5)$

ជំនួសចូល និងសម្រួល $(x-1)$ ៖

$$\lim_{x\to 1}\frac{(x-1)(x^8+\ldots+x+1-3)}{(x-1)(x^5+\ldots+x+1+5)}$$
$$=\lim_{x\to 1}\frac{x^8+\ldots+x+1-3}{x^5+\ldots+x+1+5}$$

ជំនួស $x=1$ ៖ ដោយកន្សោមពី x^n រហូតដល់ 1 មាន $n+1$ តួ (សុទ្ធតែស្មើ ១)

$$=\frac{9-3}{6+5}=\frac{6}{11}$$

ដូចនេះ $\lim\limits_{x\to 1}\frac{x^9-3x+2}{x^6+5x-6}=\frac{6}{11}$។

ទ. $\lim\limits_{x\to 2}\frac{x-2}{x^3-2x^2-x-2}$

ជំនួស $x=2$ យើងបាន៖

ភាគយក៖ $2-2=0$

ភាគបែង៖ $2^3-2(2^2)-2-2=8-8-2-2=-4$

ដោយភាគយកស្មើ 0 តែភាគបែងជាចំនួនខុសពី 0 នោះប្រភាគទាំងមូលស្មើ 0។

$$\lim_{x\to 2}\frac{x-2}{x^3-2x^2-x-2}$$
$$=\frac{0}{-4}$$
$$=0$$

ដូចនេះ លីមីតនេះស្មើនឹង 0។

ឃ. $\lim\limits_{x\to 1}\frac{x^m-1}{x^n-1}$

(នេះជាលំហាត់ដូចនឹងលំហាត់ទី ១៩ ក្នុងប្រធានទី ៣)

ប្រើរូបមន្តពន្លាតទូទៅនៃ $A^k - B^k$ ចំពោះភាគយកនិងភាគបែង រួចសម្រួលកត្តា $(x-1)$៖

$$\lim_{x\to 1}\frac{(x-1)(x^{m-1}+\dots+x+1)}{(x-1)(x^{n-1}+\dots+x+1)}$$
$$=\lim_{x\to 1}\frac{x^{m-1}+\dots+x+1}{x^{n-1}+\dots+x+1}$$

ជួស $x=1$ វាស្មើនឹងការបូកលេខ ១ ចំនួន m ដងនៅភាគយក និង n ដងនៅភាគបែង៖

$$=\frac{m}{n}$$

ដូចនេះ $\lim\limits_{x\to 1}\frac{x^m-1}{x^n-1}=\frac{m}{n}$។

ង. $\lim\limits_{x\to 1}\frac{x^n-1}{x-1}$

(នេះជាលំហាត់ដូចនឹងលំហាត់ទី ១៨ ក្នុងប្រធានទី ៣)

ប្រើរូបមន្តពន្លាតទូទៅ រួចសម្រួលកត្តា $(x-1)$ ៖

$$\lim_{x\to 1}(x^{n-1}+x^{n-2}+\dots+x+1)$$

ជួស $x=1$ យើងនឹងបានផលបូកនៃលេខ ១ ចំនួន n ដង (ដោយសារពី x^1 ដល់ x^{n-1} មាន $n-1$ តួ បូកបន្ថែមតួ ១ ចុងក្រោយ គឺ n តួ)។

ដូចនេះ លីមីតស្មើនឹង n។

ដំណោះស្រាយលំហាត់ទី៥

ប្រធាន៖ ចូរដោះស្រាយលីមីតខាងក្រោមឲ្យបានត្រឹមត្រូវ៖

ក. $\lim\limits_{x\to+\infty}\frac{\sqrt{x+\sqrt{x+\sqrt{x}}}}{\sqrt{x}}$

កាលណា $x\to+\infty$ ទាំងភាគយកនិងភាគបែងខិតជិត $+\infty$ ដែលជារាងមិនកំណត់ $\frac{\infty}{\infty}$។
យើងទាញ x ជាកត្តាពីក្នុងរ៉ាឌីកាល់នៃភាគយក ដើម្បីអាចសម្រួលជាមួយ $\sqrt{x}$ នៅភាគបែងបាន៖

$$\sqrt{x+\sqrt{x+\sqrt{x}}}=\sqrt{x\left(1+\frac{\sqrt{x+\sqrt{x}}}{x}\right)}$$
$$=\sqrt{x}\sqrt{1+\sqrt{\frac{x+\sqrt{x}}{x^2}}}$$
$$=\sqrt{x}\sqrt{1+\sqrt{\frac{1}{x}+\sqrt{\frac{x}{x^4}}}}$$
$$=\sqrt{x}\sqrt{1+\sqrt{\frac{1}{x}+\frac{1}{x\sqrt{x}}}}$$

ជំនួសចូលក្នុងលីមីត និងសម្រួល $\sqrt{x}$ ៖

$$\lim_{x\to+\infty}\frac{\sqrt{x}\sqrt{1+\sqrt{\frac{1}{x}+\frac{1}{x\sqrt{x}}}}}{\sqrt{x}}$$
$$=\lim_{x\to+\infty}\sqrt{1+\sqrt{\frac{1}{x}+\frac{1}{x\sqrt{x}}}}$$

កាលណា $x\to+\infty$ នោះ $\frac{1}{x}\to 0$ និង $\frac{1}{x\sqrt{x}}\to 0$។ ដូច្នេះ៖

$$=\sqrt{1+\sqrt{0+0}}=\sqrt{1+0}=1$$

ដូចនេះ $\lim\limits_{x\to+\infty}\frac{\sqrt{x+\sqrt{x+\sqrt{x}}}}{\sqrt{x}}=1$។

ខ. $\lim\limits_{x\to+\infty}\frac{\sqrt{x}+\sqrt[3]{x}+\sqrt[4]{x}}{\sqrt{2x+3}}$

រាងមិនកំណត់ $\frac{\infty}{\infty}$។ យើងទាញ x ជាកត្តាដែលមានដឺក្រេខ្ពស់ជាងគេពីភាគយក និងភាគបែង។
នៅភាគយក ដឺក្រេខ្ពស់គេគឺ $\sqrt{x}=x^{\frac{1}{2}}$ ចំណែកឯនៅភាគបែង ដឺក្រេខ្ពស់គេគឺ $\sqrt{x}$ ដូចគ្នា។
យើងចែកទាំងភាគយក និងភាគបែងនឹង $\sqrt{x}$៖

$$\lim_{x\to+\infty}\frac{\frac{\sqrt{x}}{\sqrt{x}}+\frac{\sqrt[3]{x}}{\sqrt{x}}+\frac{\sqrt[4]{x}}{\sqrt{x}}}{\frac{\sqrt{2x+3}}{\sqrt{x}}}$$

$$=\lim_{x\to+\infty}\frac{1+x^{\frac{1}{3}-\frac{1}{2}}+x^{\frac{1}{4}-\frac{1}{2}}}{\sqrt{\frac{2x+3}{x}}}$$
$$=\lim_{x\to+\infty}\frac{1+x^{-\frac{1}{6}}+x^{-\frac{1}{4}}}{\sqrt{2+\frac{3}{x}}}$$

ដោយសារ $x^{-\frac{1}{6}}=\frac{1}{\sqrt[6]{x}}\to 0$ និង $x^{-\frac{1}{4}}=\frac{1}{\sqrt[4]{x}}\to 0$ ព្រមទាំង $\frac{3}{x}\to 0$៖

$$=\frac{1+0+0}{\sqrt{2+0}}$$
$$=\frac{1}{\sqrt{2}}$$
$$=\frac{\sqrt{2}}{2}$$

ដូចនេះ លីមីតស្មើនឹង $\frac{\sqrt{2}}{2}$។

គ. $\lim\limits_{x\to+\infty}\frac{\sqrt{x^2+x-3}+2x}{\sqrt{x^2+4}+x}$

រាង មិនកំណត់ $\frac{\infty}{\infty}$។ យើង ទាញ x^2 ជា កត្តា ចេញពី រ៉ាឌីកាល់៖

$$\sqrt{x^2+x-3}=\sqrt{x^2\left(1+\frac{1}{x}-\frac{3}{x^2}\right)}$$
$$=|x|\sqrt{1+\frac{1}{x}-\frac{3}{x^2}}$$

ដោយសារ $x\to+\infty$ នោះ $|x|=x$។ ដូចនេះភាគយក ក្លាយជា $x\sqrt{1+\frac{1}{x}-\frac{3}{x^2}}+2x$។

ភាគបែង៖ $\sqrt{x^2+4}+x=x\sqrt{1+\frac{4}{x^2}}+x$។

ជំនួសចូលលីមីត និងទាញ x ជាកត្តារួមសម្រួលចោល៖

$$\lim_{x\to+\infty}\frac{x\left(\sqrt{1+\frac{1}{x}-\frac{3}{x^2}}+2\right)}{x\left(\sqrt{1+\frac{4}{x^2}}+1\right)}$$
$$=\lim_{x\to+\infty}\frac{\sqrt{1+\frac{1}{x}-\frac{3}{x^2}}+2}{\sqrt{1+\frac{4}{x^2}}+1}$$

ជំនួស $x\to+\infty$ (តួដែលមាន x នៅភាគបែងនឹងទៅជា 0)៖

$$=\frac{\sqrt{1+0-0}+2}{\sqrt{1+0}+1}$$
$$=\frac{1+2}{1+1}$$
$$=\frac{3}{2}$$

ដូចនេះ លីមីតស្មើនឹង $\frac{3}{2}$។

ឃ. $\lim\limits_{x\to\infty}\frac{x^2-1}{2x^2+1}$

រាងមិនកំណត់ $\frac{\infty}{\infty}$។ សម្រាប់អនុគមន៍សនិទាន កាលណា $x\to\infty$ លីមីតនៃអនុគមន៍នោះស្មើនឹងផលធៀបនៃតួដែលមានដឺក្រេខ្ពស់ជាងគេនៃភាគយកនិងភាគបែង៖

$$\lim_{x\to\infty}\frac{x^2-1}{2x^2+1}$$
$$=\lim_{x\to\infty}\frac{x^2}{2x^2}$$

សម្រួល x^2 ចោល៖

$$=\frac{1}{2}$$

ដូចនេះ លីមីតស្មើនឹង $\frac{1}{2}$។

ង. $\lim\limits_{x\to-\infty}\frac{x^3+x^2-4}{2x^3+x+11}$

រាងមិនកំណត់ $\frac{\infty}{\infty}$។ យកតែតួដែលមានដឺក្រេខ្ពស់គេនៅភាគយក និងភាគបែង៖

$$\lim_{x\to-\infty}\frac{x^3+x^2-4}{2x^3+x+11}=\lim_{x\to-\infty}\frac{x^3}{2x^3}$$
$$=\frac{1}{2}$$

ដូចនេះ លីមីតស្មើនឹង $\frac{1}{2}$។

ច. $\lim\limits_{x\to\infty}\frac{3x^2+2x-1}{x^3-x+2}$

រាងមិនកំណត់ $\frac{\infty}{\infty}$។ យកតែតួដឺក្រេធំជាងគេ៖

$$\lim_{x\to\infty}\frac{3x^2+2x-1}{x^3-x+2}$$
$$=\lim_{x\to\infty}\frac{3x^2}{x^3}$$

សម្រួល x^2 ចោល៖

$$=\lim_{x\to\infty}\frac{3}{x}=0$$

ដូចនេះ លីមីតស្មើនឹង 0។

ឆ. $\lim_{x\to\infty}\left(\frac{x^3}{x^2+2}-x\right)$

រាង $\infty-\infty$។ ដើម្បីដោះស្រាយ យើងត្រូវតម្រូវភាគបែងរួម៖

$$\begin{aligned}\frac{x^3}{x^2+2}-x &= \frac{x^3-x(x^2+2)}{x^2+2}\\ &= \frac{x^3-x^3-2x}{x^2+2}\\ &= \frac{-2x}{x^2+2}\end{aligned}$$

ឥឡូវយើងបានរាង $\frac{\infty}{\infty}$។ យកតួដឺក្រេធំជាងគេ៖

$$\begin{aligned}\lim_{x\to\infty}\frac{-2x}{x^2+2} &= \lim_{x\to\infty}\frac{-2x}{x^2}\\ &= \lim_{x\to\infty}\frac{-2}{x}\\ &= 0\end{aligned}$$

ដូចនេះ លីមីតស្មើនឹង 0។

ជ. $\lim_{x\to\infty}\frac{x^2+3x-4}{3x^2-2x+5}$

រាងមិនកំណត់ $\frac{\infty}{\infty}$។ យកតែតួដឺក្រេធំជាងគេ៖

$$\begin{aligned}&\lim_{x\to\infty}\frac{x^2}{3x^2}\\ &= \frac{1}{3}\end{aligned}$$

ដូចនេះ លីមីតស្មើនឹង $\frac{1}{3}$។

ឈ. $\lim_{x\to\infty}\frac{x(x-1)(x-2)}{x^2+6x-9}$

រាងមិនកំណត់ $\frac{\infty}{\infty}$។ យកតួដឺក្រេធំជាងគេ៖

នៅភាគយក ដឺក្រេធំជាងគេគឺពេលគុណតួ x បញ្ចូលគ្នា គឺ $x\cdot x\cdot x=x^3$។

នៅភាគបែង ដឺក្រេធំជាងគេគឺ x^2។

$$\begin{aligned}&\lim_{x\to\infty}\frac{x^3}{x^2}\\ &= \lim_{x\to\infty}x\end{aligned}$$

បើសិនជា $x\to+\infty$ នោះលីមីតស្មើ $+\infty$ ហើយបើ $x\to-\infty$ នោះលីមីតស្មើ $-\infty$។ ជាទូទៅ វាស្មើនឹង ∞។

ញ. $\lim_{x\to\infty}\frac{\sqrt{x^2+9}}{x+3}$

រាងមិនកំណត់ $\frac{\infty}{\infty}$។ ទាញ x^2 ជាកត្តាចេញពីរ៉ាឌីកាល់៖

$$\begin{aligned}\sqrt{x^2+9} &= \sqrt{x^2\left(1+\frac{9}{x^2}\right)}\\ &= |x|\sqrt{1+\frac{9}{x^2}}\end{aligned}$$

ភាគបែងអាចទាញ x ជាកត្តា គឺ $x\left(1+\frac{3}{x}\right)$។

$$\begin{aligned}&\lim_{x\to\infty}\frac{|x|\sqrt{1+\frac{9}{x^2}}}{x\left(1+\frac{3}{x}\right)}\\ &= \lim_{x\to\infty}\frac{|x|}{x}\cdot\frac{\sqrt{1+\frac{9}{x^2}}}{1+\frac{3}{x}}\end{aligned}$$

បើសិន $x\to+\infty$ នោះ $\frac{|x|}{x}=1$ លីមីតស្មើ $1\cdot\frac{\sqrt{1}}{1}=1$។

បើសិន $x\to-\infty$ នោះ $\frac{|x|}{x}=-1$ លីមីតស្មើ $-1\cdot\frac{\sqrt{1}}{1}=-1$។

ដ. $\lim\limits_{x\to\infty}\left(\frac{x^2+x-1}{2x^2-x+1}\right)^3$

យើងគណនាលីមីតនៅក្នុងវង់ក្រចកសិន ដោយយកតួដឺក្រេធំជាងគេ៖

$$\lim_{x\to\infty}\frac{x^2+x-1}{2x^2-x+1}=\lim_{x\to\infty}\frac{x^2}{2x^2}$$
$$=\frac{1}{2}$$

បន្ទាប់មកលើកជាស្វ័យគុណ ៣ ៖

$$\left(\frac{1}{2}\right)^3=\frac{1}{8}$$

ដូចនេះ លីមីតស្មើនឹង $\frac{1}{8}$។

ឋ. $\lim\limits_{x\to\infty}\frac{x^2+2x+1}{5x}$

យកតួដឺក្រេធំជាងគេ៖

$$\lim_{x\to\infty}\frac{x^2}{5x}=\lim_{x\to\infty}\frac{x}{5}$$
$$=\infty$$

ឌ. $\lim\limits_{x\to\infty}\frac{x^3+x^4-1}{2x^5-x^2+x}$

ភាគយកដឺក្រេធំគេគឺ x^4 (ដោយសារ $x^4>x^3$) ហើយភាគបែងគឺ $2x^5$។

$$\lim_{x\to\infty}\frac{x^4}{2x^5}=\lim_{x\to\infty}\frac{1}{2x}$$
$$=0$$

ដូចនេះ លីមីតស្មើនឹង 0។

ឍ. $\lim\limits_{x\to\infty}\frac{(\sqrt{x^2+1}+x)^2}{\sqrt[3]{x^6+1}}$

(ករណី $x\to+\infty$)

យកតែតួធំៗចេញពីរ៉ាឌីកាល់៖ ភាគយក $\sqrt{x^2+1}\approx\sqrt{x^2}=x$។

ដូចនេះក្នុងវង់ក្រចកភាគយកគឺប្រហែល $x+x=2x$ រួចលើកជាការេបាន $(2x)^2=4x^2$។

ភាគបែង $\sqrt[3]{x^6+1}\approx\sqrt[3]{x^6}=x^2$។

$$\lim_{x\to+\infty}\frac{4x^2}{x^2}$$
$$=4$$

(បើ $x\to-\infty$ លីមីតស្មើ 0 ព្រោះភាគយកខិតជិត 0)។

ណ. $\lim\limits_{x\to-\infty}\frac{x^6+7x^4-40}{1-x-5x^7}$

យកតួដឺក្រេធំជាងគេ៖ ភាគយកគឺ x^6 និងភាគបែងគឺ $-5x^7$។

$$\lim_{x\to-\infty}\frac{x^6}{-5x^7}=\lim_{x\to-\infty}\frac{1}{-5x}$$
$$=0$$

ដូចនេះ លីមីតស្មើនឹង 0។

ត. $\lim\limits_{x\to\infty}\frac{(x+1)(x-2)}{3x^2+6x-5}$

ពន្លាតភាគយកបន្តិច ឬយកតែតួធំ $x\cdot x=x^2$។ ភាគបែងគឺ $3x^2$។

$$\lim_{x\to\infty}\frac{x^2}{3x^2}$$
$$=\frac{1}{3}$$

ដូចនេះ លីមីតស្មើនឹង $\frac{1}{3}$។

ឋ. $\lim_{x\to\infty} \frac{\sqrt{1+x^2}}{x}$

ដូចលំហាត់ទី ១០ ដែរ យើងទាញ x^2 ចេញពីឫសបាន $|x|$។

$$\lim_{x\to\infty} \frac{|x|}{x}$$

ស្មើ 1 (បើ $x \to +\infty$) និង -1 (បើ $x \to -\infty$)។

ឌ. $\lim_{x\to-\infty} \frac{5x^3-x^2+x}{1+x^2+3x^3}$

យកតួដឺក្រេធំជាងគេ៖

$$\lim_{x\to-\infty} \frac{5x^3}{3x^3}$$
$$= \frac{5}{3}$$

ឍ. $\lim_{x\to-\infty} \left(\frac{x^3-8}{x^4+16}\right)^{10}$

គណនាលីមីតក្នុងវង់ក្រចកសិន៖ $\lim_{x\to-\infty} \frac{x^3}{x^4} = \lim_{x\to-\infty} \frac{1}{x} = 0$។

រួចស្វ័យគុណ ១០៖ $0^{10} = 0$។

ដូចនេះ លីមីតស្មើនឹង ០។

ណ. $\lim_{x\to-\infty} \frac{8x-2x^5+x^6}{11x+5x^3+3x^5}$

យកតួធំជាងគេ៖ ភាគយកគឺ x^6 ភាគបែងគឺ $3x^5$។

$$\lim_{x\to-\infty} \frac{x^6}{3x^5} = \lim_{x\to-\infty} \frac{x}{3}$$
$$= -\infty$$

ត. $\lim_{x\to\infty} \left(\frac{3x^2+2x+1}{x^2-3x+1}\right)^4$

គណនាលីមីតក្នុងវង់ក្រចកសិន៖ $\lim_{x\to\infty} \frac{3x^2}{x^2} = 3$។

រួចលើកជាស្វ័យគុណ ៤ ៖ $3^4 = 81$។

ដូចនេះ លីមីតស្មើនឹង 81។

ថ. $\lim_{x\to\infty} \frac{1+x-3x^3}{1+x^2+3x^3}$

យកតួដឺក្រេធំជាងគេ៖

$$\lim_{x\to\infty} \frac{-3x^3}{3x^3}$$
$$= -1$$

ទ. $\lim_{x\to\infty} \frac{(x+3)(x+4)(x+5)}{x^4+x-11}$

ផលគុណភាគយកមានដឺក្រេធំបំផុតគឺ $x \cdot x \cdot x = x^3$។

ដឺក្រេភាគបែងគឺ x^4។

$$\lim_{x\to\infty} \frac{x^3}{x^4} = \lim_{x\to\infty} \frac{1}{x}$$
$$= 0$$

ធ. $\lim_{x\to\infty} \left(\frac{x^3}{2x^2-1} - \frac{x^2}{2x+1}\right)$

តម្រូវភាគបែងរួម៖

$$\frac{x^3(2x+1)-x^2(2x^2-1)}{(2x^2-1)(2x+1)} = \frac{2x^4+x^3-2x^4+x^2}{4x^3+2x^2-2x-1}$$
$$= \frac{x^3+x^2}{4x^3+\ldots}$$

យកតួដឺក្រេធំជាងគេ៖

$$\lim_{x\to\infty} \frac{x^3}{4x^3}$$
$$= \frac{1}{4}$$

ម. $\lim\limits_{x\to\infty}\left(x^2-\frac{x^4-1}{x^2-2}\right)$

តម្រូវភាគបែងរួម៖

$$\frac{x^2(x^2-2)-(x^4-1)}{x^2-2}=\frac{x^4-2x^2-x^4+1}{x^2-2}$$
$$=\frac{-2x^2+1}{x^2-2}$$

យកតួដឺក្រេធំជាងគេ៖

$$\lim_{x\to\infty}\frac{-2x^2}{x^2}$$
$$=-2$$

យ. $\lim\limits_{x\to\infty}\frac{(x-1)^{100}(6x+1)^{200}}{(3x+5)^{300}}$

ទាញយកតែតួ x ដែលមានដឺក្រេធំជាងគេពីក្នុងវង់ក្រចកនីមួយៗរួចស្វ័យគុណ៖

ភាគយក៖ ធំគេគឺ $x^{100}\cdot(6x)^{200}=6^{200}\cdot x^{300}$

ភាគបែង៖ ធំគេគឺ $(3x)^{300}=3^{300}\cdot x^{300}$

$$\lim_{x\to\infty}\frac{6^{200}\cdot x^{300}}{3^{300}\cdot x^{300}}=\frac{6^{200}}{3^{300}}$$
$$=\frac{(2\cdot3)^{200}}{3^{300}}$$
$$=\frac{2^{200}\cdot3^{200}}{3^{300}}$$
$$=\frac{2^{200}}{3^{100}}$$

ដូចនេះ លីមីតស្មើនឹង $\frac{2^{200}}{3^{100}}$។

រ. $\lim\limits_{x\to\infty}\frac{\sqrt[4]{x^5}+\sqrt[5]{x^3}+\sqrt[6]{x^8}}{\sqrt[3]{x^4+2}}$

យើងបំប្លែងរ៉ាឌីកាល់ជាស្វ័យគុណដើម្បីងាយប្រៀបធៀបដឺក្រេធំជាងគេ។

ភាគយកមាន៖ $x^{\frac{5}{4}},x^{\frac{3}{5}}$ និង $x^{\frac{8}{6}}=x^{\frac{4}{3}}$។ ដោយ $\frac{4}{3}>\frac{5}{4}>\frac{3}{5}$ (ព្រោះ៖ $\frac{4}{3}\approx1.33$ និង $\frac{5}{4}=1.25$) នាំឲ្យតួដឺក្រេធំគេគឺ $x^{\frac{4}{3}}$។

ភាគបែងមាន៖ $\sqrt[3]{x^4+2}\approx\sqrt[3]{x^4}=x^{\frac{4}{3}}$។

$$\lim_{x\to\infty}\frac{x^{\frac{4}{3}}}{x^{\frac{4}{3}}}$$
$$=1$$

ល. $\lim\limits_{x\to-\infty}\frac{x^2(2x+1)(3x-2)}{2x^2(5x-8)(x+6)}$

យកតែផលគុណនៃតួដឺក្រេធំៗ៖

ភាគយក៖ $x^2\cdot(2x)\cdot(3x)=6x^4$

ភាគបែង៖ $2x^2\cdot(5x)\cdot(x)=10x^4$

$$\lim_{x\to-\infty}\frac{6x^4}{10x^4}=\frac{6}{10}$$
$$=\frac{3}{5}$$

វ. $\lim\limits_{x\to\infty}\left(\frac{2x^8+8x^6+6x^4}{4x^8-x^6+12x^4}\right)^5$

គណនាលីមីតក្នុងវង់ក្រចកសិន៖ $\lim\limits_{x\to\infty}\frac{2x^8}{4x^8}=\frac{2}{4}=\frac{1}{2}$។

រួចលើកជាស្វ័យគុណ ៥ ៖ $\left(\frac{1}{2}\right)^5=\frac{1}{32}$។

ស. $\lim\limits_{x\to-\infty}\frac{(2x-3)^{20}(3x+2)^{30}}{(2x+1)^{50}}$

យកតួធំៗស្វ័យគុណ៖

ភាគយក៖ $(2x)^{20}\cdot(3x)^{30}=2^{20}x^{20}\cdot3^{30}x^{30}=2^{20}3^{30}x^{50}$

ភាគបែង៖ $(2x)^{50}=2^{50}x^{50}$

$$\lim_{x\to-\infty}\frac{2^{20}\cdot3^{30}\cdot x^{50}}{2^{50}\cdot x^{50}}=\frac{2^{20}\cdot3^{30}}{2^{50}}$$
$$=\frac{3^{30}}{2^{30}}$$
$$=\left(\frac{3}{2}\right)^{30}$$

ដំណោះស្រាយលំហាត់ទី៦

ប្រធាន៖ គណនាលីមីតខាងក្រោម៖

ក. $\lim\limits_{x\to1}\frac{\sqrt[3]{x}-1}{x-1}$

ជំនួស $x=1$ ចូលក្នុងលីមីតយើងបានរាងមិនកំណត់ $\frac{0}{0}$ ។
យើងអាចប្រើរូបមន្ត $a^3-b^3=(a-b)(a^2+ab+b^2)$ ដើម្បីបំបាត់រាងមិនកំណត់។ តាង $t=\sqrt[3]{x}\Rightarrow t^3=x$ កាលណា $x\to1$ នោះ $t\to1$។

$$\lim_{x\to1}\frac{\sqrt[3]{x}-1}{x-1}=\lim_{t\to1}\frac{t-1}{t^3-1}=\lim_{t\to1}\frac{t-1}{(t-1)(t^2+t+1)}=\lim_{t\to1}\frac{1}{t^2+t+1}=\frac{1}{1+1+1}=\frac{1}{3}$$

ដូចនេះ $\lim\limits_{x\to1}\frac{\sqrt[3]{x}-1}{x-1}=\frac{1}{3}$។

ខ. $\lim\limits_{x\to0}\frac{\sqrt[n]{a+x}-\sqrt[n]{a}}{x}$

ជំនួស $x=0$ ចូលយើងបានរាងមិនកំណត់ $\frac{0}{0}$ ។
តាង $u=\sqrt[n]{a+x}$ និង $v=\sqrt[n]{a}$ នោះ $u^n=a+x$ និង $v^n=a$ នាំឲ្យ $x=u^n-v^n$។ ពេល $x\to0$ នោះ $u\to v$។

$$\lim_{x\to0}\frac{\sqrt[n]{a+x}-\sqrt[n]{a}}{x}=\lim_{u\to v}\frac{u-v}{u^n-v^n}=\lim_{u\to v}\frac{u-v}{(u-v)(u^{n-1}+u^{n-2}v+\cdots}$$

$$=\lim_{u\to v}\frac{1}{u^{n-1}+u^{n-2}v+\cdots+v^{n-1}}=\frac{1}{nv^{n-1}}=\frac{1}{n(\sqrt[n]{a})^{n-1}}=\frac{1}{n}a^{\frac{1-n}{n}}$$

ដូចនេះ $\lim\limits_{x\to0}\frac{\sqrt[n]{a+x}-\sqrt[n]{a}}{x}=\frac{1}{n}a^{\frac{1-n}{n}}$។

គ. $\lim\limits_{x\to1}\frac{1-\sqrt[2021]{x}}{1-x}$

ជំនួស $x=1$ ចូលយើងបានរាងមិនកំណត់ $\frac{0}{0}$ ។
តាង $t=\sqrt[2021]{x}\Rightarrow x=t^{2021}$។ ពេល $x\to1$ នោះ $t\to1$។

$$\lim_{x\to1}\frac{1-\sqrt[2021]{x}}{1-x}=\lim_{t\to1}\frac{1-t}{1-t^{2021}}=\lim_{t\to1}\frac{1-t}{(1-t)(1+t+t^2+\cdots+t^{2020})}=\lim_{t\to1}\frac{1}{1+t+t^2+\cdots+t^{2020}}=\frac{1}{2021}$$

ដូចនេះ $\lim\limits_{x\to1}\frac{1-\sqrt[2021]{x}}{1-x}=\frac{1}{2021}$។

ឃ. $\lim\limits_{x\to0}\frac{\sqrt[5]{(x+1)^3}-1}{x}$

ជំនួស $x=0$ ចូលយើងបានរាងមិនកំណត់ $\frac{0}{0}$ ។
តាង $t=\sqrt[5]{x+1}\Rightarrow t^5=x+1$ នាំឲ្យ $x=t^5-1$។ ពេល $x\to0$ នោះ $t\to1$។

$$\lim_{x\to0}\frac{\sqrt[5]{(x+1)^3}-1}{x}=\lim_{t\to1}\frac{t^3-1}{t^5-1}=\lim_{t\to1}\frac{(t-1)(t^2+t+1)}{(t-1)(t^4+t^3+t^2+t+1}$$

ដូចនេះ $\lim\limits_{x\to0}\frac{\sqrt[5]{(x+1)^3}-1}{x}=\frac{3}{5}$។

ង. $\lim\limits_{x\to1}\frac{1-\sqrt[3]{2-x}}{1-x}$

ជំនួស $x=1$ ចូលយើងបានរាងមិនកំណត់ $\frac{0}{0}$ ។
តាង $t=\sqrt[3]{2-x}\Rightarrow t^3=2-x$ នាំឲ្យ $1-x=t^3-1$។
ពេល $x\to1$ នោះ $t\to1$។

$$\lim_{x\to1}\frac{1-\sqrt[3]{2-x}}{1-x}=\lim_{t\to1}\frac{1-t}{t^3-1}=\lim_{t\to1}\frac{-(t-1)}{(t-1)(t^2+t+1)}=\lim_{t\to1}\frac{-1}{t^2+t+1}=-\frac{1}{3}$$

ដូចនេះ $\lim\limits_{x\to1}\frac{1-\sqrt[3]{2-x}}{1-x}=-\frac{1}{3}$។

ច. $\lim\limits_{x\to0}\frac{\sqrt[3]{x+1}-1}{x(x+2)}$

ជំនួស $x=0$ ចូលយើងបានរាងមិនកំណត់ $\frac{0}{0}$ ។
តាង $t=\sqrt[3]{x+1}\Rightarrow t^3=x+1$ នាំឲ្យ $x=t^3-1$ និង $x+2=t^3+1$។ ពេល $x\to0$ នោះ $t\to1$។

$$\lim_{x\to0}\frac{\sqrt[3]{x+1}-1}{x(x+2)}=\lim_{t\to1}\frac{t-1}{(t^3-1)(t^3+1)}=\lim_{t\to1}\frac{t-1}{(t-1)(t^2+t+1)(}$$

ដូចនេះ $\lim\limits_{x\to0}\frac{\sqrt[3]{x+1}-1}{x(x+2)}=\frac{1}{6}$។

ឆ. $\lim\limits_{x\to 2}\frac{x^2-x-2}{\sqrt[3]{x+6}-2}$

ជំនួស $x=2$ ចូលយើងបានរាងមិនកំណត់ $\frac{0}{0}$ ។ កភាគយក $x^2-x-2=(x-2)(x+1)$។ តាង $t=\sqrt[3]{x+6}\Rightarrow x=t^3-6$ និង $x+1=t^3-5$។ ពេល $x\to 2$ នោះ $t\to 2$។

$$\lim_{x\to 2}\frac{x^2-x-2}{\sqrt[3]{x+6}-2}=\lim_{t\to 2}\frac{(t^3-8)(t^3-5)}{t-2}=\lim_{t\to 2}\frac{(t-2)(t^2+2t+4)(t^3-5)}{t-2}=\lim_{t\to 2}(t^2+2t+4)(t^3-5)=12\times 3=36$$

ដូចនេះ $\lim\limits_{x\to 2}\frac{x^2-x-2}{\sqrt[3]{x+6}-2}=36$។

ជ. $\lim\limits_{x\to 1}\frac{\sqrt[5]{x^2+x-1}-1}{1+\sqrt[3]{1-2x}}$

ជំនួស $x=1$ ចូលយើងបានរាងមិនកំណត់ $\frac{0}{0}$ ។ យើងអាចចែកនិងគុណកន្សោមដើម្បីប្តូរអថេរ៖

$$\lim_{x\to 1}\frac{\sqrt[5]{x^2+x-1}-1}{1+\sqrt[3]{1-2x}}=\lim_{x\to 1}\left(\frac{\sqrt[5]{x^2+x-1}-1}{x^2+x-2}\cdot\frac{x^2+x-2}{x-1}\cdot\frac{x}{1+}\right.$$

គណនាលីមីតនីមួយៗ៖

- ទី១៖ តាង $u=\sqrt[5]{x^2+x-1}\Rightarrow x^2+x-2=u^5-1$ លីមីតគឺ $\lim\limits_{u\to 1}\frac{u-1}{u^5-1}=\frac{1}{5}$ ។
- ទី២៖ $\lim\limits_{x\to 1}\frac{x^2+x-2}{x-1}=\lim\limits_{x\to 1}\frac{(x-1)(x+2)}{x-1}=3$ ។
- ទី៣៖ តាង $t=\sqrt[3]{1-2x}\Rightarrow x=\frac{1-t^3}{2}$ ពេល $x\to 1\Rightarrow t\to -1$ លីមីតគឺ $\lim\limits_{t\to -1}\frac{\frac{1-t^3}{2}-1}{1+t}=\lim\limits_{t\to -1}\frac{-(t+1)(t^2-t+1)}{2(t+1)}=-\frac{3}{2}$ ។

គុណលទ្ធផលទាំងបីបញ្ចូលគ្នា៖

$$\lim_{x\to 1}\frac{\sqrt[5]{x^2+x-1}-1}{1+\sqrt[3]{1-2x}}=\frac{1}{5}\cdot 3\cdot\left(-\frac{3}{2}\right)=-\frac{9}{10}$$

ដូចនេះ $\lim\limits_{x\to 1}\frac{\sqrt[5]{x^2+x-1}-1}{1+\sqrt[3]{1-2x}}=-\frac{9}{10}$។

ឈ. $\lim\limits_{x\to 0}\frac{\sqrt{1+2x}-1}{3x}$

ជំនួស $x=0$ ចូលយើងបានរាងមិនកំណត់ $\frac{0}{0}$ ។ គុណភាគយកនិងភាគបែងនឹងកន្សោមឆ្លាស់ $\sqrt{1+2x}+1$៖

$$\lim_{x\to 0}\frac{\sqrt{1+2x}-1}{3x}=\lim_{x\to 0}\frac{(\sqrt{1+2x}-1)(\sqrt{1+2x}+1)}{3x(\sqrt{1+2x}+1)}=\lim_{x\to 0}\frac{1+2x-1}{3x(\sqrt{1+2x}+1)}=\lim_{x\to 0}\frac{2x}{3x(\sqrt{1+2x}+1)}$$

$$=\lim_{x\to 0}\frac{2}{3(\sqrt{1+2x}+1)}=\frac{2}{3(1+1)}=\frac{1}{3}$$

ដូចនេះ $\lim\limits_{x\to 0}\frac{\sqrt{1+2x}-1}{3x}=\frac{1}{3}$។

ញ. $\lim\limits_{x\to 0}\frac{\sqrt{1+x}-\sqrt{1-x}}{x}$

ជំនួស $x=0$ ចូលយើងបានរាងមិនកំណត់ $\frac{0}{0}$ ។ គុណនឹងកន្សោមឆ្លាស់ $\sqrt{1+x}+\sqrt{1-x}$៖

$$\lim_{x\to 0}\frac{\sqrt{1+x}-\sqrt{1-x}}{x}=\lim_{x\to 0}\frac{(1+x)-(1-x)}{x(\sqrt{1+x}+\sqrt{1-x})}=\lim_{x\to 0}\frac{}{x(\sqrt{1+}}$$

ដូចនេះ $\lim\limits_{x\to 0}\frac{\sqrt{1+x}-\sqrt{1-x}}{x}=1$។

ដ. $\lim\limits_{x\to 0}\frac{x-\sqrt{x}}{\sqrt{x}}$

ដោយ x នៅក្នុងរ៉ាឌីកាល់ នោះ $x>0$ ឬ $x\to 0^+$ ។ ជំនួស $x=0$ ចូលយើងបានរាងមិនកំណត់ $\frac{0}{0}$ ។ ចាប់ $\sqrt{x}$ ជាកត្តា៖

$$\lim_{x\to 0}\frac{x-\sqrt{x}}{\sqrt{x}}=\lim_{x\to 0}\frac{\sqrt{x}(\sqrt{x}-1)}{\sqrt{x}}=\lim_{x\to 0}(\sqrt{x}-1)=0-1=-1$$

ដូចនេះ $\lim\limits_{x\to 0}\frac{x-\sqrt{x}}{\sqrt{x}}=-1$ ។

ឋ. $\lim\limits_{x\to 5}\frac{\sqrt{x-1}-2}{x^2-25}$

ជំនួស $x=5$ ចូលយើងបានរាងមិនកំណត់ $\frac{0}{0}$ ។ គុណកន្សោមឆ្លាស់ $\sqrt{x-1}+2$ នឹងពន្លាត $x^2-25=(x-5)(x+5)$ ៖

$$\lim_{x\to 5}\frac{\sqrt{x-1}-2}{x^2-25}=\lim_{x\to 5}\frac{(x-1)-4}{(x-5)(x+5)(\sqrt{x-1}+2)}=\lim_{x\to 5}$$

$$=\lim_{x\to 5}\frac{1}{(x+5)(\sqrt{x-1}+2)}=\frac{1}{(5+5)(\sqrt{4}+2)}=\frac{1}{10\times 4}=\frac{1}{40}$$

ដូចនេះ $\lim\limits_{x\to 5}\frac{\sqrt{x-1}-2}{x^2-25}=\frac{1}{40}$ ។

ឌ. $\lim\limits_{x\to 3}\frac{\sqrt{x+6}-3}{x^3-5x^2+3x+9}$

ជំនួស $x=3$ ចូលយើងបានរាងមិនកំណត់ $\frac{0}{0}$ ។ ភាគបែងបំបែកជាកត្តា $x^3-5x^2+3x+9=(x-3)^2(x+1)$ ៖

$$\lim_{x\to 3}\frac{\sqrt{x+6}-3}{x^3-5x^2+3x+9}=\lim_{x\to 3}\frac{(x+6)-9}{(x-3)^2(x+1)(\sqrt{x+6}+3)}=\lim_{x\to 3}\frac{1}{(x-3)(x+1)(\sqrt{x+6}+3)}$$

នៅពេល $x\to 3$ ភាគបែងខិតជិត 0 (ដោយសារកត្តា $x-3$) ឯភាគយកស្មើ 1 ។ ដូចនេះលីមីតនេះគ្មានតម្លៃកំណត់ទេ (ស្មើ $\pm\infty$) ។

ដូចនេះ $\lim\limits_{x\to 3}\frac{\sqrt{x+6}-3}{x^3-5x^2+3x+9}=\pm\infty$ ។

ឍ. $\lim\limits_{x\to 9}\frac{3-\sqrt{x}}{27-\sqrt{x^3}}$

ជំនួស $x=9$ ចូលយើងបានរាងមិនកំណត់ $\frac{0}{0}$ ។ តាង $t=\sqrt{x}$ ពេល $x\to 9$ នោះ $t\to 3$ ៖

$$\lim_{x\to 9}\frac{3-\sqrt{x}}{27-(\sqrt{x})^3}=\lim_{t\to 3}\frac{3-t}{27-t^3}=\lim_{t\to 3}\frac{3-t}{(3-t)(9+3t+t^2)}=\lim_{t\to 3}$$

ដូចនេះ $\lim\limits_{x\to 9}\frac{3-\sqrt{x}}{27-\sqrt{x^3}}=\frac{1}{27}$ ។

ណ. $\lim\limits_{x\to 0}\frac{\sqrt[3]{1+x}-\sqrt[3]{1-x}}{x}$

ជំនួស $x=0$ ចូលយើងបានរាងមិនកំណត់ $\frac{0}{0}$ ។ ប្រើរូបមន្ត a^3-b^3 ដោយគុណកន្សោមបង្គប់៖

$$\lim_{x\to 0}\frac{\sqrt[3]{1+x}-\sqrt[3]{1-x}}{x}=\lim_{x\to 0}\frac{(1+x)-(1-x)}{x\left(\sqrt[3]{(1+x)^2}+\sqrt[3]{(1+x)(1-x)}+\sqrt[3]{(1-x)^2}\right)}$$

$$=\lim_{x\to 0}\frac{2x}{x\left(\sqrt[3]{(1+x)^2}+\sqrt[3]{1-x^2}+\sqrt[3]{(1-x)^2}\right)}=\frac{2}{1+1+1}=\frac{2}{3}$$

ដូចនេះ $\lim\limits_{x\to 0}\frac{\sqrt[3]{1+x}-\sqrt[3]{1-x}}{x}=\frac{2}{3}$ ។

ត. $\lim\limits_{x\to 1}\frac{x^{\frac{2}{3}}-1}{x^{\frac{3}{5}}-1}$

ជាទម្រង់មិនកំណត់ $\frac{0}{0}$ ។

ចែកភាគយក និងភាគបែងនឹង $(x-1)$ ៖

$$\lim_{x\to 1}\frac{x^{\frac{2}{3}}-1}{x^{\frac{3}{5}}-1}=\lim_{x\to 1}\frac{\frac{x^{\frac{2}{3}}-1}{x-1}}{\frac{x^{\frac{3}{5}}-1}{x-1}}$$

តាមរូបមន្ត $\lim\limits_{x\to a}\frac{x^n-a^n}{x-a}=na^{n-1}$ យើងបាន៖

$$\frac{\frac{2}{3}(1)^{\frac{2}{3}-1}}{\frac{3}{5}(1)^{\frac{3}{5}-1}}=\frac{\frac{2}{3}}{\frac{3}{5}}=\frac{10}{9}$$

ដូចនេះ $\lim\limits_{x\to 1}\frac{x^{\frac{2}{3}}-1}{x^{\frac{3}{5}}-1}=\frac{10}{9}$ ។

ថ. $\lim\limits_{x\to 1}\frac{1-\sqrt[n]{x}}{1-\sqrt[m]{x}}$

ជាទម្រង់មិនកំណត់ $\frac{0}{0}$ ។

ចែកភាគយក និងភាគបែងនឹង $(1-x)$ ៖

$$\lim_{x\to 1}\frac{1-x^{\frac{1}{n}}}{1-x^{\frac{1}{m}}}=\lim_{x\to 1}\frac{\frac{1-x^{\frac{1}{n}}}{1-x}}{\frac{1-x^{\frac{1}{m}}}{1-x}}=\frac{\frac{1}{n}(1)^{\frac{1}{n}-1}}{\frac{1}{m}(1)^{\frac{1}{m}-1}}=\frac{\frac{1}{n}}{\frac{1}{m}}=\frac{m}{n}$$

ដូចនេះ $\lim\limits_{x\to 1}\frac{1-\sqrt[n]{x}}{1-\sqrt[m]{x}}=\frac{m}{n}$ ។

ទ. $\lim\limits_{x\to 1}\frac{1-\sqrt{x}}{1-x^2}$

ជាទម្រង់មិនកំណត់ $\frac{0}{0}$ ។

បំបែកភាគបែងជាកត្តា ៖

$$\lim_{x\to 1}\frac{1-\sqrt{x}}{1-x^2}=\lim_{x\to 1}\frac{1-\sqrt{x}}{(1-x)(1+x)}=\lim_{x\to 1}\frac{1-\sqrt{x}}{(1-\sqrt{x})(1+\sqrt{x})(1+x)}$$

$$=\lim_{x\to 1}\frac{1}{(1+\sqrt{x})(1+x)}=\frac{1}{(1+\sqrt{1})(1+1)}=\frac{1}{(2)(2)}=\frac{1}{4}$$

ដូចនេះ $\lim\limits_{x\to 1}\frac{1-\sqrt{x}}{1-x^2}=\frac{1}{4}$ ។

ឃ. $\lim\limits_{x\to 4}\frac{3-\sqrt{5+x}}{1-\sqrt{5-x}}$

ជាទម្រង់មិនកំណត់ $\frac{0}{0}$ ។

គុណភាគយក និងភាគបែងនឹងកន្សោមឆ្លាស់៖

$$\lim_{x\to 4}\frac{(3-\sqrt{5+x})(3+\sqrt{5+x})(1+\sqrt{5-x})}{(1-\sqrt{5-x})(1+\sqrt{5-x})(3+\sqrt{5+x})}=\lim_{x\to 4}\frac{(9-(5+x))(1+\sqrt{5-x})}{(1-(5-x))(3+\sqrt{5+x})}$$

$$=\lim_{x\to 4}\frac{(4-x)(1+\sqrt{5-x})}{(x-4)(3+\sqrt{5+x})}=\lim_{x\to 4}\frac{-(x-4)(1+\sqrt{5-x})}{(x-4)(3+\sqrt{5+x})}$$

$$=\lim_{x\to 4}\frac{-(1+\sqrt{5-x})}{3+\sqrt{5+x}}=\frac{-(1+\sqrt{1})}{3+\sqrt{9}}=\frac{-2}{6}=-\frac{1}{3}$$

ដូចនេះ $\lim\limits_{x\to 4}\frac{3-\sqrt{5+x}}{1-\sqrt{5-x}}=-\frac{1}{3}$ ។

ង. $\lim\limits_{x\to 1}\frac{\sqrt{x+8}-\sqrt{8x+1}}{\sqrt{5-x}-\sqrt{7x-3}}$

ជាទម្រង់មិនកំណត់ $\frac{0}{0}$ ។

គុណភាគយក និងភាគបែងនឹងកន្សោមឆ្លាស់រៀងគ្នា៖

$$\lim_{x\to 1}\frac{(\sqrt{x+8}-\sqrt{8x+1})(\sqrt{x+8}+\sqrt{8x+1})(\sqrt{5-x}+\sqrt{7x-3})}{(\sqrt{5-x}-\sqrt{7x-3})(\sqrt{5-x}+\sqrt{7x-3})(\sqrt{x+8}+\sqrt{8x+1})}$$

$$=\lim_{x\to 1}\frac{((x+8)-(8x+1))(\sqrt{5-x}+\sqrt{7x-3})}{((5-x)-(7x-3))(\sqrt{x+8}+\sqrt{8x+1})}$$

$$=\lim_{x\to 1}\frac{(7-7x)(\sqrt{5-x}+\sqrt{7x-3})}{(8-8x)(\sqrt{x+8}+\sqrt{8x+1})}=\lim_{x\to 1}\frac{-7(x-1)(\sqrt{5-x}+\sqrt{7x-3})}{-8(x-1)(\sqrt{x+8}+\sqrt{8x+1})}$$

$$=\lim_{x\to 1}\frac{7(\sqrt{5-x}+\sqrt{7x-3})}{8(\sqrt{x+8}+\sqrt{8x+1})}=\frac{7(\sqrt{4}+\sqrt{4})}{8(\sqrt{9}+\sqrt{9})}=\frac{7(4)}{8(6)}=\frac{28}{48}=\frac{7}{12}$$

ដូចនេះ $\lim\limits_{x\to 1}\frac{\sqrt{x+8}-\sqrt{8x+1}}{\sqrt{5-x}-\sqrt{7x-3}}=\frac{7}{12}$ ។

មេរៀនទី 5 ដំណោះស្រាយលំហាត់ប្រតិបត្តិ

ប. $\lim\limits_{x\to 0}\frac{\sqrt{4+x}-\sqrt{4-x}}{4x}$

ជាទម្រង់មិនកំណត់ $\frac{0}{0}$ ។

គុណភាគយក និងភាគបែងនឹងកន្សោមឆ្លាស់៖

$$\lim_{x\to 0}\frac{(\sqrt{4+x}-\sqrt{4-x})(\sqrt{4+x}+\sqrt{4-x})}{4x(\sqrt{4+x}+\sqrt{4-x})}$$

$$=\lim_{x\to 0}\frac{(4+x)-(4-x)}{4x(\sqrt{4+x}+\sqrt{4-x})}=\lim_{x\to 0}\frac{2x}{4x(\sqrt{4+x}+\sqrt{4-x})}$$

$$=\lim_{x\to 0}\frac{1}{2(\sqrt{4+x}+\sqrt{4-x})}=\frac{1}{2(\sqrt{4}+\sqrt{4})}=\frac{1}{2(4)}=\frac{1}{8}$$

ដូចនេះ $\lim\limits_{x\to 0}\frac{\sqrt{4+x}-\sqrt{4-x}}{4x}=\frac{1}{8}$ ។

ផ. $\lim\limits_{x\to 7}\frac{2-\sqrt{x-3}}{x^2-49}$

ជាទម្រង់មិនកំណត់ $\frac{0}{0}$ ។

គុណនឹងកន្សោមឆ្លាស់ ព្រមទាំងបំបែកភាគបែង៖

$$\lim_{x\to 7}\frac{(2-\sqrt{x-3})(2+\sqrt{x-3})}{(x-7)(x+7)(2+\sqrt{x-3})}$$

$$=\lim_{x\to 7}\frac{4-(x-3)}{(x-7)(x+7)(2+\sqrt{x-3})}=\lim_{x\to 7}\frac{7-x}{(x-7)(x+7)(2+\sqrt{x-3})}$$

$$=\lim_{x\to 7}\frac{-(x-7)}{(x-7)(x+7)(2+\sqrt{x-3})}=\lim_{x\to 7}\frac{-1}{(x+7)(2+\sqrt{x-3})}$$

$$=\frac{-1}{(7+7)(2+\sqrt{4})}=\frac{-1}{14\times 4}=-\frac{1}{56}$$

ដូចនេះ $\lim\limits_{x\to 7}\frac{2-\sqrt{x-3}}{x^2-49}=-\frac{1}{56}$ ។

ព. $\lim\limits_{x\to 0}\frac{\sqrt{1+x}-\sqrt{1-x}}{x}$
ជាទម្រង់មិនកំណត់ $\frac{0}{0}$ ។
គុណនឹងកន្សោមឆ្លាស់៖

$$\lim_{x\to 0}\frac{(\sqrt{1+x}-\sqrt{1-x})(\sqrt{1+x}+\sqrt{1-x})}{x(\sqrt{1+x}+\sqrt{1-x})}$$

$$=\lim_{x\to 0}\frac{(1+x)-(1-x)}{x(\sqrt{1+x}+\sqrt{1-x})}=\lim_{x\to 0}\frac{2x}{x(\sqrt{1+x}+\sqrt{1-x})}$$

$$=\lim_{x\to 0}\frac{2}{\sqrt{1+x}+\sqrt{1-x}}=\frac{2}{1+1}=1$$

ដូចនេះ $\lim\limits_{x\to 0}\frac{\sqrt{1+x}-\sqrt{1-x}}{x}=1$។

ភ. $\lim\limits_{x\to 3}\frac{\sqrt{x^2-2x+6}-\sqrt{x^2+2x-6}}{x^2-4x+3}$
ជាទម្រង់មិនកំណត់ $\frac{0}{0}$ ។
គុណនឹងកន្សោមឆ្លាស់ និងបំបែកភាគបែង៖

$$\lim_{x\to 3}\frac{(\sqrt{x^2-2x+6}-\sqrt{x^2+2x-6})(\sqrt{x^2-2x+6}+\sqrt{x^2+2x-}}{(x-1)(x-3)(\sqrt{x^2-2x+6}+\sqrt{x^2+2x-6})}$$

$$=\lim_{x\to 3}\frac{(x^2-2x+6)-(x^2+2x-6)}{(x-1)(x-3)(\sqrt{x^2-2x+6}+\sqrt{x^2+2x-6})}$$

$$=\lim_{x\to 3}\frac{-4x+12}{(x-1)(x-3)(\sqrt{x^2-2x+6}+\sqrt{x^2+2x-6})}$$

$$=\lim_{x\to 3}\frac{-4(x-3)}{(x-1)(x-3)(\sqrt{x^2-2x+6}+\sqrt{x^2+2x-6})}$$

$$=\lim_{x\to 3}\frac{-4}{(x-1)(\sqrt{x^2-2x+6}+\sqrt{x^2+2x-6})}=\frac{-4}{(3-1)(\sqrt{9}+\sqrt{}}$$

ដូចនេះ $\lim\limits_{x\to 3}\frac{\sqrt{x^2-2x+6}-\sqrt{x^2+2x-6}}{x^2-4x+3}=-\frac{1}{3}$។

មេរៀនទី 5 ដំណោះស្រាយលំហាត់ប្រតិបត្តិ

ម. $\lim\limits_{x\to-1}\frac{x+1}{2-\sqrt{4+x+x^2}}$

ជាទម្រង់មិនកំណត់ $\frac{0}{0}$ ។

គុណនឹងកន្សោមឆ្លាស់នៃភាគបែង៖

$$\lim_{x\to-1}\frac{(x+1)(2+\sqrt{4+x+x^2})}{(2-\sqrt{4+x+x^2})(2+\sqrt{4+x+x^2})}$$

$$=\lim_{x\to-1}\frac{(x+1)(2+\sqrt{4+x+x^2})}{4-(4+x+x^2)}=\lim_{x\to-1}\frac{(x+1)(2+\sqrt{4+x+x^2})}{-x^2-x}$$

$$=\lim_{x\to-1}\frac{(x+1)(2+\sqrt{4+x+x^2})}{-x(x+1)}=\lim_{x\to-1}\frac{2+\sqrt{4+x+x^2}}{-x}$$

$$=\frac{2+\sqrt{4-1+1}}{-(-1)}=\frac{2+2}{1}=4$$

ដូចនេះ $\lim\limits_{x\to-1}\frac{x+1}{2-\sqrt{4+x+x^2}}=4$។

យ. $\lim\limits_{x\to0}\frac{\sqrt{x^2+1}-1}{\sqrt{x^2+16}-4}$

ជាទម្រង់មិនកំណត់ $\frac{0}{0}$ ។

គុណនឹងកន្សោមឆ្លាស់នៃភាគយក និងភាគបែង៖

$$\lim_{x\to0}\frac{(\sqrt{x^2+1}-1)(\sqrt{x^2+1}+1)(\sqrt{x^2+16}+4)}{(\sqrt{x^2+16}-4)(\sqrt{x^2+16}+4)(\sqrt{x^2+1}+1)}$$

$$=\lim_{x\to0}\frac{(x^2+1-1)(\sqrt{x^2+16}+4)}{(x^2+16-16)(\sqrt{x^2+1}+1)}=\lim_{x\to0}\frac{x^2(\sqrt{x^2+16}+4)}{x^2(\sqrt{x^2+1}+1)}$$

$$=\lim_{x\to0}\frac{\sqrt{x^2+16}+4}{\sqrt{x^2+1}+1}=\frac{\sqrt{16}+4}{\sqrt{1}+1}=\frac{4+4}{1+1}=\frac{8}{2}=4$$

ដូចនេះ $\lim\limits_{x\to0}\frac{\sqrt{x^2+1}-1}{\sqrt{x^2+16}-4}=4$។

រ. $\lim\limits_{x\to 0}\frac{\sqrt{x^2+1}-1}{\sqrt{x^2+9}-3}$

ជាទម្រង់មិនកំណត់ $\frac{0}{0}$ ។

គុណនឹងកន្សោមឆ្លាស់នៃភាគយក និងភាគបែង៖

$$\lim_{x\to 0}\frac{(\sqrt{x^2+1}-1)(\sqrt{x^2+1}+1)(\sqrt{x^2+9}+3)}{(\sqrt{x^2+9}-3)(\sqrt{x^2+9}+3)(\sqrt{x^2+1}+1)}$$

$$=\lim_{x\to 0}\frac{(x^2+1-1)(\sqrt{x^2+9}+3)}{(x^2+9-9)(\sqrt{x^2+1}+1)}=\lim_{x\to 0}\frac{x^2(\sqrt{x^2+9}+3)}{x^2(\sqrt{x^2+1}+1)}$$

$$=\lim_{x\to 0}\frac{\sqrt{x^2+9}+3}{\sqrt{x^2+1}+1}=\frac{\sqrt{9}+3}{\sqrt{1}+1}=\frac{3+3}{1+1}=\frac{6}{2}=3$$

ដូចនេះ $\lim\limits_{x\to 0}\frac{\sqrt{x^2+1}-1}{\sqrt{x^2+9}-3}=3$។

ល. $\lim\limits_{x\to 2}\frac{\sqrt[3]{10-x}-2}{x-2}$

ជាទម្រង់មិនកំណត់ $\frac{0}{0}$ ។

ប្រើរូបមន្ត $a^3-b^3=(a-b)(a^2+ab+b^2)$ គុណនឹងកន្សោមឆ្លាស់គូប៖

$$\lim_{x\to 2}\frac{(\sqrt[3]{10-x}-2)(\sqrt[3]{(10-x)^2}+2\sqrt[3]{10-x}+4)}{(x-2)(\sqrt[3]{(10-x)^2}+2\sqrt[3]{10-x}+4)}$$

$$=\lim_{x\to 2}\frac{(10-x)-8}{(x-2)(\sqrt[3]{(10-x)^2}+2\sqrt[3]{10-x}+4)}=\lim_{x\to 2}$$

$$=\lim_{x\to 2}\frac{-(x-2)}{(x-2)(\sqrt[3]{(10-x)^2}+2\sqrt[3]{10-x}+4)}=\lim_{x\to 2}$$

$$=\frac{-1}{\sqrt[3]{8^2}+2\sqrt[3]{8}+4}=\frac{-1}{4+4+4}=-\frac{1}{12}$$

ដូចនេះ $\lim\limits_{x\to 2}\frac{\sqrt[3]{10-x}-2}{x-2}=-\frac{1}{12}$។

មេរៀនទី 5 ដំណោះស្រាយលំហាត់ប្រតិបត្តិ

 រ. $\lim\limits_{x\to 1}\frac{x-1}{\sqrt[3]{26+x}-3}$

ជាទម្រង់មិនកំណត់ $\frac{0}{0}$ ។

គុណនឹងកន្សោមឆ្លាស់គូបនៃភាគបែង៖

$$\lim_{x\to 1}\frac{(x-1)(\sqrt[3]{(26+x)^2}+3\sqrt[3]{26+x}+9)}{(\sqrt[3]{26+x}-3)(\sqrt[3]{(26+x)^2}+3\sqrt[3]{26+x}+9)}$$

$$=\lim_{x\to 1}\frac{(x-1)(\sqrt[3]{(26+x)^2}+3\sqrt[3]{26+x}+9)}{26+x-27}$$

$$=\lim_{x\to 1}\frac{(x-1)(\sqrt[3]{(26+x)^2}+3\sqrt[3]{26+x}+9)}{x-1}$$

$$=\lim_{x\to 1}(\sqrt[3]{(26+x)^2}+3\sqrt[3]{26+x}+9)=\sqrt[3]{27^2}+3\sqrt[3]{27}+9=9+9+9=27$$

ដូចនេះ $\lim\limits_{x\to 1}\frac{x-1}{\sqrt[3]{26+x}-3}=27$។

ស. $\lim\limits_{x\to 0}\frac{\sqrt[3]{x^2+1}-1}{x^2}$

ជាទម្រង់មិនកំណត់ $\frac{0}{0}$ ។

(ចំណាំ៖ ប្រធានដើមមានការភាន់ច្រឡំបន្តិច ដូច្នេះយើងកែសម្រួលទៅជា $\frac{\sqrt[3]{x^2+1}-1}{x^2}$) គុណនឹងកន្សោមឆ្លាស់គូប៖

$$\lim_{x\to 0}\frac{(\sqrt[3]{x^2+1}-1)(\sqrt[3]{(x^2+1)^2}+\sqrt[3]{x^2+1}+1)}{x^2(\sqrt[3]{(x^2+1)^2}+\sqrt[3]{x^2+1}+1)}$$

$$=\lim_{x\to 0}\frac{(x^2+1)-1}{x^2(\sqrt[3]{(x^2+1)^2}+\sqrt[3]{x^2+1}+1)}=\lim_{x\to 0}\frac{x^2}{x^2(\sqrt[3]{(x^2+1)^2}+\sqrt[3]{}$$

$$=\lim_{x\to 0}\frac{1}{\sqrt[3]{(x^2+1)^2}+\sqrt[3]{x^2+1}+1}=\frac{1}{1+1+1}=\frac{1}{3}$$

ដូចនេះ $\lim\limits_{x\to 0}\frac{\sqrt[3]{x^2+1}-1}{x^2}=\frac{1}{3}$។

ដំណោះស្រាយលំហាត់ទី៧

ប្រធាន៖ គណនាលីមីតខាងក្រោម៖

ក. $\lim_{x\to\infty}(-3x^2+5x+2)$

កាលណា $x\to\infty$ លីមីតមានរាងមិនកំណត់ $\infty-\infty$។ ក្នុងករណីនេះ យើងត្រូវទាញតួដែលមានដឺក្រេធំជាងគេគឺ x^2 ជាកត្តា។

$$\lim_{x\to\infty}(-3x^2+5x+2)=\lim_{x\to\infty}x^2\left(-3+\frac{5}{x}+\frac{2}{x^2}\right)$$

ដោយ $\lim_{x\to\infty}x^2=+\infty$

និង $\lim_{x\to\infty}\left(-3+\frac{5}{x}+\frac{2}{x^2}\right)=-3$

ហេតុនេះ $(+\infty)\times(-3)=-\infty$។

ដូចនេះ $\lim_{x\to\infty}(-3x^2+5x+2)=-\infty$។

ខ. $\lim_{x\to\infty}(-2+x-x^4)$

កាលណា $x\to\infty$ លីមីតមានរាងមិនកំណត់ $\infty-\infty$។ យើងទាញតួដែលមានដឺក្រេធំជាងគេគឺ x^4 ជាកត្តា។

$$\lim_{x\to\infty}(-2+x-x^4)=\lim_{x\to\infty}x^4\left(-\frac{2}{x^4}+\frac{1}{x^3}-1\right)$$

ដោយ $\lim_{x\to\infty}x^4=+\infty$

និង $\lim_{x\to\infty}\left(-\frac{2}{x^4}+\frac{1}{x^3}-1\right)=-1$

ហេតុនេះ $(+\infty)\times(-1)=-\infty$។

ដូចនេះ $\lim_{x\to\infty}(-2+x-x^4)=-\infty$។

គ. $\lim_{x\to\infty}(-x^3+x^2-4x+5)$

កាលណា $x\to\infty$ ក្នុងទីនេះយើងសន្មតថាជា $+\infty$។ លីមីតមានរាងមិនកំណត់ $\infty-\infty$។ យើងទាញតួ x^3 ជាកត្តា។

$$\lim_{x\to\infty}(-x^3+x^2-4x+5)=\lim_{x\to\infty}x^3\left(-1+\frac{1}{x}-\frac{4}{x^2}+\frac{5}{x^3}\right)$$

ដោយ $\lim_{x\to\infty}x^3=+\infty$

និង $\lim_{x\to\infty}\left(-1+\frac{1}{x}-\frac{4}{x^2}+\frac{5}{x^3}\right)=-1$

ហេតុនេះ $(+\infty)\times(-1)=-\infty$។

ដូចនេះ $\lim_{x\to\infty}(-x^3+x^2-4x+5)=-\infty$។

ឃ. $\lim_{x\to+\infty}(1+x-x^5)$

កាលណា $x\to+\infty$ លីមីតមានរាងមិនកំណត់ $\infty-\infty$។ យើងទាញតួ x^5 ជាកត្តា។

$$\lim_{x\to+\infty}(1+x-x^5)=\lim_{x\to+\infty}x^5\left(\frac{1}{x^5}+\frac{1}{x^4}-1\right)$$

ដោយ $\lim_{x\to+\infty}x^5=+\infty$

និង $\lim_{x\to+\infty}\left(\frac{1}{x^5}+\frac{1}{x^4}-1\right)=-1$

ហេតុនេះ $(+\infty)\times(-1)=-\infty$។

ដូចនេះ $\lim_{x\to+\infty}(1+x-x^5)=-\infty$។

ង. $\lim_{x\to-\infty}(1-3x-x^3)$

កាលណា $x\to-\infty$ លីមីតមានរាងមិនកំណត់ $\infty-\infty$។ យើងទាញតួ x^3 ជាកត្តា។

$$\lim_{x\to-\infty}(1-3x-x^3)=\lim_{x\to-\infty}x^3\left(\frac{1}{x^3}-\frac{3}{x^2}-1\right)$$

ដោយ $\lim_{x\to-\infty}x^3=-\infty$

និង $\lim_{x\to-\infty}\left(\frac{1}{x^3}-\frac{3}{x^2}-1\right)=-1$

ហេតុនេះ $(-\infty)\times(-1)=+\infty$។

ដូចនេះ $\lim_{x\to-\infty}(1-3x-x^3)=+\infty$។

ច. $\lim_{x\to\infty}(\sqrt{x-2}-\sqrt{x})$

កាលណា $x\to\infty$ លីមីតមានរាងមិនកំណត់ $\infty-\infty$។ យើងគុណ និងចែកនឹងកន្សោមឆ្លាស់ $(\sqrt{x-2}+\sqrt{x})$។

$$\lim_{x\to\infty}(\sqrt{x-2}-\sqrt{x})=\lim_{x\to\infty}\frac{(\sqrt{x-2}-\sqrt{x})(\sqrt{x-2}+\sqrt{x})}{\sqrt{x-2}+\sqrt{x}}$$

$$=\lim_{x\to\infty}\frac{(x-2)-x}{\sqrt{x-2}+\sqrt{x}}$$

$$=\lim_{x\to\infty}\frac{-2}{\sqrt{x-2}+\sqrt{x}}$$

ដោយ ភាគបែង $\lim_{x\to\infty}(\sqrt{x-2}+\sqrt{x})=+\infty$ នោះ ផលចែកស្មើ 0។

ដូចនេះ $\lim_{x\to\infty}(\sqrt{x-2}-\sqrt{x})=0$។

មេរៀនទី 5 ដំណោះស្រាយលំហាត់ប្រតិបត្តិ

ឆ. $\lim\limits_{x\to\infty}(\sqrt{x^2+x}-x)$

កាលណា $x\to\infty$ លីមីតមានរាងមិនកំណត់ $\infty-\infty$។ យើងគុណ និងចែកនឹងកន្សោមឆ្លាស់។

$$\lim_{x\to\infty}(\sqrt{x^2+x}-x)=\lim_{x\to\infty}\frac{(\sqrt{x^2+x}-x)(\sqrt{x^2+x}+x)}{\sqrt{x^2+x}+x}$$
$$=\lim_{x\to\infty}\frac{(x^2+x)-x^2}{\sqrt{x^2+x}+x}$$
$$=\lim_{x\to\infty}\frac{x}{\sqrt{x^2+x}+x}$$
$$=\lim_{x\to\infty}\frac{x}{x\left(\sqrt{1+\frac{1}{x}}+1\right)}$$
$$=\lim_{x\to\infty}\frac{1}{\sqrt{1+\frac{1}{x}}+1}$$

កាលណា $x\to\infty$, $\frac{1}{x}\to 0$ នាំឲ្យលីមីតស្មើ $\frac{1}{\sqrt{1+0}+1}=\frac{1}{2}$។

ដូចនេះ $\lim\limits_{x\to\infty}(\sqrt{x^2+x}-x)=\frac{1}{2}$។

ជ. $\lim\limits_{x\to-\infty}(\sqrt{x^2+x}-x)$

កាលណា $x\to-\infty$ តួ $\sqrt{x^2+x}\to+\infty$ និង $-x\to+\infty$ ដូច្នេះលីមីតគ្មានរាងមិនកំណត់ទេ គឺវាមានរាង $+\infty+\infty$។

$$\lim_{x\to-\infty}(\sqrt{x^2+x}-x)=\lim_{x\to-\infty}\left(\sqrt{x^2(1+\frac{1}{x})}-x\right)$$
$$=\lim_{x\to-\infty}\left(|x|\sqrt{1+\frac{1}{x}}-x\right)$$
$$=\lim_{x\to-\infty}\left(-x\sqrt{1+\frac{1}{x}}-x\right)$$
$$=\lim_{x\to-\infty}-x\left(\sqrt{1+\frac{1}{x}}+1\right)$$

ដោយ $\lim\limits_{x\to-\infty}-x=+\infty$ និងតួក្នុងវង់ក្រចកខិតជិត 2។

ដូចនេះ $\lim\limits_{x\to-\infty}(\sqrt{x^2+x}-x)=+\infty$។

ឈ. $\lim\limits_{x\to+\infty}(\sqrt{x-3}-\sqrt{x})$

កាលណា $x\to+\infty$ លីមីតមានរាងមិនកំណត់ $\infty-\infty$។ យើងគុណ និងចែកនឹងកន្សោមឆ្លាស់។

$$\lim_{x\to+\infty}(\sqrt{x-3}-\sqrt{x})=\lim_{x\to+\infty}\frac{(\sqrt{x-3}-\sqrt{x})(\sqrt{x-3}+\sqrt{x})}{\sqrt{x-3}+\sqrt{x}}$$
$$=\lim_{x\to+\infty}\frac{(x-3)-x}{\sqrt{x-3}+\sqrt{x}}$$
$$=\lim_{x\to+\infty}\frac{-3}{\sqrt{x-3}+\sqrt{x}}$$

ដោយភាគបែងខិតជិត $+\infty$ នោះប្រភាគស្មើ 0។

ដូចនេះ $\lim\limits_{x\to+\infty}(\sqrt{x-3}-\sqrt{x})=0$។

ញ. $\lim\limits_{x\to+\infty}\sqrt{x}(\sqrt{x-3}-\sqrt{x})$

កាលណា $x\to+\infty$ លីមីតមានរាងមិនកំណត់ $\infty\times 0$។ យើងគុណ និងចែកកន្សោមក្នុងវង់ក្រចកនឹងកន្សោមឆ្លាស់របស់វា។

$$\lim_{x\to+\infty}\sqrt{x}(\sqrt{x-3}-\sqrt{x})=\lim_{x\to+\infty}\sqrt{x}\frac{(\sqrt{x-3}-\sqrt{x})(\sqrt{x-3}+\sqrt{x})}{\sqrt{x-3}+\sqrt{x}}$$
$$=\lim_{x\to+\infty}\sqrt{x}\frac{(x-3)-x}{\sqrt{x-3}+\sqrt{x}}$$
$$=\lim_{x\to+\infty}\frac{-3\sqrt{x}}{\sqrt{x}\left(\sqrt{1-\frac{3}{x}}+1\right)}$$
$$=\lim_{x\to+\infty}\frac{-3}{\sqrt{1-\frac{3}{x}}+1}$$

កាលណា $x\to+\infty$, $\frac{3}{x}\to 0$។ ដូច្នេះ លីមីត ស្មើ $\frac{-3}{\sqrt{1-0}+1}=-\frac{3}{2}$។

ដូចនេះ $\lim\limits_{x\to+\infty}\sqrt{x}(\sqrt{x-3}-\sqrt{x})=-\frac{3}{2}$។

ដ. $\lim\limits_{x\to+\infty} x(\sqrt{x^2+1}-x)$

មានរាងមិនកំណត់ $\infty-\infty$ ក្នុងវង់ក្រចក។ គុណនិងចែកកន្សោមឆ្លាស់៖

$$\begin{aligned}\lim_{x\to+\infty} x(\sqrt{x^2+1}-x) &= \lim_{x\to+\infty} x\frac{(\sqrt{x^2+1}-x)(\sqrt{x^2+1}+x)}{\sqrt{x^2+1}+x}\\ &= \lim_{x\to+\infty}\frac{x(x^2+1-x^2)}{\sqrt{x^2(1+\frac{1}{x^2})}+x}\\ &= \lim_{x\to+\infty}\frac{x}{x\sqrt{1+\frac{1}{x^2}}+x}\\ &= \lim_{x\to+\infty}\frac{x}{x(\sqrt{1+\frac{1}{x^2}}+1)}\\ &= \lim_{x\to+\infty}\frac{1}{\sqrt{1+\frac{1}{x^2}}+1}=\frac{1}{1+1}=\frac{1}{2}\end{aligned}$$

ដូចនេះ $\lim\limits_{x\to+\infty} x(\sqrt{x^2+1}-x)=\frac{1}{2}$។

ឋ. $\lim\limits_{x\to-\infty} x(\sqrt{x^2+1}-x)$

ជំនួសតម្លៃ $x\to-\infty$ ចូលក្នុងលីមីតដោយផ្ទាល់ យើងបានរាងគុណនៃ $-\infty$ និង $+\infty$៖

$$\begin{aligned}\lim_{x\to-\infty} x(\sqrt{x^2+1}-x) &= (-\infty)\cdot(+\infty-(-\infty))\\ &= (-\infty)\cdot(+\infty)=-\infty\end{aligned}$$

ដូចនេះ $\lim\limits_{x\to-\infty} x(\sqrt{x^2+1}-x)=-\infty$។

ឌ. $\lim\limits_{x\to+\infty} (\sqrt{x^2+1}-x)$

មានរាងមិនកំណត់ $\infty-\infty$។ គុណនិងចែកកន្សោមឆ្លាស់៖

$$\begin{aligned}\lim_{x\to+\infty} (\sqrt{x^2+1}-x) &= \lim_{x\to+\infty}\frac{(\sqrt{x^2+1}-x)(\sqrt{x^2+1}+x)}{\sqrt{x^2+1}+x}\\ &= \lim_{x\to+\infty}\frac{x^2+1-x^2}{\sqrt{x^2+1}+x}\\ &= \lim_{x\to+\infty}\frac{1}{\sqrt{x^2+1}+x}=0\end{aligned}$$

ដូចនេះ $\lim\limits_{x\to+\infty} (\sqrt{x^2+1}-x)=0$។

ឍ. $\lim\limits_{x\to-\infty} (\sqrt{x^2+1}-x)$

ដោយ $x\to-\infty$ នោះ $\sqrt{x^2+1}\to+\infty$ និង $-x\to+\infty$៖

$$\begin{aligned}\lim_{x\to-\infty} (\sqrt{x^2+1}-x) &= +\infty-(-\infty)\\ &= +\infty+\infty=+\infty\end{aligned}$$

ដូចនេះ $\lim\limits_{x\to-\infty} (\sqrt{x^2+1}-x)=+\infty$។

ណ. $\lim\limits_{x\to+\infty} (\sqrt{x+1}-\sqrt{x^2+3})$

ចាប់ x ជាកត្តា ព្រោះដឺក្រេក្នុងឫសមិនស្មើគ្នា៖

$$\begin{aligned}\lim_{x\to+\infty} (\sqrt{x+1}-\sqrt{x^2+3}) &= \lim_{x\to+\infty}\left(\sqrt{x^2(\frac{1}{x}+\frac{1}{x^2})}-\sqrt{x^2(1+\frac{3}{x^2})}\right)\\ &= \lim_{x\to+\infty} x\left(\sqrt{\frac{1}{x}+\frac{1}{x^2}}-\sqrt{1+\frac{3}{x^2}}\right)\\ &= (+\infty)\cdot(0-1)=-\infty\end{aligned}$$

ដូចនេះ $\lim\limits_{x\to+\infty} (\sqrt{x+1}-\sqrt{x^2+3})=-\infty$។

ត. $\lim\limits_{x\to\pm\infty} (x\sqrt{x^2+1}-x)$

ចាប់ x ជាកត្តារួម៖

$$\lim_{x\to\pm\infty} (x\sqrt{x^2+1}-x)=\lim_{x\to\pm\infty} x(\sqrt{x^2+1}-1)$$

បើ $x\to+\infty \Rightarrow (+\infty)\cdot(+\infty-1)=+\infty$

បើ $x\to-\infty \Rightarrow (-\infty)\cdot(+\infty-1)=-\infty$

ដូចនេះ $\lim\limits_{x\to\pm\infty} (x\sqrt{x^2+1}-x)=\pm\infty$។

មេរៀនទី 5 ដំណោះស្រាយលំហាត់ប្រតិបត្តិ

ប. $\lim\limits_{x\to-\infty}(\sqrt{2x^2+x}+3x-1)$

ចាប់ x ជាកត្តា ដោយប្រយ័ត្នសញ្ញាពេលបញ្ចេញពីឫស (ដោយ $x\to-\infty$ នាំឲ្យ $\sqrt{x^2}=|x|=-x$)៖

$$\begin{aligned}\lim_{x\to-\infty}(\sqrt{2x^2+x}+3x-1) &= \lim_{x\to-\infty}\left(\sqrt{x^2(2+\frac{1}{x})}+x(3-\frac{1}{x})\right)\\ &= \lim_{x\to-\infty}\left(-x\sqrt{2+\frac{1}{x}}+x(3-\frac{1}{x})\right)\\ &= \lim_{x\to-\infty}x\left(-\sqrt{2+\frac{1}{x}}+3-\frac{1}{x}\right)\\ &= (-\infty)\cdot(-\sqrt{2}+3)=-\infty \quad (\text{ព្រោះ } 3-\sqrt{2}>0)\end{aligned}$$

ដូចនេះ $\lim\limits_{x\to-\infty}(\sqrt{2x^2+x}+3x-1)=-\infty$។

ផ. $\lim\limits_{x\to\infty}(\sqrt{x^2-1}-\sqrt{x^2+3x})$

សន្មត $x\to+\infty$ ព្រោះវាជារាង $\infty-\infty$។ គុណនិងចែកកន្សោមឆ្លាស់៖

$$\begin{aligned}\lim_{x\to+\infty}(\sqrt{x^2-1}-\sqrt{x^2+3x}) &= \lim_{x\to+\infty}\frac{(\sqrt{x^2-1}-\sqrt{x^2+3x})(\sqrt{x^2-1}+\sqrt{x^2+3x})}{\sqrt{x^2-1}+\sqrt{x^2+3x}}\\ &= \lim_{x\to+\infty}\frac{(x^2-1)-(x^2+3x)}{\sqrt{x^2(1-\frac{1}{x^2})}+\sqrt{x^2(1+\frac{3}{x})}}\\ &= \lim_{x\to+\infty}\frac{-3x-1}{x\sqrt{1-\frac{1}{x^2}}+x\sqrt{1+\frac{3}{x}}}\\ &= \lim_{x\to+\infty}\frac{x(-3-\frac{1}{x})}{x(\sqrt{1-\frac{1}{x^2}}+\sqrt{1+\frac{3}{x}})}\\ &= \frac{-3}{1+1}=-\frac{3}{2}\end{aligned}$$

ដូចនេះ $\lim\limits_{x\to\infty}(\sqrt{x^2-1}-\sqrt{x^2+3x})=-\frac{3}{2}$។

ព. $\lim\limits_{x\to+\infty}(\sqrt[3]{x^3+3x^2+1}-x)$

គុណនឹងកន្សោមឆ្លាស់ទី៣ $(a-b)(a^2+ab+b^2)=a^3-b^3$ ៖

$$\begin{aligned}\lim_{x\to+\infty}(\sqrt[3]{x^3+3x^2+1}-x) &= \lim_{x\to+\infty}\frac{x^3+3x^2+1-x^3}{(\sqrt[3]{x^3+3x^2+1})^2+x\sqrt[3]{x^3+3x^2+1}+x^2}\\ &= \lim_{x\to+\infty}\frac{3x^2+1}{x^2(\sqrt[3]{1+\frac{3}{x}+\frac{1}{x^3}})^2+x^2\sqrt[3]{1+\frac{3}{x}+\frac{1}{x^3}}+x^2}\\ &= \lim_{x\to+\infty}\frac{x^2(3+\frac{1}{x^2})}{x^2\left((\sqrt[3]{1+\frac{3}{x}+\frac{1}{x^3}})^2+\sqrt[3]{1+\frac{3}{x}+\frac{1}{x^3}}+1\right)}\\ &= \frac{3}{1^2+1+1}=\frac{3}{3}=1\end{aligned}$$

ដូចនេះ $\lim\limits_{x\to+\infty}(\sqrt[3]{x^3+3x^2+1}-x)=1$។

ភ. $\lim\limits_{x\to-\infty}(\sqrt[4]{x^4+4}-x)$

ជំនួសតម្លៃ $x\to-\infty$ ចូលផ្ទាល់៖

$$\begin{aligned}\lim_{x\to-\infty}(\sqrt[4]{x^4+4}-x) &= (+\infty)-(-\infty)\\ &= +\infty+\infty=+\infty\end{aligned}$$

ដូចនេះ $\lim\limits_{x\to-\infty}(\sqrt[4]{x^4+4}-x)=+\infty$។

ដំណោះស្រាយលំហាត់ទី៨

ប្រធាន៖ គណនាលីមីតខាងក្រោម៖

ក. $\lim_{x\to\infty} \frac{2x-7}{3x}$

ជួសរាងមិនកំណត់ $\frac{\infty}{\infty}$

យើងទាញយកតួដែលមានដឺក្រេខ្ពស់ជាងគេជាកត្តា៖

$$\begin{aligned}\lim_{x\to\infty}\frac{2x-7}{3x} &= \lim_{x\to\infty}\frac{x\left(2-\frac{7}{x}\right)}{3x}\\ &= \lim_{x\to\infty}\frac{2-\frac{7}{x}}{3}\\ &= \frac{2-0}{3}=\frac{2}{3}\end{aligned}$$

ដូចនេះ $\lim_{x\to\infty} \frac{2x-7}{3x} = \frac{2}{3}$។

ខ. $\lim_{x\to\infty} \frac{2x+1}{x}$

ជួសរាងមិនកំណត់ $\frac{\infty}{\infty}$

យើងទាញយកតួដែលមានដឺក្រេខ្ពស់ជាងគេជាកត្តា៖

$$\begin{aligned}\lim_{x\to\infty}\frac{2x+1}{x} &= \lim_{x\to\infty}\frac{x\left(2+\frac{1}{x}\right)}{x}\\ &= \lim_{x\to\infty}\left(2+\frac{1}{x}\right)\\ &= 2+0=2\end{aligned}$$

ដូចនេះ $\lim_{x\to\infty} \frac{2x+1}{x} = 2$។

គ. $\lim_{x\to\infty} \frac{7+x^2}{-3+x^2}$

ជួសរាងមិនកំណត់ $\frac{\infty}{\infty}$

យើងទាញយកតួដែលមានដឺក្រេខ្ពស់ជាងគេជាកត្តា៖

$$\begin{aligned}\lim_{x\to\infty}\frac{7+x^2}{-3+x^2} &= \lim_{x\to\infty}\frac{x^2\left(\frac{7}{x^2}+1\right)}{x^2\left(-\frac{3}{x^2}+1\right)}\\ &= \lim_{x\to\infty}\frac{\frac{7}{x^2}+1}{-\frac{3}{x^2}+1}\\ &= \frac{0+1}{0+1}=1\end{aligned}$$

ដូចនេះ $\lim_{x\to\infty} \frac{7+x^2}{-3+x^2} = 1$។

ឃ. $\lim_{x\to\infty} \frac{2x}{3+(x-1)^2}$

ជួសរាងមិនកំណត់ $\frac{\infty}{\infty}$

យើងពន្លាតភាគបែងនិងទាញតួដឺក្រេខ្ពស់ជាងគេជាកត្តា៖

$$\begin{aligned}\lim_{x\to\infty}\frac{2x}{3+(x-1)^2} &= \lim_{x\to\infty}\frac{2x}{3+x^2-2x+1}\\ &= \lim_{x\to\infty}\frac{2x}{x^2-2x+4}\\ &= \lim_{x\to\infty}\frac{x(2)}{x^2\left(1-\frac{2}{x}+\frac{4}{x^2}\right)}\\ &= \lim_{x\to\infty}\frac{2}{x\left(1-\frac{2}{x}+\frac{4}{x^2}\right)}\\ &= \frac{2}{\infty(1-0+0)}=0\end{aligned}$$

ដូចនេះ $\lim_{x\to\infty} \frac{2x}{3+(x-1)^2} = 0$។

ង. $\lim\limits_{x\to\infty}\frac{3x^2-4x+4}{(x-1)^2}$

ជួសរាងមិនកំណត់ $\frac{\infty}{\infty}$

$$\begin{aligned}\lim_{x\to\infty}\frac{3x^2-4x+4}{(x-1)^2}&=\lim_{x\to\infty}\frac{3x^2-4x+4}{x^2-2x+1}\\&=\lim_{x\to\infty}\frac{x^2\left(3-\frac{4}{x}+\frac{4}{x^2}\right)}{x^2\left(1-\frac{2}{x}+\frac{1}{x^2}\right)}\\&=\lim_{x\to\infty}\frac{3-\frac{4}{x}+\frac{4}{x^2}}{1-\frac{2}{x}+\frac{1}{x^2}}\\&=\frac{3-0+0}{1-0+0}=3\end{aligned}$$

ដូចនេះ $\lim\limits_{x\to\infty}\frac{3x^2-4x+4}{(x-1)^2}=3$។

ច. $\lim\limits_{x\to\infty}\frac{\sqrt{x+2}-\sqrt{2}}{x}$

ជួសរាងមិនកំណត់ $\frac{\infty}{\infty}$

$$\begin{aligned}\lim_{x\to\infty}\frac{\sqrt{x+2}-\sqrt{2}}{x}&=\lim_{x\to\infty}\frac{\sqrt{x\left(1+\frac{2}{x}\right)}-\sqrt{2}}{x}\\&=\lim_{x\to\infty}\frac{\sqrt{x}\sqrt{1+\frac{2}{x}}-\sqrt{2}}{x}\\&=\lim_{x\to\infty}\frac{\sqrt{x}\left(\sqrt{1+\frac{2}{x}}-\frac{\sqrt{2}}{\sqrt{x}}\right)}{(\sqrt{x})^2}\\&=\lim_{x\to\infty}\frac{\sqrt{1+\frac{2}{x}}-\frac{\sqrt{2}}{\sqrt{x}}}{\sqrt{x}}\\&=\frac{\sqrt{1+0}-0}{\infty}=0\end{aligned}$$

ដូចនេះ $\lim\limits_{x\to\infty}\frac{\sqrt{x+2}-\sqrt{2}}{x}=0$។

ឆ. $\lim\limits_{x\to\infty}\frac{\sqrt{x+5}-\sqrt{5}}{\sqrt{x}-5}$

ជួសរាងមិនកំណត់ $\frac{\infty}{\infty}$

$$\begin{aligned}\lim_{x\to\infty}\frac{\sqrt{x+5}-\sqrt{5}}{\sqrt{x}-5}&=\lim_{x\to\infty}\frac{\sqrt{x\left(1+\frac{5}{x}\right)}-\sqrt{5}}{\sqrt{x}-5}\\&=\lim_{x\to\infty}\frac{\sqrt{x}\left(\sqrt{1+\frac{5}{x}}-\frac{\sqrt{5}}{\sqrt{x}}\right)}{\sqrt{x}\left(1-\frac{5}{\sqrt{x}}\right)}\\&=\lim_{x\to\infty}\frac{\sqrt{1+\frac{5}{x}}-\frac{\sqrt{5}}{\sqrt{x}}}{1-\frac{5}{\sqrt{x}}}\\&=\frac{\sqrt{1+0}-0}{1-0}=1\end{aligned}$$

ដូចនេះ $\lim\limits_{x\to\infty}\frac{\sqrt{x+5}-\sqrt{5}}{\sqrt{x}-5}=1$។

ជ. $\lim\limits_{x\to\infty}\frac{\sqrt{x^2+9}-\sqrt{x^2-9}}{6x}$

នៅពេល $x\to\infty$, ភាគយកមានរាង $\infty-\infty$ យើងត្រូវគុណនឹងកន្សោមឆ្លាស់៖

$$\begin{aligned}\lim_{x\to\infty}\frac{\sqrt{x^2+9}-\sqrt{x^2-9}}{6x}&=\lim_{x\to\infty}\frac{(\sqrt{x^2+9}-\sqrt{x^2-9})(\sqrt{x^2+9}+\sqrt{x^2-9})}{6x(\sqrt{x^2+9}+\sqrt{x^2-9})}\\&=\lim_{x\to\infty}\frac{(x^2+9)-(x^2-9)}{6x(\sqrt{x^2+9}+\sqrt{x^2-9})}\\&=\lim_{x\to\infty}\frac{18}{6x(\sqrt{x^2+9}+\sqrt{x^2-9})}\\&=\lim_{x\to\infty}\frac{3}{x(\sqrt{x^2+9}+\sqrt{x^2-9})}\\&=\frac{3}{\infty(\infty+\infty)}=0\end{aligned}$$

ដូចនេះ $\lim\limits_{x\to\infty}\frac{\sqrt{x^2+9}-\sqrt{x^2-9}}{6x}=0$។

ឈ. $\lim\limits_{x\to\infty}\frac{\sqrt{x-1}-2x}{x-7}$

ជួសរាងមិនកំណត់ $\frac{\infty}{\infty}$

$$\begin{aligned}\lim_{x\to\infty}\frac{\sqrt{x-1}-2x}{x-7}&=\lim_{x\to\infty}\frac{x\left(\frac{\sqrt{x-1}}{x}-2\right)}{x\left(1-\frac{7}{x}\right)}\\&=\lim_{x\to\infty}\frac{\sqrt{\frac{x-1}{x^2}}-2}{1-\frac{7}{x}}\\&=\lim_{x\to\infty}\frac{\sqrt{\frac{1}{x}-\frac{1}{x^2}}-2}{1-\frac{7}{x}}\\&=\frac{\sqrt{0-0}-2}{1-0}=-2\end{aligned}$$

ដូចនេះ $\lim\limits_{x\to\infty}\frac{\sqrt{x-1}-2x}{x-7}=-2$។

ញ. $\lim\limits_{x\to\infty}\frac{\sqrt{x}-6x}{3x+1}$

ជួសរាងមិនកំណត់ $\frac{\infty}{\infty}$

$$\begin{aligned}\lim_{x\to\infty}\frac{\sqrt{x}-6x}{3x+1}&=\lim_{x\to\infty}\frac{x\left(\frac{\sqrt{x}}{x}-6\right)}{x\left(3+\frac{1}{x}\right)}\\&=\lim_{x\to\infty}\frac{\frac{1}{\sqrt{x}}-6}{3+\frac{1}{x}}\\&=\frac{0-6}{3+0}=-2\end{aligned}$$

ដូចនេះ $\lim\limits_{x\to\infty}\frac{\sqrt{x}-6x}{3x+1}=-2$។

ដ. $\lim\limits_{x\to\infty}\frac{\sqrt{x^2+1}+\sqrt{x}}{\sqrt[4]{x^3+x}-x}$

ជួសជួស $x=\infty$ នាំឲ្យរាងមិនកំណត់ $\frac{\infty}{\infty}$

យើងទាញ x ជាកត្តា ៖

$$\lim_{x\to\infty}\frac{x\left(\sqrt{1+\frac{1}{x^2}}+\sqrt{\frac{1}{x}}\right)}{x\left(\sqrt[4]{\frac{1}{x}+\frac{1}{x^3}}-1\right)}=\frac{1+0}{0-1}=-1$$

ដូចនេះ $\lim\limits_{x\to\infty}\frac{\sqrt{x^2+1}+\sqrt{x}}{\sqrt[4]{x^3+x}-x}=-1$។

ឋ. $\lim\limits_{x\to\infty}\frac{\sqrt{x^2+1}+\sqrt{x}}{\sqrt[4]{x^2+1}-x}$

ជួសជួស $x=\infty$ នាំឲ្យរាងមិនកំណត់ $\frac{\infty}{\infty}$

យើងទាញ x ជាកត្តា ៖

$$\lim_{x\to\infty}\frac{x\left(\sqrt{1+\frac{1}{x^2}}+\sqrt{\frac{1}{x}}\right)}{x\left(\sqrt[4]{\frac{1}{x^2}+\frac{1}{x^4}}-1\right)}=\frac{1+0}{0-1}=-1$$

ដូចនេះ $\lim\limits_{x\to\infty}\frac{\sqrt{x^2+1}+\sqrt{x}}{\sqrt[4]{x^2+1}-x}=-1$។

ឌ. $\lim\limits_{x\to\infty}\frac{\sqrt[3]{x}-2\sqrt{x^3}}{\sqrt[4]{x^5}+x\sqrt{x}}$

ជួសជួស $x=\infty$ នាំឲ្យរាងមិនកំណត់ $\frac{\infty}{\infty}$

យើងចែកភាគយក និងភាគបែងនឹង $x^{\frac{3}{2}}$ ឬ $x\sqrt{x}$ ដែលជាតួមានដឺក្រេខ្ពស់បំផុត ៖

$$\lim_{x\to\infty}\frac{x^{\frac{3}{2}}\left(x^{-\frac{7}{6}}-2\right)}{x^{\frac{3}{2}}\left(x^{-\frac{1}{4}}+1\right)}=\frac{0-2}{0+1}=-2$$

ដូចនេះ $\lim\limits_{x\to\infty}\frac{\sqrt[3]{x}-2\sqrt{x^3}}{\sqrt[4]{x^5}+x\sqrt{x}}=-2$។

ឍ. $\lim\limits_{x\to-\infty}\frac{\sqrt{x^2+1}-\sqrt[3]{x^2+1}}{2\sqrt[4]{x^4+1}-\sqrt[5]{x^4+1}}$

ជួសជួស $x=-\infty$ នាំឲ្យរាងមិនកំណត់ $\frac{\infty}{\infty}$

ដោយ $x\to-\infty$ យើងមាន $\sqrt{x^2+1}=|x|\sqrt{1+\frac{1}{x^2}}=-x\sqrt{1+\frac{1}{x^2}}$ និង $\sqrt[4]{x^4+1}=|x|\sqrt[4]{1+\frac{1}{x^4}}=-x\sqrt[4]{1+\frac{1}{x^4}}$

$$\lim_{x\to-\infty}\frac{-x\sqrt{1+\frac{1}{x^2}}-x^{\frac{2}{3}}\sqrt[3]{1+\frac{1}{x^2}}}{-2x\sqrt[4]{1+\frac{1}{x^4}}-x^{\frac{4}{5}}\sqrt[5]{1+\frac{1}{x^4}}}=\lim_{x\to-\infty}\frac{-x\left(\sqrt{1+\frac{1}{x^2}}+x^{-\frac{1}{3}}\sqrt{\ldots}\right.}{-x\left(2\sqrt[4]{1+\frac{1}{x^4}}+x^{-\frac{1}{5}}\ldots\right.}$$

ដូចនេះ $\lim\limits_{x\to-\infty}\frac{\sqrt{x^2+1}-\sqrt[3]{x^2+1}}{2\sqrt[4]{x^4+1}-\sqrt[5]{x^4+1}}=\frac{1}{2}$។

ណ. $\lim\limits_{x\to+\infty}\frac{\sqrt{x}}{\sqrt{x+\sqrt{x}}}$

ជួសជួស $x=+\infty$ នាំឲ្យរាងមិនកំណត់ $\frac{\infty}{\infty}$

ចែកភាគយក និងភាគបែងនឹង $\sqrt{x}$ ៖

$$\lim_{x\to+\infty}\frac{\frac{\sqrt{x}}{\sqrt{x}}}{\sqrt{\frac{x+\sqrt{x}}{x}}}=\lim_{x\to+\infty}\frac{1}{\sqrt{1+\frac{1}{\sqrt{x}}}}=\frac{1}{\sqrt{1+0}}=1$$

ដូចនេះ $\lim\limits_{x\to+\infty}\frac{\sqrt{x}}{\sqrt{x+\sqrt{x}}}=1$។

ត. $\lim\limits_{x\to+\infty}\frac{\sqrt{x+\sqrt{x+\sqrt{x}}}}{\sqrt{x}}$

ជួសជួស $x=+\infty$ នាំឲ្យរាងមិនកំណត់ $\frac{\infty}{\infty}$

ចែកភាគយក និងភាគបែងនឹង $\sqrt{x}$ ៖

$$\lim_{x\to+\infty}\sqrt{\frac{x+\sqrt{x+\sqrt{x}}}{x}}=\lim_{x\to+\infty}\sqrt{1+\sqrt{\frac{x+\sqrt{x}}{x^2}}}=\lim_{x\to+\infty}\sqrt{1+\sqrt{}}$$

ដូចនេះ $\lim\limits_{x\to+\infty}\frac{\sqrt{x+\sqrt{x+\sqrt{x}}}}{\sqrt{x}}=1$។

ថ. $\lim\limits_{x\to\infty}\frac{\sqrt{x^2+2x-3}+2x}{\sqrt{x^2+4}+x}$

ជួសជួស $x=\infty$ (យក $x\to+\infty$) នាំឲ្យរាងមិនកំណត់ $\frac{\infty}{\infty}$

ទាញ x ជាកត្តា ៖

$$\lim_{x\to\infty}\frac{x\left(\sqrt{1+\frac{2}{x}-\frac{3}{x^2}}+2\right)}{x\left(\sqrt{1+\frac{4}{x^2}}+1\right)}=\frac{1+2}{1+1}=\frac{3}{2}$$

ដូចនេះ $\lim\limits_{x\to\infty}\frac{\sqrt{x^2+2x-3}+2x}{\sqrt{x^2+4}+x}=\frac{3}{2}$។

ទ. $\lim\limits_{x\to\infty}\frac{\sqrt[4]{x^4+2}}{\sqrt[3]{5+27x^3}}$

ជួសជួស $x=\infty$ (យក $x\to+\infty$) នាំឲ្យរាងមិនកំណត់ $\frac{\infty}{\infty}$

ទាញ x ជាកត្តា ៖

$$\lim_{x\to\infty}\frac{x\sqrt[4]{1+\frac{2}{x^4}}}{3x\sqrt[3]{\frac{5}{27x^3}+1}}=\lim_{x\to\infty}\frac{\sqrt[4]{1+\frac{2}{x^4}}}{3\sqrt[3]{1+\frac{5}{27x^3}}}=\frac{1}{3}$$

ដូចនេះ $\lim\limits_{x\to\infty}\frac{\sqrt[4]{x^4+2}}{\sqrt[3]{5+27x^3}}=\frac{1}{3}$។

ធ. $\lim\limits_{x\to+\infty}\frac{\sqrt{x+\sqrt{x+\sqrt{x}}}}{\sqrt{x+1}}$

ជួសជួស $x=+\infty$ នាំឲ្យរាងមិនកំណត់ $\frac{\infty}{\infty}$

បញ្ចូនរ៉ាឌីកាល់ និងចែកនឹង x ៖

$$\lim_{x\to+\infty}\sqrt{\frac{x+\sqrt{x+\sqrt{x}}}{x+1}}=\lim_{x\to+\infty}\sqrt{\frac{1+\sqrt{\frac{1}{x}+\frac{1}{x\sqrt{x}}}}{1+\frac{1}{x}}}=\sqrt{\frac{1+0}{1+0}}=1$$

ដូចនេះ $\lim\limits_{x\to+\infty}\frac{\sqrt{x+\sqrt{x+\sqrt{x}}}}{\sqrt{x+1}}=1$។

ន. $\lim\limits_{x\to+\infty}\frac{\sqrt{x}-\sqrt{x+1}}{x+1}$

ជួសជួស $x=+\infty$ នាំឲ្យរាងមិនកំណត់

យើងគុណនឹងកន្សោមឆ្លាស់ ៖

$$\lim_{x\to+\infty}\frac{x-(x+1)}{(x+1)(\sqrt{x}+\sqrt{x+1})}=\lim_{x\to+\infty}\frac{-1}{(x+1)(\sqrt{x}+\sqrt{x+1})}$$

ដូចនេះ $\lim\limits_{x\to+\infty}\frac{\sqrt{x}-\sqrt{x+1}}{x+1}=0$។

ដំណោះស្រាយលំហាត់ទី៩

ប្រធាន៖ គណនាលីមីតខាងក្រោម៖

ក. $\lim\limits_{x\to\frac{\pi}{6}}(\sin^2 x+\cos^2 x)$

ជំនួសតម្លៃ $x=\frac{\pi}{6}$ ចូលក្នុងកន្សោម៖

$$\lim_{x\to\frac{\pi}{6}}(\sin^2 x+\cos^2 x)=\left(\sin\frac{\pi}{6}\right)^2+\left(\cos\frac{\pi}{6}\right)^2=\left(\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2=\frac{1}{4}+\frac{3}{4}=\frac{4}{4}$$

ដូចនេះ $\lim\limits_{x\to\frac{\pi}{6}}(\sin^2 x+\cos^2 x)=1$។

ខ. $\lim\limits_{x\to\frac{\pi}{6}}(\sin x-1)^2$

ជំនួសតម្លៃ $x=\frac{\pi}{6}$ ចូលក្នុងកន្សោម៖

$$\lim_{x\to\frac{\pi}{6}}(\sin x-1)^2=\left(\sin\frac{\pi}{6}-1\right)^2=\left(\frac{1}{2}-1\right)^2=\left(-\frac{1}{2}\right)^2=\frac{1}{4}$$

ដូចនេះ $\lim\limits_{x\to\frac{\pi}{6}}(\sin x-1)^2=\frac{1}{4}$។

គ. $\lim\limits_{x\to\pi}\tan x$

ជំនួសតម្លៃ $x=\pi$ ចូលក្នុងកន្សោម៖

$$\lim_{x\to\pi}\tan x=\tan\pi=0$$

ដូចនេះ $\lim\limits_{x\to\pi}\tan x=0$។

ឃ. $\lim\limits_{x\to7\pi}\cos\frac{x}{3}$

ជំនួសតម្លៃ $x=7\pi$ ចូលក្នុងកន្សោម៖

$$\lim_{x\to7\pi}\cos\frac{x}{3}=\cos\frac{7\pi}{3}=\cos\left(2\pi+\frac{\pi}{3}\right)=\cos\frac{\pi}{3}=\frac{1}{2}$$

ដូចនេះ $\lim\limits_{x\to7\pi}\cos\frac{x}{3}=\frac{1}{2}$។

ង. $\lim\limits_{x\to\frac{13\pi}{4}}(\cot x-\tan x)$

ជំនួសតម្លៃ $x=\frac{13\pi}{4}$ ចូលក្នុងកន្សោម៖

$$\lim_{x\to\frac{13\pi}{4}}(\cot x-\tan x)=\cot\frac{13\pi}{4}-\tan\frac{13\pi}{4}$$

ដោយ $\frac{13\pi}{4}=3\pi+\frac{\pi}{4}$ នោះ $\tan\left(3\pi+\frac{\pi}{4}\right)=\tan\frac{\pi}{4}=1$

និង $\cot\left(3\pi+\frac{\pi}{4}\right)=\cot\frac{\pi}{4}=1$។

$$=1-1=0$$

ដូចនេះ $\lim\limits_{x\to\frac{13\pi}{4}}(\cot x-\tan x)=0$។

ច. $\lim\limits_{x\to0}\frac{\sin x+\cos^2 x}{x^2+1}$

ជំនួសតម្លៃ $x=0$ ចូលក្នុងកន្សោម៖

$$\lim_{x\to0}\frac{\sin x+\cos^2 x}{x^2+1}=\frac{\sin0+\cos^2 0}{0^2+1}=\frac{0+1^2}{0+1}=\frac{1}{1}=1$$

ដូចនេះ $\lim\limits_{x\to0}\frac{\sin x+\cos^2 x}{x^2+1}=1$។

ឆ. $\lim\limits_{x\to\frac{\pi}{4}}\frac{\cos x-\tan x}{\sin^2 x}$

ជំនួសតម្លៃ $x=\frac{\pi}{4}$ ចូលក្នុងកន្សោម៖

$$\lim_{x\to\frac{\pi}{4}}\frac{\cos x-\tan x}{\sin^2 x}=\frac{\cos\frac{\pi}{4}-\tan\frac{\pi}{4}}{\left(\sin\frac{\pi}{4}\right)^2}=\frac{\frac{\sqrt{2}}{2}-1}{\left(\frac{\sqrt{2}}{2}\right)^2}=\frac{\frac{\sqrt{2}-2}{2}}{\frac{2}{4}}=\frac{\frac{\sqrt{2}-2}{2}}{\frac{1}{2}}=\sqrt{2}-2$$

ដូចនេះ $\lim\limits_{x\to\frac{\pi}{4}}\frac{\cos x-\tan x}{\sin^2 x}=\sqrt{2}-2$។

ជ. $\lim\limits_{x\to\frac{\pi}{3}}\frac{\tan x-\sin x}{\cos x-1}$

ជំនួសតម្លៃ $x=\frac{\pi}{3}$ ចូលក្នុងកន្សោម៖

$$\lim_{x\to\frac{\pi}{3}}\frac{\tan x-\sin x}{\cos x-1}=\frac{\tan\frac{\pi}{3}-\sin\frac{\pi}{3}}{\cos\frac{\pi}{3}-1}=\frac{\sqrt{3}-\frac{\sqrt{3}}{2}}{\frac{1}{2}-1}=\frac{\frac{2\sqrt{3}-\sqrt{3}}{2}}{\frac{1-2}{2}}=\frac{\frac{\sqrt{3}}{2}}{-\frac{1}{2}}$$

ដូចនេះ $\lim\limits_{x\to\frac{\pi}{3}}\frac{\tan x-\sin x}{\cos x-1}=-\sqrt{3}$។

ឈ. $\lim\limits_{x\to\frac{\pi}{6}}\frac{\sin^2 x+\tan^2 x}{2\sin x}$

ជំនួសតម្លៃ $x=\frac{\pi}{6}$ ចូលក្នុងកន្សោម៖

$$\lim_{x\to\frac{\pi}{6}}\frac{\sin^2 x+\tan^2 x}{2\sin x}=\frac{\left(\sin\frac{\pi}{6}\right)^2+\left(\tan\frac{\pi}{6}\right)^2}{2\sin\frac{\pi}{6}}=\frac{\left(\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{3}\right)^2}{2\cdot\frac{1}{2}}=\frac{\frac{1}{4}+\frac{3}{9}}{1}=\frac{1}{4}+\frac{1}{3}=\frac{3+4}{12}=\frac{7}{12}$$

ដូចនេះ $\lim\limits_{x\to\frac{\pi}{6}}\frac{\sin^2 x+\tan^2 x}{2\sin x}=\frac{7}{12}$។

ញ. $\lim\limits_{x\to 0}\frac{\cos x+\tan x}{\tan^2 x+3}$

ជំនួសតម្លៃ $x=0$ ចូលក្នុងកន្សោម៖

$$\lim_{x\to 0}\frac{\cos x+\tan x}{\tan^2 x+3}=\frac{\cos 0+\tan 0}{\tan^2 0+3}=\frac{1+0}{0+3}=\frac{1}{3}$$

ដូចនេះ $\lim\limits_{x\to 0}\frac{\cos x+\tan x}{\tan^2 x+3}=\frac{1}{3}$។

ដ. $\lim\limits_{x\to\frac{\pi}{4}}\frac{\cos x+\sin x}{\cos^2 2x}$

នៅពេល $x\to\frac{\pi}{4}$ ភាគយកគឺ $\cos\frac{\pi}{4}+\sin\frac{\pi}{4}=\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}=\sqrt{2}>0$។ ចំពោះភាគបែង នៅពេល $x\to\frac{\pi}{4}$ គឺ $\cos^2\left(2\cdot\frac{\pi}{4}\right)=\cos^2\frac{\pi}{2}=0$។ ដោយភាគបែងមានការេ វាតែងតែមានតម្លៃវិជ្ជមានធំជាងសូន្យជានិច្ច (0^+)។

$$\lim_{x\to\frac{\pi}{4}}\frac{\cos x+\sin x}{\cos^2 2x}=\frac{\sqrt{2}}{0^+}=+\infty$$

ដូចនេះ $\lim\limits_{x\to\frac{\pi}{4}}\frac{\cos x+\sin x}{\cos^2 2x}=+\infty$។

ឋ. $\lim\limits_{x\to 0}\frac{\sin 10x}{10x}$

ជាទម្រង់មិនកំណត់ $\frac{0}{0}$។ យើងប្រើរូបមន្ត $\lim\limits_{u\to 0}\frac{\sin u}{u}=1$៖

$$\lim_{x\to 0}\frac{\sin 10x}{10x}=1$$

ដូចនេះ $\lim\limits_{x\to 0}\frac{\sin 10x}{10x}=1$។

ឌ. $\lim\limits_{x\to 0}\frac{\sin 3x}{2x}$

ជាទម្រង់មិនកំណត់ $\frac{0}{0}$។ យើងគុណនិងចែកជាមួយ ៣ ដើម្បីបង្កើតរូបមន្ត៖

$$\lim_{x\to 0}\frac{\sin 3x}{2x}=\lim_{x\to 0}\frac{\sin 3x}{3x}\cdot\frac{3}{2}=1\cdot\frac{3}{2}=\frac{3}{2}$$

ដូចនេះ $\lim\limits_{x\to 0}\frac{\sin 3x}{2x}=\frac{3}{2}$។

ឍ. $\lim\limits_{x\to 0}\frac{\tan 8x}{x}$

ជាទម្រង់មិនកំណត់ $\frac{0}{0}$។ យើងគុណនិងចែកជាមួយ ៨ ដើម្បីបង្កើតរូបមន្ត៖

$$\lim_{x\to 0}\frac{\tan 8x}{x}=\lim_{x\to 0}\frac{\tan 8x}{8x}\cdot 8=1\cdot 8=8$$

ដូចនេះ $\lim\limits_{x\to 0}\frac{\tan 8x}{x}=8$។

ណ. $\lim\limits_{x\to 0}\frac{\sin 3x}{5x}$

ជាទម្រង់មិនកំណត់ $\frac{0}{0}$។ យើងគុណនិងចែកជាមួយ ៣ ដើម្បីបង្កើតរូបមន្តៈ

$$\lim_{x\to 0}\frac{\sin 3x}{5x}=\lim_{x\to 0}\frac{\sin 3x}{3x}\cdot\frac{3}{5}=1\cdot\frac{3}{5}=\frac{3}{5}$$

ដូចនេះ $\lim\limits_{x\to 0}\frac{\sin 3x}{5x}=\frac{3}{5}$។

ត. $\lim\limits_{x\to 0}\frac{\tan 5x}{\sin 4x}$

ជាទម្រង់មិនកំណត់ $\frac{0}{0}$។ យើងចែកភាគយកនិងភាគបែងនឹង x ព្រមទាំងតម្រូវមេគុណៈ

$$\lim_{x\to 0}\frac{\tan 5x}{\sin 4x}=\lim_{x\to 0}\frac{\frac{\tan 5x}{5x}\cdot 5x}{\frac{\sin 4x}{4x}\cdot 4x}=\lim_{x\to 0}\frac{\frac{\tan 5x}{5x}}{\frac{\sin 4x}{4x}}\cdot\frac{5}{4}=\frac{1}{1}\cdot\frac{5}{4}=\frac{5}{4}$$

ដូចនេះ $\lim\limits_{x\to 0}\frac{\tan 5x}{\sin 4x}=\frac{5}{4}$។

ថ. $\lim\limits_{x\to 0}\frac{\tan 5x}{\tan 6x}$

ជាទម្រង់មិនកំណត់ $\frac{0}{0}$។ យើងចែកភាគយកនិងភាគបែងនឹង x ព្រមទាំងតម្រូវមេគុណៈ

$$\lim_{x\to 0}\frac{\tan 5x}{\tan 6x}=\lim_{x\to 0}\frac{\frac{\tan 5x}{5x}\cdot 5x}{\frac{\tan 6x}{6x}\cdot 6x}=\lim_{x\to 0}\frac{\frac{\tan 5x}{5x}}{\frac{\tan 6x}{6x}}\cdot\frac{5}{6}=\frac{1}{1}\cdot\frac{5}{6}=\frac{5}{6}$$

ដូចនេះ $\lim\limits_{x\to 0}\frac{\tan 5x}{\tan 6x}=\frac{5}{6}$។

ទ. $\lim\limits_{x\to 0}\frac{\tan x}{3x}$

ជាទម្រង់មិនកំណត់ $\frac{0}{0}$។ យើងប្រើរូបមន្ត $\lim\limits_{x\to 0}\frac{\tan x}{x}=1$៖

$$\lim_{x\to 0}\frac{\tan x}{3x}=\frac{1}{3}\cdot\lim_{x\to 0}\frac{\tan x}{x}=\frac{1}{3}\cdot 1=\frac{1}{3}$$

ដូចនេះ $\lim\limits_{x\to 0}\frac{\tan x}{3x}=\frac{1}{3}$។

ឆ. $\lim_{x\to 0}\frac{1-\cos x}{x}$

ជាទម្រង់មិនកំណត់ $\frac{0}{0}$។ យើងប្រើរូបមន្តត្រីកោណមាត្រ $1-\cos x = 2\sin^2\frac{x}{2}$៖

$$\lim_{x\to 0}\frac{1-\cos x}{x} = \lim_{x\to 0}\frac{2\sin^2\frac{x}{2}}{x} = \lim_{x\to 0}\frac{\sin\frac{x}{2}}{\frac{x}{2}}\cdot\sin\frac{x}{2} = 1\cdot\sin 0 = 0$$

ដូចនេះ $\lim_{x\to 0}\frac{1-\cos x}{x} = 0$។

ឧ. $\lim_{x\to 0}\frac{x}{\sqrt{1-\cos x}}$

ជាទម្រង់មិនកំណត់ $\frac{0}{0}$ ។

យើងប្រើរូបមន្ត $1-\cos x = 2\sin^2\frac{x}{2}$

$$\lim_{x\to 0}\frac{x}{\sqrt{1-\cos x}} = \lim_{x\to 0}\frac{x}{\sqrt{2\sin^2\frac{x}{2}}} = \lim_{x\to 0}\frac{x}{\sqrt{2}\left|\sin\frac{x}{2}\right|}$$

ពេល $x\to 0^+$ នោះ $\sin\frac{x}{2} > 0 \Rightarrow \left|\sin\frac{x}{2}\right| = \sin\frac{x}{2}$

$$\lim_{x\to 0^+}\frac{x}{\sqrt{2}\sin\frac{x}{2}} = \lim_{x\to 0^+}\frac{2\cdot\frac{x}{2}}{\sqrt{2}\sin\frac{x}{2}} = \frac{2}{\sqrt{2}} = \sqrt{2}$$

ពេល $x\to 0^-$ នោះ $\sin\frac{x}{2} < 0 \Rightarrow \left|\sin\frac{x}{2}\right| = -\sin\frac{x}{2}$

$$\lim_{x\to 0^-}\frac{x}{-\sqrt{2}\sin\frac{x}{2}} = \lim_{x\to 0^-}\frac{2\cdot\frac{x}{2}}{-\sqrt{2}\sin\frac{x}{2}} = -\frac{2}{\sqrt{2}} = -\sqrt{2}$$

ដោយលីមីតឆ្វេងនិងលីមីតស្តាំមិនស្មើគ្នា នោះលីមីតនេះគ្មានតម្លៃទេ។

ដូចនេះ $\lim_{x\to 0}\frac{x}{\sqrt{1-\cos x}}$ គ្មានលីមីត។

ប. $\lim_{x\to 0}\frac{\sin x}{x^3}$

ជាទម្រង់មិនកំណត់ $\frac{0}{0}$ ។

យើងបំបែកកន្សោមដូចខាងក្រោម៖

$$\lim_{x\to 0}\frac{\sin x}{x^3} = \lim_{x\to 0}\left(\frac{\sin x}{x}\cdot\frac{1}{x^2}\right)$$

ដោយ $\lim_{x\to 0}\frac{\sin x}{x} = 1$ និង $\lim_{x\to 0}\frac{1}{x^2} = +\infty$

$$= 1\cdot(+\infty) = +\infty$$

ដូចនេះ $\lim_{x\to 0}\frac{\sin x}{x^3} = +\infty$។

ផ. $\lim_{x\to 0}\frac{1-\cos 2x}{x\sin x}$

ជាទម្រង់មិនកំណត់ $\frac{0}{0}$ ។

យើងប្រើរូបមន្ត $1-\cos 2x = 2\sin^2 x$ ជំនួសក្នុងលីមីត៖

$$\lim_{x\to 0}\frac{1-\cos 2x}{x\sin x} = \lim_{x\to 0}\frac{2\sin^2 x}{x\sin x}$$

$$= \lim_{x\to 0}\frac{2\sin x}{x} = 2\cdot 1 = 2$$

ដូចនេះ $\lim_{x\to 0}\frac{1-\cos 2x}{x\sin x} = 2$។

ព. $\lim\limits_{x\to0}\frac{\sin^3\frac{x}{2}}{x^3}$

ជាទម្រង់មិនកំណត់ $\frac{0}{0}$ ។

យើងអាចរៀបចំកន្សោមដើម្បីប្រើរូបមន្ត $\lim\limits_{u\to0}\frac{\sin u}{u}=1$៖

$$\lim_{x\to0}\frac{\sin^3\frac{x}{2}}{x^3}=\lim_{x\to0}\frac{\sin^3\frac{x}{2}}{8\cdot\left(\frac{x}{2}\right)^3}$$

$$=\frac{1}{8}\lim_{x\to0}\left(\frac{\sin\frac{x}{2}}{\frac{x}{2}}\right)^3=\frac{1}{8}\cdot1^3=\frac{1}{8}$$

ដូចនេះ $\lim\limits_{x\to0}\frac{\sin^3\frac{x}{2}}{x^3}=\frac{1}{8}$។

ភ. $\lim\limits_{x\to0}\frac{\sin4x+\sin7x}{\sin3x}$

ជាទម្រង់មិនកំណត់ $\frac{0}{0}$ ។

យើងចែកភាគយក និងភាគបែងនឹង x៖

$$\lim_{x\to0}\frac{\sin4x+\sin7x}{\sin3x}=\lim_{x\to0}\frac{\frac{\sin4x}{x}+\frac{\sin7x}{x}}{\frac{\sin3x}{x}}$$

$$=\frac{4\lim\limits_{x\to0}\frac{\sin4x}{4x}+7\lim\limits_{x\to0}\frac{\sin7x}{7x}}{3\lim\limits_{x\to0}\frac{\sin3x}{3x}}=\frac{4(1)+7(1)}{3(1)}=\frac{11}{3}$$

ដូចនេះ $\lim\limits_{x\to0}\frac{\sin4x+\sin7x}{\sin3x}=\frac{11}{3}$។

ម. $\lim\limits_{x\to0}\frac{\sqrt{1-\cos x^2}}{1-\cos x}$

ជាទម្រង់មិនកំណត់ $\frac{0}{0}$ ។

យើងប្រើរូបមន្ត $1-\cos u=2\sin^2\frac{u}{2}$៖

$$\lim_{x\to0}\frac{\sqrt{1-\cos x^2}}{1-\cos x}=\lim_{x\to0}\frac{\sqrt{2\sin^2\frac{x^2}{2}}}{2\sin^2\frac{x}{2}}$$

ដោយសារ $x\to0$ នោះ $\frac{x^2}{2}>0$ នាំឲ្យ $\sin\frac{x^2}{2}>0$ ដែល $\sqrt{\sin^2\frac{x^2}{2}}=\sin\frac{x^2}{2}$៖

$$=\lim_{x\to0}\frac{\sqrt{2}\sin\frac{x^2}{2}}{2\sin^2\frac{x}{2}}=\frac{\sqrt{2}}{2}\lim_{x\to0}\frac{\frac{\sin\frac{x^2}{2}}{\frac{x^2}{2}}\cdot\frac{x^2}{2}}{\left(\frac{\sin\frac{x}{2}}{\frac{x}{2}}\right)^2\cdot\frac{x^2}{4}}$$

$$=\frac{\sqrt{2}}{2}\lim_{x\to0}\frac{\frac{\sin\frac{x^2}{2}}{\frac{x^2}{2}}}{\left(\frac{\sin\frac{x}{2}}{\frac{x}{2}}\right)^2}\cdot\frac{\frac{x^2}{2}}{\frac{x^2}{4}}=\frac{\sqrt{2}}{2}\cdot\frac{1}{1^2}\cdot2=\sqrt{2}$$

ដូចនេះ $\lim\limits_{x\to0}\frac{\sqrt{1-\cos x^2}}{1-\cos x}=\sqrt{2}$។

យ. $\lim\limits_{x\to0}\frac{\tan x-\sin x}{x^3}$

ជាទម្រង់មិនកំណត់ $\frac{0}{0}$ ។

ដោយ $\tan x=\frac{\sin x}{\cos x}$៖

$$\lim_{x\to0}\frac{\frac{\sin x}{\cos x}-\sin x}{x^3}=\lim_{x\to0}\frac{\sin x(1-\cos x)}{x^3\cos x}$$

$$=\lim_{x\to0}\frac{\sin x}{x}\cdot\frac{1-\cos x}{x^2}\cdot\frac{1}{\cos x}$$

ប្រើរូបមន្ត $1-\cos x=2\sin^2\frac{x}{2}$ នោះ $\lim\limits_{x\to0}\frac{1-\cos x}{x^2}=\frac{1}{2}$៖

$$=1\cdot\frac{1}{2}\cdot\frac{1}{1}=\frac{1}{2}$$

ដូចនេះ $\lim\limits_{x\to0}\frac{\tan x-\sin x}{x^3}=\frac{1}{2}$។

រ. $\lim\limits_{x\to\infty} x\sin\frac{\pi}{x}$

ជាទម្រង់មិនកំណត់ $\infty\cdot 0$ ។

តាង $t=\frac{\pi}{x}$ នោះពេល $x\to\infty$ យើងបាន $t\to 0$ ហើយ $x=\frac{\pi}{t}$ ៖

$$\lim_{x\to\infty} x\sin\frac{\pi}{x} = \lim_{t\to 0}\frac{\pi}{t}\sin t = \lim_{t\to 0}\pi\cdot\frac{\sin t}{t}$$

$$= \pi\cdot 1 = \pi$$

ដូចនេះ $\lim\limits_{x\to\infty} x\sin\frac{\pi}{x} = \pi$ ។

ល. $\lim\limits_{x\to -1}\frac{x^3+1}{\sin(x+1)}$

ជាទម្រង់មិនកំណត់ $\frac{0}{0}$ ។

យើងបំបែកភាគយកជាកត្តា តាមរូបមន្ត $a^3+b^3=(a+b)(a^2-ab+b^2)$ ៖

$$\lim_{x\to -1}\frac{(x+1)(x^2-x+1)}{\sin(x+1)} = \lim_{x\to -1}\frac{x+1}{\sin(x+1)}\cdot(x^2-x+1)$$

តាង $u=x+1$ ពេល $x\to -1$ នោះ $u\to 0$ ៖

$$=\lim_{u\to 0}\frac{u}{\sin u}\cdot\lim_{x\to -1}(x^2-x+1) = 1\cdot((-1)^2-(-1)+1) = 1\cdot 3 = 3$$

ដូចនេះ $\lim\limits_{x\to -1}\frac{x^3+1}{\sin(x+1)} = 3$ ។

វ. $\lim\limits_{x\to 0^+}\frac{|\sin x|}{x}$

ជាទម្រង់មិនកំណត់ $\frac{0}{0}$ ។

នៅពេល $x\to 0^+$ (តម្លៃ $x>0$) នោះ $\sin x>0$ នាំឲ្យ $|\sin x|=\sin x$ ៖

$$\lim_{x\to 0^+}\frac{|\sin x|}{x} = \lim_{x\to 0^+}\frac{\sin x}{x} = 1$$

ដូចនេះ $\lim\limits_{x\to 0^+}\frac{|\sin x|}{x} = 1$ ។

ស. $\lim\limits_{x\to 1}\frac{\tan(x-1)}{\sqrt{x}-1}$

ជាទម្រង់មិនកំណត់ $\frac{0}{0}$ ។

យើងគុណភាគយក និងភាគបែង នឹងកន្សោមឆ្លាស់នៃភាគបែង គឺ $\sqrt{x}+1$ ៖

$$\lim_{x\to 1}\frac{\tan(x-1)(\sqrt{x}+1)}{(\sqrt{x}-1)(\sqrt{x}+1)} = \lim_{x\to 1}\frac{\tan(x-1)(\sqrt{x}+1)}{x-1}$$

$$=\lim_{x\to 1}\frac{\tan(x-1)}{x-1}\cdot\lim_{x\to 1}(\sqrt{x}+1)$$

តាង $u=x-1$ ពេល $x\to 1$ នោះ $u\to 0$ ហើយ $\lim\limits_{u\to 0}\frac{\tan u}{u}=1$ ៖

$$=1\cdot(\sqrt{1}+1)=2$$

ដូចនេះ $\lim\limits_{x\to 1}\frac{\tan(x-1)}{\sqrt{x}-1} = 2$ ។

ហ. $\lim\limits_{x\to0}\frac{\sqrt{\cos x}-1}{\sin^2 x}$

ជាទម្រង់មិនកំណត់ $\frac{0}{0}$ ។

គុណនឹងកន្សោមឆ្លាស់ភាគយក គឺ $\sqrt{\cos x}+1$ ៖

$$\lim_{x\to0}\frac{(\sqrt{\cos x}-1)(\sqrt{\cos x}+1)}{\sin^2 x(\sqrt{\cos x}+1)}=\lim_{x\to0}\frac{\cos x-1}{(1-\cos^2 x)(\sqrt{\cos x}+1)}$$

$$=\lim_{x\to0}\frac{-(1-\cos x)}{(1-\cos x)(1+\cos x)(\sqrt{\cos x}+1)}$$

សម្រួល $(1-\cos x)$ ចោល (ដោយ $x\neq0$)៖

$$=\lim_{x\to0}\frac{-1}{(1+\cos x)(\sqrt{\cos x}+1)}$$

ជំនួស $x=0$ ចូល៖

$$=\frac{-1}{(1+\cos0)(\sqrt{\cos0}+1)}=\frac{-1}{(1+1)(1+1)}=-\frac{1}{4}$$

ដូចនេះ $\lim\limits_{x\to0}\frac{\sqrt{\cos x}-1}{\sin^2 x}=-\frac{1}{4}$ ។

ឡ. $\lim\limits_{x\to0}\frac{\sin3x+\sin5x}{\sin2x}$

ជាទម្រង់មិនកំណត់ $\frac{0}{0}$ ។

ចែកភាគយកនិងភាគបែងនឹង x ៖

$$\lim_{x\to0}\frac{\frac{\sin3x}{x}+\frac{\sin5x}{x}}{\frac{\sin2x}{x}}=\frac{3\lim\limits_{x\to0}\frac{\sin3x}{3x}+5\lim\limits_{x\to0}\frac{\sin5x}{5x}}{2\lim\limits_{x\to0}\frac{\sin2x}{2x}}$$

$$=\frac{3(1)+5(1)}{2(1)}=\frac{8}{2}=4$$

ដូចនេះ $\lim\limits_{x\to0}\frac{\sin3x+\sin5x}{\sin2x}=4$ ។

អ. $\lim\limits_{x\to 0}\frac{\cos x-\cos^3 x}{x^2}$

ជាទម្រង់មិនកំណត់ $\frac{0}{0}$ ។

ទាញ $\cos x$ ជាកត្តា រួចប្រើរូបមន្ត $1-\cos^2 x=\sin^2 x$៖

$$\lim_{x\to 0}\frac{\cos x(1-\cos^2 x)}{x^2}=\lim_{x\to 0}\frac{\cos x\cdot\sin^2 x}{x^2}$$

$$=\lim_{x\to 0}\cos x\cdot\left(\frac{\sin x}{x}\right)^2$$

$$=\cos 0\cdot 1^2=1\cdot 1=1$$

ដូចនេះ $\lim\limits_{x\to 0}\frac{\cos x-\cos^3 x}{x^2}=1$។

ដំណោះស្រាយលំហាត់ទី១០

ប្រធាន៖ ចូរដោះស្រាយលីមីតខាងក្រោមឲ្យបានត្រឹមត្រូវ៖

ក. $\lim\limits_{x\to 0}\frac{2\sin x}{3x}$

ប្រើរូបមន្ត $\lim\limits_{x\to 0}\frac{\sin x}{x}=1$៖

$$\lim_{x\to 0}\frac{2\sin x}{3x}=\frac{2}{3}\lim_{x\to 0}\frac{\sin x}{x}$$
$$=\frac{2}{3}(1)$$
$$=\frac{2}{3}$$

ខ. $\lim\limits_{x\to 0}\frac{1-\cos x}{2x}$

គុណភាគយកនិងភាគបែងនឹង $(1+\cos x)$ ឬប្រើរូបមន្ត $1-\cos x=2\sin^2\frac{x}{2}$៖

$$\lim_{x\to 0}\frac{2\sin^2\frac{x}{2}}{2x}=\lim_{x\to 0}\frac{\sin\frac{x}{2}}{x}\cdot\sin\frac{x}{2}$$
$$=\left(\frac{1}{2}\right)\cdot(0)$$
$$=0$$

គ. $\lim\limits_{x\to 0}\frac{2\tan x}{x}$

ប្រើរូបមន្ត $\lim\limits_{x\to 0}\frac{\tan x}{x}=1$៖

$$\lim_{x\to 0}2\left(\frac{\tan x}{x}\right)$$
$$=2(1)$$
$$=2$$

ឃ. $\lim\limits_{x\to 0}\frac{\sin ax}{\sin bx}$

ចែកភាគយក និងភាគបែងនឹង x ៖

$$\lim_{x\to 0}\frac{\frac{\sin ax}{x}}{\frac{\sin bx}{x}}$$
$$=\frac{a}{b}$$

ង. $\lim_{x\to 0} \frac{\sin 4x}{3x}$

គុណ និងចែកនឹង ៤៖

$$\lim_{x\to 0} \frac{4}{3}\frac{\sin 4x}{4x} = \frac{4}{3}(1) = \frac{4}{3}$$

ច. $\lim_{x\to 0} \frac{\tan mx}{\sin nx}$

ចែកនឹង x ទាំងលើទាំងក្រោម៖

$$\lim_{x\to 0} \frac{\frac{\tan mx}{x}}{\frac{\sin nx}{x}} = \frac{m}{n}$$

ឆ. $\lim_{x\to 0} \frac{1-\cos x}{x\sin x}$

ប្រើរូបមន្ត $1-\cos x = 2\sin^2\frac{x}{2}$ ៖

$$\lim_{x\to 0} \frac{2\sin^2\frac{x}{2}}{x(2\sin\frac{x}{2}\cos\frac{x}{2})} = \lim_{x\to 0} \frac{\sin\frac{x}{2}}{x\cos\frac{x}{2}} = \lim \frac{\frac{\sin(x/2)}{x}}{\cos(x/2)} = \frac{1/2}{1} = \frac{1}{2}$$

ជ. $\lim_{x\to \frac{\pi}{4}} \frac{\cos x - \sin x}{\cos 2x}$

ប្រើ រូបមន្ត $\cos 2x = \cos^2 x - \sin^2 x = (\cos x - \sin x)(\cos x + \sin x)$ ៖

$$\lim_{x\to \frac{\pi}{4}} \frac{\cos x - \sin x}{(\cos x - \sin x)(\cos x + \sin x)} = \lim_{x\to \frac{\pi}{4}} \frac{1}{\cos x + \sin x} = \frac{1}{\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

ឈ. $\lim_{x\to 0} \frac{\sin 3x}{x}$

$$= \lim_{x\to 0} 3\frac{\sin 3x}{3x} = 3(1) = 3$$

ញ. $\lim_{x\to 0} \frac{\tan 3x}{2x}$

$$= \lim_{x\to 0} \frac{3}{2}\frac{\tan 3x}{3x} = \frac{3}{2}(1) = \frac{3}{2}$$

ដ. $\lim_{x\to 0} \frac{\sin 2x}{\sin 5x}$

$$= \lim \frac{\frac{\sin 2x}{x}}{\frac{\sin 5x}{x}} = \frac{2}{5}$$

ឋ. $\lim_{x\to 0} \frac{1-\cos 2x}{x^2}$

ប្រើ $1-\cos 2x = 2\sin^2 x$ ៖

$$\lim_{x\to 0} \frac{2\sin^2 x}{x^2} = 2(1)^2 = 2$$

ឌ. $\lim_{x\to 0}\frac{\cos x-\cos^2 x}{x}$

ទាញ $\cos x$ ជាកត្តា ៖

$$\begin{aligned}\lim_{x\to 0}\frac{\cos x(1-\cos x)}{x} &= \lim_{x\to 0}\cos x\cdot\frac{2\sin^2(x/2)}{x}\\ &= 1\cdot 0\\ &= 0\end{aligned}$$

ឍ. $\lim_{x\to 0}\frac{\cos 3x-\cos^2 3x}{6x}$

ទាញ $\cos 3x$ ជាកត្តា ៖

$$\begin{aligned}\lim_{x\to 0}\frac{\cos 3x(1-\cos 3x)}{6x} &= 1\cdot\frac{0}{0} \implies \lim\frac{2\sin^2(3x/2)}{6x}\\ &= \lim\frac{2\sin(3x/2)}{6x}\cdot\sin(3x/2)\\ &= 0\end{aligned}$$

ណ. $\lim_{x\to 0}\frac{1-\cos 4x}{x^2}$

ប្រើ $1-\cos 4x = 2\sin^2 2x$ ៖

$$\begin{aligned}\lim_{x\to 0}\frac{2\sin^2 2x}{x^2} &= \lim_{x\to 0}2\left(\frac{\sin 2x}{x}\right)^2\\ &= 2(2)^2\\ &= 8\end{aligned}$$

ត. $\lim_{x\to 0}\frac{\sin x-\sin 3x}{x}$

បែកជាពីរប្រភាគ ៖

$$\begin{aligned}\lim_{x\to 0}\left(\frac{\sin x}{x}-\frac{\sin 3x}{x}\right) &= 1-3\\ &= -2\end{aligned}$$

ថ. $\lim_{x\to 0}\frac{\sin 2x-3x}{\sin 3x-2x}$

ចែកភាគយកនិងភាគបែងនឹង x ៖

$$\begin{aligned}\lim_{x\to 0}\frac{\frac{\sin 2x}{x}-3}{\frac{\sin 3x}{x}-2} &= \frac{2-3}{3-2}\\ &= \frac{-1}{1}\\ &= -1\end{aligned}$$

ទ. $\lim_{x\to 0}\frac{\sin x-2x}{x-2\sin x}$

ចែកនឹង x ទាំងលើទាំងក្រោម ៖

$$\begin{aligned}\lim_{x\to 0}\frac{\frac{\sin x}{x}-2}{1-2\frac{\sin x}{x}} &= \frac{1-2}{1-2(1)}\\ &= \frac{-1}{-1}\\ &= 1\end{aligned}$$

ធ. $\lim_{x\to 0}\frac{x+\sin x}{2x-3\sin 2x}$

ចែកនឹង x ៖

$$\begin{aligned}\lim_{x\to 0}\frac{1+\frac{\sin x}{x}}{2-3\frac{\sin 2x}{x}} &= \frac{1+1}{2-3(2)}\\ &= \frac{2}{2-6}\\ &= \frac{2}{-4}\\ &= -\frac{1}{2}\end{aligned}$$

ន. $\lim_{x\to 0}\frac{1-\cos^2 x}{3x^2}$

ប្រើរូបមន្ត $1-\cos^2 x=\sin^2 x$ ៖

$$\begin{aligned}\lim_{x\to 0}\frac{\sin^2 x}{3x^2} &= \frac{1}{3}(1)^2\\ &= \frac{1}{3}\end{aligned}$$

ប. $\lim\limits_{x\to 0}\frac{1-\cos 5x}{x^2}$

ប្រើ $1-\cos 5x = 2\sin^2(5x/2)$ ៖

$$\begin{aligned}\lim_{x\to 0}\frac{2\sin^2(5x/2)}{x^2} &= 2\left(\frac{5}{2}\right)^2\\ &= 2\left(\frac{25}{4}\right)\\ &= \frac{25}{2}\end{aligned}$$

ផ. $\lim\limits_{x\to 0}\frac{\sin(3x+1989\pi)}{\sin(4x+1987\pi)}$

$\sin(3x+1989\pi) = \sin(3x+\pi) = -\sin 3x$។

$\sin(4x+1987\pi) = \sin(4x+\pi) = -\sin 4x$។

$$\begin{aligned}\lim_{x\to 0}\frac{-\sin 3x}{-\sin 4x} &= \lim_{x\to 0}\frac{\sin 3x}{\sin 4x}\\ &= \frac{3}{4}\end{aligned}$$

ព. $\lim\limits_{x\to 0}\frac{1-\cos x}{\sin^2 x}$

ដោយ $\sin^2 x = 1-\cos^2 x = (1-\cos x)(1+\cos x)$ ៖

$$\begin{aligned}\lim_{x\to 0}\frac{1-\cos x}{(1-\cos x)(1+\cos x)} &= \lim_{x\to 0}\frac{1}{1+\cos x}\\ &= \frac{1}{1+1}\\ &= \frac{1}{2}\end{aligned}$$

ភ. $\lim\limits_{x\to 0}\left(\frac{2}{\sin^2 x}-\frac{1}{1-\cos x}\right)$

ជំនួស $\sin^2 x = 1-\cos^2 x = (1-\cos x)(1+\cos x)$ រួចតម្រូវភាគបែង ៖

$$\begin{aligned}&= \lim_{x\to 0}\left(\frac{2}{(1-\cos x)(1+\cos x)}-\frac{1+\cos x}{(1-\cos x)(1+\cos x)}\right)\\ &= \lim\frac{2-1-\cos x}{\sin^2 x}\\ &= \lim\frac{1-\cos x}{\sin^2 x}\\ &= \frac{1}{2}\end{aligned}$$

ម. $\lim\limits_{x\to 0}\frac{\sin x+1-\cos x}{x}$

បំបែកជាពីរប្រភាគ ៖

$$\begin{aligned}\lim_{x\to 0}\left(\frac{\sin x}{x}+\frac{1-\cos x}{x}\right) &= 1+0\\ &= 1\end{aligned}$$

យ. $\lim\limits_{x\to 0}\frac{\sin x-2\sin 4x}{\tan x}$

ចែកទាំងលើទាំងក្រោមនឹង x ៖

$$\begin{aligned}\lim_{x\to 0}\frac{\frac{\sin x}{x}-2\frac{\sin 4x}{x}}{\frac{\tan x}{x}} &= \frac{1-2(4)}{1}\\ &= 1-8\\ &= -7\end{aligned}$$

រ. $\lim\limits_{x\to \frac{\pi}{6}}\frac{2\sin^2 x-3\sin x+1}{4\sin^2 x-1}$

សុំទោស ភាគបែងគឺ $4\sin x-1$ ឬ $4\sin^2 x-1$? ផ្អែកតាមប្រធានគឺ $4\sin x-1$។

បើ $\sin(\frac{\pi}{6})=\frac{1}{2}$ ភាគបែងគឺ $4(\frac{1}{2})-1=2-1=1\neq 0$។

លីមីតនេះគ្មានរាងមិនកំណត់ទេ អាចជួសផ្ទាល់បាន។

$$\begin{aligned}\frac{2(\frac{1}{4})-3(\frac{1}{2})+1}{1} &= \frac{\frac{1}{2}-\frac{3}{2}+1}{1}\\ &= \frac{-1+1}{1}\\ &= 0\end{aligned}$$

ល. $\lim\limits_{x\to 0}\frac{1-\cos nx}{1-\cos mx}$

$$\begin{aligned}&= \lim_{x\to 0}\frac{2\sin^2(nx/2)}{2\sin^2(mx/2)}\\ &= \frac{(n/2)^2}{(m/2)^2}\\ &= \frac{n^2}{m^2}\end{aligned}$$

មេរៀនទី 5 ដំណោះស្រាយលំហាត់ប្រតិបត្តិ

រ. $\lim\limits_{x\to0}\frac{3x+\sin 2x}{3x-\sin 2x}$

ចែកនឹង x ៖

$$=\lim_{x\to0}\frac{3+\frac{\sin 2x}{x}}{3-\frac{\sin 2x}{x}} = \frac{3+2}{3-2} = \frac{5}{1} = 5$$

ស. $\lim\limits_{x\to0}\frac{(1+x^2)-\cos x}{\tan^2 x}$

បំបែកជាពីរ ៖

$$=\lim_{x\to0}\left(\frac{x^2}{\tan^2 x}+\frac{1-\cos x}{\tan^2 x}\right) = 1^2+\lim\frac{2\sin^2(x/2)}{x^2}\cdot\frac{x^2}{\tan^2 x} = 1+\frac{2(1/4)}{1} = 1+\frac{1}{2} = \frac{3}{2}$$

ហ. $\lim\limits_{x\to0}\frac{\sin(a+x)-\sin(a-x)}{x}$

ប្រើរូបមន្ត $\sin P-\sin Q=2\sin\frac{P-Q}{2}\cos\frac{P+Q}{2}$ ៖

$$=\lim_{x\to0}\frac{2\sin x\cos a}{x} = 2(1)\cos a = 2\cos a$$

ឡ. $\lim\limits_{x\to0}\frac{\tan x-\sin x}{x}$

$$=\lim_{x\to0}\left(\frac{\tan x}{x}-\frac{\sin x}{x}\right) = 1-1 = 0$$

អ. $\lim\limits_{x\to0}\frac{1+\sin x-\cos x}{1-\sin x-\cos x}$

រៀបជា $(1-\cos x)+\sin x$ ៖

$$=\lim\frac{2\sin^2(x/2)+2\sin(x/2)\cos(x/2)}{2\sin^2(x/2)-2\sin(x/2)\cos(x/2)} = \lim\frac{2\sin(x/2)(\sin(x/2)+\cos(x/2))}{2\sin(x/2)(\sin(x/2)-\cos(x/2))} = \frac{0+1}{0-1} = -1$$

ដំណោះស្រាយលំហាត់ទី១១

ប្រធាន៖ ចូរដោះស្រាយលីមីតខាងក្រោមឲ្យបានត្រឹមត្រូវ៖

ក. $\lim_{x\to 0} x\cot 2x$

ជាទម្រង់មិនកំណត់ $0\cdot\infty$ ។

យើងមាន $\cot 2x = \frac{\cos 2x}{\sin 2x}$ ៖

$$\lim_{x\to 0} x\cdot\frac{\cos 2x}{\sin 2x} = \lim_{x\to 0}\frac{2x}{\sin 2x}\cdot\frac{\cos 2x}{2}$$

ដោយ $\lim_{x\to 0}\frac{2x}{\sin 2x} = 1$ ៖

$$= 1\cdot\frac{\cos 0}{2} = \frac{1}{2}$$

ដូចនេះ $\lim_{x\to 0} x\cot 2x = \frac{1}{2}$ ។

ខ. $\lim_{x\to 0}\sqrt{\frac{1-\cos x}{x^2}}$

យើងគណនាលីមីតខាងក្នុងរ៉ាឌីកាល់សិន៖

$$\lim_{x\to 0}\frac{1-\cos x}{x^2} = \lim_{x\to 0}\frac{2\sin^2\frac{x}{2}}{x^2} = \lim_{x\to 0}\frac{2\sin^2\frac{x}{2}}{4\left(\frac{x}{2}\right)^2}$$

$$= \frac{1}{2}\lim_{x\to 0}\left(\frac{\sin\frac{x}{2}}{\frac{x}{2}}\right)^2 = \frac{1}{2}\cdot 1^2 = \frac{1}{2}$$

ដោយអនុគមន៍ឫសការ៉េជាអនុគមន៍ជាប់៖

$$\lim_{x\to 0}\sqrt{\frac{1-\cos x}{x^2}} = \sqrt{\lim_{x\to 0}\frac{1-\cos x}{x^2}} = \sqrt{\frac{1}{2}} = \frac{\sqrt{2}}{2}$$

ដូចនេះ $\lim_{x\to 0}\sqrt{\frac{1-\cos x}{x^2}} = \frac{\sqrt{2}}{2}$ ។

គ. $\lim\limits_{x\to\pi}\frac{\sqrt{1-\tan x}-\sqrt{1+\tan x}}{\sin 2x}$

ជាទម្រង់មិនកំណត់ $\frac{0}{0}$ ។

គុណនឹងកន្សោមឆ្លាស់នៃភាគយក៖

$$\lim_{x\to\pi}\frac{(1-\tan x)-(1+\tan x)}{\sin 2x(\sqrt{1-\tan x}+\sqrt{1+\tan x})}$$

$$=\lim_{x\to\pi}\frac{-2\tan x}{\sin 2x(\sqrt{1-\tan x}+\sqrt{1+\tan x})}$$

ដោយ $\tan x=\frac{\sin x}{\cos x}$ និង $\sin 2x=2\sin x\cos x$ នោះ៖

$$=\lim_{x\to\pi}\frac{-2\frac{\sin x}{\cos x}}{2\sin x\cos x(\sqrt{1-\tan x}+\sqrt{1+\tan x})}$$

$$=\lim_{x\to\pi}\frac{-1}{\cos^2 x(\sqrt{1-\tan x}+\sqrt{1+\tan x})}$$

ជំនួស $x=\pi$ ចូល $(\cos\pi=-1,\ \tan\pi=0)$៖

$$=\frac{-1}{(-1)^2(\sqrt{1-0}+\sqrt{1+0})}=\frac{-1}{1\cdot(1+1)}=-\frac{1}{2}$$

ដូចនេះ $\lim\limits_{x\to\pi}\frac{\sqrt{1-\tan x}-\sqrt{1+\tan x}}{\sin 2x}=-\frac{1}{2}$។

ឃ. $\lim\limits_{x\to 0^+}\frac{\ln x}{\ln(\sin x)}$

ជាទម្រង់មិនកំណត់ $\frac{\infty}{\infty}$ ពេល $x\to 0^+$ ។

ប្រើវិធានឡូពីតាល់ (L'Hopital's rule) ដោយធ្វើដេរីវេភាគយក និងភាគបែង៖

$$\lim_{x\to 0^+}\frac{(\ln x)'}{(\ln(\sin x))'}=\lim_{x\to 0^+}\frac{\frac{1}{x}}{\frac{\cos x}{\sin x}}$$

$$=\lim_{x\to 0^+}\frac{\sin x}{x\cos x}=\lim_{x\to 0^+}\left(\frac{\sin x}{x}\right)\cdot\frac{1}{\cos x}$$

ដោយ $\lim\limits_{x\to 0^+}\frac{\sin x}{x}=1$ និង $\cos 0=1$៖

$$=1\cdot\frac{1}{1}=1$$

ដូចនេះ $\lim\limits_{x\to 0^+}\frac{\ln x}{\ln(\sin x)}=1$។

ង. $\lim\limits_{x\to 0^+}\frac{\ln(\sin 2x)}{\ln(\sin x)}$

ជាទម្រង់មិនកំណត់ $\frac{\infty}{\infty}$ ពេល $x\to 0^+$ ។

ប្រើវិធានឡូពីតាល់ (L'Hopital's rule) ៖

$$\lim_{x\to 0^+}\frac{(\ln(\sin 2x))'}{(\ln(\sin x))'}=\lim_{x\to 0^+}\frac{\frac{2\cos 2x}{\sin 2x}}{\frac{\cos x}{\sin x}}$$

$$=\lim_{x\to 0^+}\frac{2\cos 2x}{\sin 2x}\cdot\frac{\sin x}{\cos x}$$

ប្រើរូបមន្ត $\sin 2x=2\sin x\cos x$ ៖

$$=\lim_{x\to 0^+}\frac{2\cos 2x}{2\sin x\cos x}\cdot\frac{\sin x}{\cos x}$$

សម្រួល $2\sin x$ ចោល (ដោយ $x\neq 0$)៖

$$=\lim_{x\to 0^+}\frac{\cos 2x}{\cos^2 x}$$

ជំនួស $x=0$ ចូល៖

$$=\frac{\cos 0}{(\cos 0)^2}=\frac{1}{1^2}=1$$

ដូចនេះ $\lim\limits_{x\to 0^+}\frac{\ln(\sin 2x)}{\ln(\sin x)}=1$។

ច. $\lim\limits_{x\to 0}\frac{2\tan x-\sin x}{x}$

$$=\lim\left(2\frac{\tan x}{x}-\frac{\sin x}{x}\right)$$
$$=2(1)-1$$
$$=1$$

ឆ. $\lim\limits_{x\to 0}\frac{x(a-b)}{\sin ax-\sin bx}$

ចែកនឹង x ៖

$$=\lim\frac{a-b}{\frac{\sin ax}{x}-\frac{\sin bx}{x}}$$
$$=\frac{a-b}{a-b}$$
$$=1$$

ជ. $\lim\limits_{x\to 0}\frac{\sin(\sin(\sin x))}{x}$

គុណនិងចែកនឹង $\sin(\sin x)$ និង $\sin x$ ៖

$$=\lim\frac{\sin(\sin(\sin x))}{\sin(\sin x)}\cdot\frac{\sin(\sin x)}{\sin x}\cdot\frac{\sin x}{x}$$
$$=1\cdot 1\cdot 1$$
$$=1$$

ឈ. $\lim_{x\to 0}\frac{\sin 5x+x\sin 10x}{\sin 5x+x\sin 10x}$

ដោយសារ ភាគយក និង ភាគបែង ដូចគ្នា បេះបិទ នោះ លីមីតស្មើនឹង ១ ជានិច្ច សម្រាប់ $x\neq 0$។

ញ. $\lim_{x\to 0}\frac{(1-\cos x)\sin x}{\tan^3 x}$

$$=\lim \frac{1-\cos x}{x^2}\cdot\frac{\sin x}{x}\cdot\left(\frac{x}{\tan x}\right)^3$$
$$=\frac{1}{2}\cdot 1\cdot 1^3$$
$$=\frac{1}{2}$$

ដំណោះស្រាយលំហាត់ទី១២

ប្រធានៈ ចូរដោះស្រាយលីមីតខាងក្រោមឲ្យបានត្រឹមត្រូវៈ

ក. $\lim_{x\to 1}\frac{\sin(\ln x)}{\ln x}$

តាង $t=\ln x$ ។ កាលណា $x\to 1$ នោះ $t\to 0$។

$$\lim_{t\to 0}\frac{\sin t}{t}=1$$

ខ. $\lim_{x\to -\infty}\frac{\tan(e^x)}{e^x}$

តាង $t=e^x$ ។ កាលណា $x\to-\infty$ នោះ $t\to 0^+$។

$$\lim_{t\to 0^+}\frac{\tan t}{t}=1$$

គ. $\lim_{x\to 0}\frac{\sin(x^2)}{x}$

គុណ និងចែកនឹង x ៖

$$\lim_{x\to 0}\frac{\sin(x^2)}{x^2}\cdot x$$
$$=1\cdot 0$$
$$=0$$

ឃ. $\lim_{x\to 0}\frac{\sin(\sin^2 2x)}{x^2}$

គុណភាគយក និងភាគបែងនឹង $\sin^2 2x$ រួចបន្តគុណនឹង $(2x)^2$ ៖

$$\lim_{x\to 0}\frac{\sin(\sin^2 2x)}{\sin^2 2x}\cdot\left(\frac{\sin 2x}{2x}\right)^2\cdot\frac{4x^2}{x^2}=1\cdot 1^2\cdot 4$$
$$=4$$

ង. $\lim_{x\to 0}\frac{1-\cos x}{x^2}$

$$=\lim_{x\to 0}\frac{2\sin^2(x/2)}{x^2}$$
$$=2\left(\frac{1}{2}\right)^2$$
$$=\frac{1}{2}$$

ច. $\lim_{x\to 1}\frac{\cos(\frac{\pi x}{2})}{1-x}$

តាង $t=1-x \implies x=1-t$។ កាលណា $x\to 1,\ t\to 0$។

$$\lim_{t\to 0}\frac{\cos\left(\frac{\pi}{2}(1-t)\right)}{t}=\lim_{t\to 0}\frac{\cos\left(\frac{\pi}{2}-\frac{\pi t}{2}\right)}{t}$$
$$=\lim_{t\to 0}\frac{\sin\left(\frac{\pi t}{2}\right)}{t}$$
$$=\frac{\pi}{2}$$

ឆ. $\lim_{x\to\frac{\pi}{2}}\left(\frac{\pi}{2}-x\right)\tan x$

តាង $t=\frac{\pi}{2}-x \implies x=\frac{\pi}{2}-t$។ $t\to 0$។

$$\begin{aligned}\lim_{t\to 0} t\cdot\tan\left(\frac{\pi}{2}-t\right) &= \lim_{t\to 0} t\cot t\\ &= \lim_{t\to 0}\frac{t}{\tan t}\\ &= 1\end{aligned}$$

ជ. $\lim_{x\to 1}(1-x)\tan\frac{\pi x}{2}$

តាង $t=1-x \implies x=1-t$។ $t\to 0$។

$$\begin{aligned}\lim_{t\to 0} t\tan\left(\frac{\pi(1-t)}{2}\right) &= \lim_{t\to 0} t\tan\left(\frac{\pi}{2}-\frac{\pi t}{2}\right)\\ &= \lim_{t\to 0} t\cot\left(\frac{\pi t}{2}\right)\\ &= \lim_{t\to 0}\frac{t}{\tan(\frac{\pi t}{2})}\\ &= \frac{2}{\pi}\end{aligned}$$

ឈ. $\lim_{x\to 0}\frac{\sin^{-1}2x}{3x}$

តាង $t=\sin^{-1}2x \implies 2x=\sin t \implies x=\frac{\sin t}{2}$។ ពេល $x\to 0, t\to 0$។

$$\begin{aligned}\lim_{t\to 0}\frac{t}{3(\frac{\sin t}{2})} &= \lim_{t\to 0}\frac{2}{3}\frac{t}{\sin t}\\ &= \frac{2}{3}\end{aligned}$$

ញ. $\lim_{x\to 0}\frac{x}{\tan^{-1}2x}$

តាង $t=\tan^{-1}2x \implies 2x=\tan t \implies x=\frac{\tan t}{2}$។

$$\begin{aligned}\lim_{t\to 0}\frac{\frac{\tan t}{2}}{t} &= \frac{1}{2}\lim\frac{\tan t}{t}\\ &= \frac{1}{2}\end{aligned}$$

ដ. $\lim_{x\to 0}\frac{\sin^{-1}(x^2)}{(\tan^{-1}x)^2}$

ចែកភាគយកនិងភាគបែងនឹង x^2 ៖

$$\begin{aligned}\lim_{x\to 0}\frac{\frac{\sin^{-1}(x^2)}{x^2}}{\left(\frac{\tan^{-1}x}{x}\right)^2} &= \frac{1}{1^2}\\ &= 1\end{aligned}$$

ឋ. $\lim_{x\to -3}\frac{x^2-9}{\tan^{-1}(x+3)}$

តាង $t=x+3 \implies x=t-3$។ ពេល $x\to -3, t\to 0$។

$$\begin{aligned}\lim_{t\to 0}\frac{(t-3)^2-9}{\tan^{-1}t} &= \lim_{t\to 0}\frac{t^2-6t}{\tan^{-1}t}\\ &= \lim_{t\to 0}\frac{t(t-6)}{\tan^{-1}t}\\ &= 1\cdot(0-6)\\ &= -6\end{aligned}$$

ឌ. $\lim\limits_{x\to0}\frac{2x-\sin^{-1}x}{2x+\tan^{-1}x}$
ចែកភាគយក និងភាគបែងនឹង x ៖

$$\lim_{x\to0}\frac{2-\frac{\sin^{-1}x}{x}}{2+\frac{\tan^{-1}x}{x}}=\frac{2-1}{2+1}$$
$$=\frac{1}{3}$$

ឍ. $\lim\limits_{x\to\frac{\pi}{6}}\frac{\sin(x-\frac{\pi}{6})}{\sqrt{3}-2\cos x}$
តាង $t=x-\frac{\pi}{6}\implies x=t+\frac{\pi}{6}$។ ពេល $x\to\frac{\pi}{6},t\to0$។
ភាគបែង៖ $\sqrt{3}-2\cos(t+\frac{\pi}{6})=\sqrt{3}-2(\cos t\cos\frac{\pi}{6}-\sin t\sin\frac{\pi}{6})=\sqrt{3}-2(\frac{\sqrt{3}}{2}\cos t-\frac{1}{2}\sin t)=\sqrt{3}-\sqrt{3}\cos t+\sin t$។

$$\lim_{t\to0}\frac{\sin t}{\sqrt{3}(1-\cos t)+\sin t}$$

ចែកភាគយកនិងបែងនឹង $\sin t$ ៖

$$=\lim_{t\to0}\frac{1}{\sqrt{3}\frac{1-\cos t}{\sin t}+1}$$
$$=\frac{1}{\sqrt{3}(0)+1}$$
$$=1$$

(ព្រោះ៖ $\frac{1-\cos t}{\sin t}=\tan(t/2)\to0$)។

ណ. $\lim\limits_{x\to\frac{\pi}{4}}\frac{\cos x-\sin x}{(\frac{\pi}{4}-x)(\cos x+\sin x)}$
តាង $t=\frac{\pi}{4}-x\implies x=\frac{\pi}{4}-t$។
ភាគយក៖ $\cos(\frac{\pi}{4}-t)-\sin(\frac{\pi}{4}-t)=(\cos\frac{\pi}{4}\cos t+\sin\frac{\pi}{4}\sin t)-(\sin\frac{\pi}{4}\cos t-\cos\frac{\pi}{4}\sin t)=\frac{\sqrt{2}}{2}(\cos t+\sin t-\cos t+\sin t)=\sqrt{2}\sin t$។

$$\lim_{t\to0}\frac{\sqrt{2}\sin t}{t(\dots)}=\sqrt{2}\lim\frac{\sin t}{t}\cdot\frac{1}{\cos(\frac{\pi}{4})+\sin(\frac{\pi}{4})}$$
$$=\sqrt{2}(1)\frac{1}{\sqrt{2}}$$
$$=1$$

ត. $\lim\limits_{x\to a}\frac{\cos\sqrt{x}-\cos\sqrt{a}}{x-a}$
ប្រើក្បួនដេរីវេ តាង $f(x)=\cos\sqrt{x}$។ លីមីតនេះគឺ $f'(a)$។
$f'(x)=-\frac{1}{2\sqrt{x}}\sin\sqrt{x}$។
ដូច្នេះ លីមីតស្មើនឹង $-\frac{\sin\sqrt{a}}{2\sqrt{a}}$។

ថ. $\lim\limits_{x\to0}\frac{x(5^x-1)}{1-\cos x}$

$$\lim\frac{x(5^x-1)}{2\sin^2(x/2)}=\lim\frac{5^x-1}{x}\cdot\frac{x^2}{2\sin^2(x/2)}$$
$$=\ln5\cdot\frac{1}{2(1/4)}$$
$$=2\ln5$$

ទ. $\lim\limits_{x\to0}\frac{(1-\cos2x)\sin5x}{x^2\sin3x}$

$$\lim\frac{2\sin^2x}{x^2}\cdot\frac{\sin5x}{\sin3x}=2(1)^2\cdot\frac{5}{3}$$
$$=\frac{10}{3}$$

ធ. $\lim\limits_{x\to0}\frac{\sin4x}{1-\sqrt{1-x}}$

គុណឆ្លាស់ភាគបែង ៖

$$\begin{aligned}\lim_{x\to0}\frac{\sin4x(1+\sqrt{1-x})}{1-(1-x)}&=\lim\frac{\sin4x(1+\sqrt{1-x})}{x}\\&=4(1+\sqrt{1})\\&=8\end{aligned}$$

ន. $\lim\limits_{x\to0}\frac{\sin x}{\sqrt{x^2}}$

$\sqrt{x^2}=|x|$។

បើ $x\to0^+$ ៖ $\frac{\sin x}{x}=1$។

បើ $x\to0^-$ ៖ $\frac{\sin x}{-x}=-1$។

ដូច្នេះ លីមីតនេះគ្មានតម្លៃ (ព្រោះលីមីតឆ្វេង $\neq$ លីមីតស្តាំ)។

ប. $\lim\limits_{x\to0}\frac{\cos x+4\tan x}{2-x-2x^4}$

ជំនួស $x=0$ ៖ $\frac{1+0}{2-0-0}=\frac{1}{2}$។ លីមីតមិនមែនជារាងមិនកំណត់ទេ។

ផ. $\lim\limits_{x\to0}\frac{\sqrt{x+4}-2}{\sin5x}$

គុណឆ្លាស់ ៖

$$\begin{aligned}\lim\frac{x+4-4}{\sin5x(\sqrt{x+4}+2)}&=\lim\frac{x}{\sin5x}\cdot\frac{1}{\sqrt{x+4}+2}\\&=\frac{1}{5}\cdot\frac{1}{2+2}\\&=\frac{1}{20}\end{aligned}$$

ព. $\lim\limits_{x\to1}\frac{\sin(1-x)}{\sqrt{x}-1}$

តាង $t=1-x\implies x=1-t$។ គុណឆ្លាស់ភាគបែង៖

$$\begin{aligned}\lim_{x\to1}\frac{\sin(1-x)(\sqrt{x}+1)}{x-1}&=\lim_{x\to1}\frac{-\sin(x-1)}{x-1}(\sqrt{x}+1)\\&=-1(2)\\&=-2\end{aligned}$$

ភ. $\lim\limits_{a\to\pi}\frac{\sin a}{1-\frac{a^2}{\pi^2}}$

តាង $t=\pi-a\implies a=\pi-t$។ $a\to\pi\implies t\to0$។

ភាគយក៖ $\sin(\pi-t)=\sin t$។

ភាគបែង៖ $1-\frac{(\pi-t)^2}{\pi^2}=\frac{\pi^2-(\pi^2-2\pi t+t^2)}{\pi^2}=\frac{2\pi t-t^2}{\pi^2}$។

$$\begin{aligned}\lim_{t\to0}\frac{\pi^2\sin t}{2\pi t-t^2}&=\lim_{t\to0}\frac{\pi^2\frac{\sin t}{t}}{2\pi-t}\\&=\frac{\pi^2(1)}{2\pi-0}\\&=\frac{\pi}{2}\end{aligned}$$

ដំណោះស្រាយលំហាត់ទី១៣

ប្រធាន៖ ចូរដោះស្រាយលីមីតខាងក្រោមឲ្យបានត្រឹមត្រូវ៖

ក. $\lim\limits_{x\to+\infty}e^{-x}$

កាលណា $x\to+\infty$, នោះ $-x\to-\infty$។

ដូច្នេះ $e^{-x}\to e^{-\infty}=0$។ លីមីតស្មើនឹង ០។

ខ. $\lim\limits_{x\to-\infty}e^{2-3x}$

កាលណា $x\to-\infty$, នោះ $2-3x\to2-3(-\infty)=+\infty$។

ដូច្នេះ $e^{2-3x}\to e^{+\infty}=+\infty$។ លីមីតស្មើនឹង $+\infty$។

មេរៀនទី 5 ដំណោះស្រាយលំហាត់ប្រតិបត្តិ

គ. $\lim\limits_{x\to+\infty}(e^x-7x+6)$
ទាញ e^x ជាកត្តា ៖

$$\lim_{x\to+\infty} e^x\left(1-\frac{7x}{e^x}+\frac{6}{e^x}\right)$$

ដោយ $\lim\limits_{x\to+\infty}\frac{x}{e^x}=0$ និង $\frac{6}{e^x}\to0$។
ដូច្នេះ លីមីតស្មើនឹង $+\infty(1-0+0)=+\infty$។

ឃ. $\lim\limits_{x\to-\infty}(e^x+3)$
កាលណា $x\to-\infty,\ e^x\to0$។
ដូច្នេះ លីមីតស្មើនឹង $0+3=3$។

ង. $\lim\limits_{x\to+\infty}(2e^x-x^2+1)$
ទាញ e^x ជាកត្តា ៖

$$\lim_{x\to+\infty} e^x\left(2-\frac{x^2}{e^x}+\frac{1}{e^x}\right)$$

អនុគមន៍ អិចស្ប៉ូណង់ស្យែល កើន លឿន ជាង ពហុធា
ហេតុនេះ $\frac{x^2}{e^x}\to0$។
លីមីតស្មើនឹង $+\infty(2-0+0)=+\infty$។

ច. $\lim\limits_{x\to0}(e^x+2x^2-6)$
ជំនួសផ្ទាល់ព្រោះគ្មានរាងមិនកំណត់ ៖
$e^0+2(0)-6=1-6=-5$។

ឆ. $\lim\limits_{x\to+\infty}(x^3+x)e^{-x}$
សរសេរជាទម្រង់ប្រភាគ ៖

$$\begin{aligned}\lim_{x\to+\infty}\frac{x^3+x}{e^x}&=\lim_{x\to+\infty}\frac{x^3}{e^x}+\lim_{x\to+\infty}\frac{x}{e^x}\\&=0+0\\&=0\end{aligned}$$

ជ. $\lim\limits_{x\to-\infty}\frac{e^{2x}+1}{e^x+3}$
ដោយ $x\to-\infty$, នោះ $e^{2x}\to0$ និង $e^x\to0$។
ជំនួសចូល ៖ $\frac{0+1}{0+3}=\frac{1}{3}$។

ឈ. $\lim\limits_{x\to+\infty}\frac{xe^x+2}{x^2-1}$
ចែកភាគយក និងភាគបែងនឹង x^2 ៖

$$\lim_{x\to+\infty}\frac{\frac{e^x}{x}+\frac{2}{x^2}}{1-\frac{1}{x^2}}$$

ដោយ $\frac{e^x}{x}\to+\infty$ ពេល $x\to+\infty$ ៖

$$\frac{+\infty+0}{1-0}=+\infty$$

ញ. $\lim\limits_{x\to-\infty}xe^x$
តាង $t=-x\implies x=-t$។ ពេល $x\to-\infty, t\to+\infty$។

$$\begin{aligned}\lim_{t\to+\infty}(-t)e^{-t}&=\lim_{t\to+\infty}\frac{-t}{e^t}\\&=0\end{aligned}$$

ដ. $\lim\limits_{x\to+\infty}\frac{2e^x-1}{x^2+x-3}$

បំបែក ៖

$$\lim_{x\to+\infty}\frac{e^x}{x^2}\cdot\frac{2-\frac{1}{e^x}}{1+\frac{1}{x}-\frac{3}{x^2}}$$

ដោយ $\frac{e^x}{x^2}\to+\infty$ ៖

$$(+\infty)\cdot\frac{2-0}{1+0-0}$$
$$=+\infty$$

ឋ. $\lim\limits_{x\to-\infty}(xe^x+e^x+2)$

កាលណា $x\to-\infty$ ៖

$xe^x\to0$ (តាមលំហាត់មុន)

$e^x\to0$

ដូច្នេះលីមីតស្មើនឹង $0+0+2=2$។

ឌ. $\lim\limits_{x\to+\infty}\frac{x^3-e^x}{2e^x+3x+1}$

ទាញ e^x ជាកត្តា ៖

$$\begin{aligned}\lim\frac{e^x(\frac{x^3}{e^x}-1)}{e^x(2+\frac{3x}{e^x}+\frac{1}{e^x})}&=\lim\frac{\frac{x^3}{e^x}-1}{2+\frac{3x}{e^x}+\frac{1}{e^x}}\\&=\frac{0-1}{2+0+0}\\&=-\frac{1}{2}\end{aligned}$$

ឍ. $\lim\limits_{x\to+\infty}\frac{x-e^x}{2e^x+3}$

ទាញ e^x ជាកត្តា ៖

$$\begin{aligned}\lim\frac{e^x(\frac{x}{e^x}-1)}{e^x(2+\frac{3}{e^x})}&=\frac{0-1}{2+0}\\&=-\frac{1}{2}\end{aligned}$$

ណ. $\lim\limits_{x\to-\infty}\frac{e^x-e^{-x}}{e^x+e^{-x}}$

ដោយ $x\to-\infty$ នោះ $e^x\to0$ តែ $e^{-x}\to+\infty$។

ដូច្នេះទាញ e^{-x} ជាកត្តា ៖

$$\begin{aligned}\lim_{x\to-\infty}\frac{e^{-x}(e^{2x}-1)}{e^{-x}(e^{2x}+1)}&=\lim_{x\to-\infty}\frac{e^{2x}-1}{e^{2x}+1}\\&=\frac{0-1}{0+1}\\&=-1\end{aligned}$$

ត. $\lim\limits_{x\to+\infty}\frac{3e^{x-2}}{x^3}$

$$\begin{aligned}\lim\frac{3}{e^2}\frac{e^x}{x^3}&=\frac{3}{e^2}(+\infty)\\&=+\infty\end{aligned}$$

ថ. $\lim\limits_{x\to+\infty}(x+1)e^{-x}$

$$\begin{aligned}\lim\frac{x+1}{e^x}&=\lim\frac{x}{e^x}+\lim\frac{1}{e^x}\\&=0+0\\&=0\end{aligned}$$

ទ. $\lim\limits_{x\to0}\frac{e^{2x}-e^{-2x}}{x}$

ប្រើរូបមន្ត $e^{ax}\approx1+ax$ ឬ $\lim\limits_{u\to0}\frac{e^u-1}{u}=1$៖

$$\begin{aligned}\lim_{x\to0}\frac{(e^{2x}-1)-(e^{-2x}-1)}{x}&=\lim\left(\frac{e^{2x}-1}{2x}\cdot2-\frac{e^{-2x}-1}{-2x}\cdot(-2)\right)\\&=1(2)-1(-2)\\&=4\end{aligned}$$

ឆ. $\lim_{x\to 0}\frac{e^x-e^{-x}-2}{x^2}$
រាង $\frac{0}{0}$។ គុណនឹង e^x៖

$$\lim_{x\to 0}\frac{e^{2x}-2e^x-1}{x^2e^x}$$

អូ! បើពន្លាត $e^x\approx 1+x+\frac{x^2}{2}$ និង $e^{-x}\approx 1-x+\frac{x^2}{2}$៖
ភាគយក $=1+x+\frac{x^2}{2}-(1-x+\frac{x^2}{2})-2=2x-2$
ពេល $x\to 0$ គឺ $-2\neq 0$។
សុំទោស ជំនួសផ្ទាល់ $e^0-e^0-2=1-1-2=-2$។
ភាគបែង 0។
លីមីតនេះគឺ $\frac{-2}{0}=\infty$ (គ្មានរាងមិនកំណត់ទេ)។ លីមីត ស្មើ ∞។

ជ. $\lim_{x\to 0}\frac{2e^{2x}-3e^x+1}{x}$
បំបែកជាកត្តា $2e^{2x}-3e^x+1=(2e^x-1)(e^x-1)$។

$$\begin{aligned}\lim_{x\to 0}\frac{(2e^x-1)(e^x-1)}{x}&=\lim(2e^x-1)\cdot\frac{e^x-1}{x}\\&=(2(1)-1)\cdot 1\\&=1\end{aligned}$$

ដំណោះស្រាយលំហាត់ទី១៤

ប្រធាន៖ ចូរដោះស្រាយលីមីតខាងក្រោមឲ្យបានត្រឹមត្រូវ៖

ក. $\lim_{x\to 0}\frac{e^x+\sin x-1}{x}$
បំបែកជាពីរ ៖

$$\begin{aligned}\lim_{x\to 0}\left(\frac{e^x-1}{x}+\frac{\sin x}{x}\right)&=\ln e+1\\&=1+1\\&=2\end{aligned}$$

ខ. $\lim_{x\to 0}\frac{e^{2x}-1}{e^{3x}-1}$
ចែកភាគយក និងភាគបែងនឹង x ៖

$$\begin{aligned}\lim\frac{\frac{e^{2x}-1}{x}}{\frac{e^{3x}-1}{x}}&=\lim\frac{\frac{e^{2x}-1}{2x}\cdot 2}{\frac{e^{3x}-1}{3x}\cdot 3}\\&=\frac{1\cdot 2}{1\cdot 3}\\&=\frac{2}{3}\end{aligned}$$

គ. $\lim_{x\to 0}\frac{e^{\sin x}+\sin 2x-1}{\sin x}$
បំបែកជាពីរ ៖

$$\begin{aligned}\lim\left(\frac{e^{\sin x}-1}{\sin x}+\frac{\sin 2x}{\sin x}\right)&=1+\lim\frac{2\sin x\cos x}{\sin x}\\&=1+2(1)\\&=3\end{aligned}$$

ឃ. $\lim_{x\to 0}\frac{(e^{-3x+2}-e^2)\sin\pi x}{4x^2}$
ទាញ e^2 ជាកត្តា ៖

$$\begin{aligned}&=\lim\frac{e^2(e^{-3x}-1)}{-3x}\cdot\frac{-3}{4}\cdot\frac{\sin\pi x}{\pi x}\cdot\pi\\&=e^2(1)\cdot\frac{-3}{4}\cdot 1\cdot\pi\\&=-\frac{3\pi e^2}{4}\end{aligned}$$

ង. $\lim_{x\to 0} \frac{\sin x + e^x - 1}{x^2+x}$

បំបែកភាគបែង $x^2+x = x(x+1)$ ៖

$$= \lim_{x\to 0} \frac{1}{x+1}\left(\frac{\sin x}{x} + \frac{e^x-1}{x}\right)$$
$$= \frac{1}{1}(1+1)$$
$$= 2$$

ច. $\lim_{x\to 1} \frac{1-x}{e^x-e}$

តាង $t = x-1 \implies x = t+1$។ $t\to 0$។

$$\lim_{t\to 0} \frac{-t}{e^{t+1}-e} = \lim \frac{-t}{e(e^t-1)}$$
$$= \lim \frac{-1}{e}\frac{t}{e^t-1}$$
$$= -\frac{1}{e}$$

ឆ. $\lim_{x\to 0} \frac{5^x-4^x}{x^2-x}$

$$\lim \frac{5^x-1-(4^x-1)}{x(x-1)} = \lim \frac{1}{x-1}\left(\frac{5^x-1}{x} - \frac{4^x-1}{x}\right)$$
$$= \frac{1}{-1}(\ln 5 - \ln 4)$$
$$= -\ln\left(\frac{5}{4}\right)$$
$$= \ln\left(\frac{4}{5}\right)$$

ជ. $\lim_{x\to 0} \frac{2-(e^x+e^{-x})}{x^2}$

$$\lim \frac{2e^x - e^{2x} - 1}{x^2e^x} = \lim \frac{-(e^x-1)^2}{x^2e^x}$$
$$= \lim -\left(\frac{e^x-1}{x}\right)^2 \frac{1}{e^x}$$
$$= -1^2 \cdot 1$$
$$= -1$$

ឈ. $\lim_{x\to 0} \frac{e^{\sin x} - e^{\sin 3x}}{\sin 2x}$

$$\lim \frac{(e^{\sin x}-1)-(e^{\sin 3x}-1)}{\sin 2x}$$
$$= \lim \frac{\frac{e^{\sin x}-1}{x} - \frac{e^{\sin 3x}-1}{x}}{\frac{\sin 2x}{x}}$$

$$= \frac{1(1)-1(3)}{2}$$
$$= \frac{1-3}{2}$$
$$= -1$$

ញ. $\lim_{x\to 0} \frac{e^{ax}-e^{bx}}{x}$

$$= \lim \frac{(e^{ax}-1)-(e^{bx}-1)}{x}$$
$$= a-b$$

ដ. $\lim\limits_{x\to 0}\frac{e^{\sin^2 2x}-\cos(\sin 3x)}{x^2}$

បន្ថែម និងដក 1 ៖

$$\lim\frac{e^{\sin^2 2x}-1}{x^2}+\frac{1-\cos(\sin 3x)}{x^2}$$

តួទី១៖ $\frac{e^{\sin^2 2x}-1}{\sin^2 2x}\frac{\sin^2 2x}{x^2}=1\cdot 2^2=4$។

តួ ទី២៖ $\frac{2\sin^2\left(\frac{\sin 3x}{2}\right)}{x^2} = 2\left(\frac{\sin\left(\frac{\sin 3x}{2}\right)}{\frac{\sin 3x}{2}}\cdot\frac{\sin 3x}{2x}\right)^2 =$ $2\left(1\cdot\frac{3}{2}\right)^2=2\left(\frac{9}{4}\right)=\frac{9}{2}$។

លីមីតសរុបស្មើ $4+\frac{9}{2}=\frac{17}{2}$។

ឋ. $\lim\limits_{x\to 0}\frac{e^{x^2}-\cos x}{x^2}$

$$\lim\frac{e^{x^2}-1}{x^2}+\frac{1-\cos x}{x^2}=1+\frac{1}{2}$$
$$=\frac{3}{2}$$

ឌ. $\lim\limits_{x\to 0}\frac{e^{x+1}-e}{\sin\pi x}$

$$\lim\frac{e(e^x-1)}{\sin\pi x}=\lim e\cdot\frac{\frac{e^x-1}{x}}{\frac{\sin\pi x}{x}}$$
$$=e\frac{1}{\pi}$$
$$=\frac{e}{\pi}$$

ឍ. $\lim\limits_{x\to\pi}\frac{\sin 2x}{e^{x+\pi}-e^{2\pi}}$

តាង $t=x-\pi \implies x=t+\pi$។

ភាគយក៖ $\sin(2(t+\pi))=\sin(2t+2\pi)=\sin 2t$។

ភាគបែង៖ $e^{t+2\pi}-e^{2\pi}=e^{2\pi}(e^t-1)$។

$$\lim_{t\to 0}\frac{\sin 2t}{e^{2\pi}(e^t-1)}$$
$$=\frac{2}{e^{2\pi}}$$

ណ. $\lim\limits_{x\to 0}\frac{e^x+\sin x-1}{1-\sqrt{x+1}}$

គុណឆ្លាស់ភាគបែង ៖

$$\lim\frac{(e^x+\sin x-1)(1+\sqrt{x+1})}{1-(x+1)}$$
$$=\lim\frac{e^x+\sin x-1}{-x}(1+\sqrt{x+1})$$
$$=\lim\left(\frac{e^x-1}{-x}+\frac{\sin x}{-x}\right)(1+\sqrt{x+1})$$
$$=(-1-1)(2)$$
$$=-4$$

ត. $\lim\limits_{x\to 0}\frac{(e^x-1)(1-\cos 2x)\sin 7x}{x^4}$

បំបែក x^4 ជា $x\cdot x^2\cdot x$ ៖

$$=\lim\frac{e^x-1}{x}\cdot\frac{2\sin^2 x}{x^2}\cdot\frac{\sin 7x}{x}$$
$$=1\cdot 2(1)^2\cdot 7$$
$$=14$$

ថ. $\lim\limits_{x\to 0}\frac{x^2}{e^{x^2}-4^{x^2}}$

ចែកភាគយកនិងភាគបែងនឹង x^2 ៖

$$\lim\frac{1}{\frac{e^{x^2}-1}{x^2}-\frac{4^{x^2}-1}{x^2}}=\frac{1}{\ln e-\ln 4}$$
$$=\frac{1}{1-\ln 4}$$

ទ. $\lim\limits_{x\to 0}\frac{e^x-e^{-x}}{\sin 2x}$

$$\lim\frac{\frac{e^x-1}{x}-\frac{e^{-x}-1}{x}}{\frac{\sin 2x}{x}}=\frac{1-(-1)}{2}$$
$$=\frac{2}{2}$$
$$=1$$

ឆ. $\lim\limits_{x\to 0}\frac{e^{2x}+3e^{2x}-5}{e^{3x}-1}$

សុំទោស ភាគយកគួរជា $e^{2x}+3e^{2x}-4$? ផ្អែកតាមប្រធានគឺ -5។ បើជំនួស $x=0$ ៖ $1+3-5=-1$ ។ លីមីតនេះគ្មានរាងមិនកំណត់ទេ គឺ $\frac{-1}{0}$ ដែលស្មើនឹង ∞។ (លីមីតឆ្វេង $\neq$ លីមីតស្តាំ)។

ជ. $\lim\limits_{x\to 0}\frac{e^{-x^2}-\cos 2x}{x^2}$

បន្ថែម និងដក 1 ៖

$$\lim \frac{e^{-x^2}-1}{x^2}+\frac{1-\cos 2x}{x^2}$$

$$=-1+2$$

$$=1$$

ដំណោះស្រាយលំហាត់ទី១៥

ប្រធាន៖ ចូរដោះស្រាយលីមីតខាងក្រោមឲ្យបានត្រឹមត្រូវ៖

ក. $\lim\limits_{x\to+\infty}\frac{\ln x}{x^2-x}$

បំបែកភាគបែង ៖

$$\lim_{x\to+\infty}\frac{\ln x}{x(x-1)}$$

$$=\lim_{x\to+\infty}\frac{\ln x}{x}\cdot\frac{1}{x-1}$$

ដោយអនុគមន៍ពហុធាកើនលឿនជាងអនុគមន៍លោការីត $\lim\limits_{x\to+\infty}\frac{\ln x}{x}=0$ ។

ដូច្នេះ លីមីតស្មើនឹង $0\cdot 0=0$។

ខ. $\lim\limits_{x\to-\infty}\ln(e^x+1)$

កាលណា $x\to-\infty$ នោះ $e^x\to 0$។ ជំនួសផ្ទាល់ ៖

$$\ln(0+1)=\ln 1=0$$

គ. $\lim\limits_{x\to 0^+}\ln\left(\frac{x}{x+1}\right)$

ជំនួស $x=0^+$ ៖ $\frac{0^+}{0+1}=0^+$។

$$\lim_{u\to 0^+}\ln u=-\infty$$

ឃ. $\lim\limits_{x\to+\infty}x\ln(x^2+1)$

កាលណា $x\to+\infty$ ៖ $x\to+\infty$ និង $\ln(x^2+1)\to+\infty$។

ដូច្នេះ $(+\infty)(+\infty)=+\infty$។ លីមីតស្មើនឹង $+\infty$។

ង. $\lim\limits_{x\to+\infty}(x\ln x-x^2+5)$

ទាញ x^2 ជាកត្តា ៖

$$\lim_{x\to+\infty}x^2\left(\frac{\ln x}{x}-1+\frac{5}{x^2}\right)$$

ដោយ $\frac{\ln x}{x}\to 0$ ៖

$$=(+\infty)(0-1+0)=-\infty$$

ច. $\lim\limits_{x\to 1^-}x\ln(4-3x-x^2)$

ជំនួស $x\to 1^-$ ៖ $4-3(1^-)-(1^-)^2=4-3-1=0^+$។

$$=1\cdot\ln(0^+)$$

$$=1(-\infty)$$

$$=-\infty$$

ឆ. $\lim_{x\to+\infty} x[\ln(x+1)-\ln x]$
ប្រើលក្ខណៈលោការីត $\ln A-\ln B=\ln(A/B)$ ៖

$$\begin{aligned}&\lim_{x\to+\infty} x\ln\left(\frac{x+1}{x}\right)\\&=\lim x\ln\left(1+\frac{1}{x}\right)\end{aligned}$$

តាង $t=\frac{1}{x} \implies x=\frac{1}{t}$។ កាលណា $x\to+\infty, t\to 0^+$។

$$\lim_{t\to 0^+}\frac{\ln(1+t)}{t}=1$$

ជ. $\lim_{x\to 0^+}(x\ln x-x^2+2)$
តាមរូបមន្តគ្រឹះ $\lim_{x\to 0^+} x\ln x=0$។
ជំនួសចូល ៖ $0-0+2=2$។ លីមីតស្មើនឹង ២។

ឈ. $\lim_{x\to 0^+}\frac{x^2\ln x+2}{\ln x+1}$
ចែកភាគយក និងភាគបែងនឹង $\ln x$ ៖

$$\lim_{x\to 0^+}\frac{x^2+\frac{2}{\ln x}}{1+\frac{1}{\ln x}}$$

ដោយ $\ln x\to-\infty$ នោះ $\frac{2}{\ln x}\to 0$។

$$=\frac{0+0}{1+0}=0$$

ញ. $\lim_{x\to 0^+}\left(\frac{1}{x}-\frac{\ln x}{x}\right)$

$$\begin{aligned}&=\lim_{x\to 0^+}\frac{1-\ln x}{x}\\&=\frac{1-(-\infty)}{0^+}\\&=\frac{+\infty}{0^+}\\&=+\infty\end{aligned}$$

ដ. $\lim_{x\to+\infty}\frac{e^x\ln x+1}{x^2}$
បំបែកភាគបែង ៖

$$\lim_{x\to+\infty}\left(\frac{e^x}{x^2}\ln x+\frac{1}{x^2}\right)$$

ដោយ $\frac{e^x}{x^2}\to+\infty$ និង $\ln x\to+\infty$ ៖

$$(+\infty)(+\infty)+0=+\infty$$

ឋ. $\lim_{x\to 0}\frac{\ln(1+x)}{1-e^x}$
ចែកភាគយកនិងភាគបែងនឹង x ៖

$$\lim_{x\to 0}\frac{\frac{\ln(1+x)}{x}}{-\frac{e^x-1}{x}}=\frac{1}{-1}$$
$$=-1$$

ឌ. $\lim\limits_{x\to 1}\frac{x^2-1+\ln x}{e^x-e}$

តាង $t=x-1 \implies x=t+1$។

$$\lim_{t\to 0}\frac{(t+1)^2-1+\ln(t+1)}{e^{t+1}-e} = \lim_{t\to 0}\frac{t^2+2t+\ln(1+t)}{e(e^t-1)}$$

ចែកភាគយក និងភាគបែងនឹង t ៖

$$\lim_{t\to 0}\frac{t+2+\frac{\ln(1+t)}{t}}{e\frac{e^t-1}{t}} = \frac{0+2+1}{e(1)} = \frac{3}{e}$$

ណ. $\lim\limits_{x\to 0}\frac{\ln(1+x)}{e^{2x}-1}$

ចែកភាគយកនិងភាគបែងនឹង x ៖

$$\lim \frac{\frac{\ln(1+x)}{x}}{\frac{e^{2x}-1}{2x}\cdot 2} = \frac{1}{1\cdot 2} = \frac{1}{2}$$

ថ. $\lim\limits_{x\to 0}\frac{\ln(1+2x^2)}{1-\cos 2x}$

ប្រើរូបមន្ត $1-\cos 2x = 2\sin^2 x$ ៖

$$\lim \frac{\ln(1+2x^2)}{2x^2}\cdot\frac{2x^2}{2\sin^2 x} = 1\cdot\lim\left(\frac{x}{\sin x}\right)^2 = 1\cdot 1^2 = 1$$

ធ. $\lim\limits_{x\to 0}\frac{\ln(x^2-x+1)}{x}$

តាង $u=x^2-x$។ ពេល $x\to 0, u\to 0$។

$$\lim_{x\to 0}\frac{\ln(1+(x^2-x))}{x^2-x}\cdot\frac{x^2-x}{x} = \lim 1\cdot(x-1) = 1(-1) = -1$$

ឍ. $\lim\limits_{x\to+\infty}[x+3-3\ln(e^x+1)]$

ទាញ e^x ជាកត្តាក្នុងលោការីត ៖

$$\begin{aligned}&\lim[x+3-3\ln(e^x(1+e^{-x}))]\\ &=\lim[x+3-3(x+\ln(1+e^{-x}))]\\ &=\lim[x+3-3x-3\ln(1+e^{-x})]\\ &=\lim[-2x+3-3\ln(1+e^{-x})]\end{aligned}$$

ដោយ $e^{-x}\to 0$ នោះ $\ln(1)=0$។

លីមីតស្មើនឹង $-\infty+3-0=-\infty$។

ត. $\lim\limits_{x\to+\infty}\left(x+\frac{\ln x}{2x+1}\right)$

$$=\lim_{x\to+\infty}x+\lim_{x\to+\infty}\frac{\ln x}{x(2+\frac{1}{x})} = +\infty+0 = +\infty$$

ទ. $\lim\limits_{x\to 0}\frac{\ln(\cos 2x)}{x^2}$

តាង $\cos 2x = 1+(\cos 2x-1)$ ៖

$$\lim_{x\to 0}\frac{\ln(1+(\cos 2x-1))}{\cos 2x-1}\cdot\frac{\cos 2x-1}{x^2}$$

តួទី១ ខិតជិត ១។ តួទី២ គឺ $\frac{-2\sin^2 x}{x^2} = -2(1)^2 = -2$។

លីមីតស្មើនឹង $1\cdot(-2)=-2$។

ន. $\lim\limits_{x\to 0}\frac{\ln(1+x)+\ln(1+2x)}{x}$

បំបែកជាពីរ ៖

$$\lim_{x\to 0}\left(\frac{\ln(1+x)}{x}+\frac{\ln(1+2x)}{2x}\cdot 2\right) = 1+1(2) = 3$$

មេរៀនទី 5 ដំណោះស្រាយលំហាត់ប្រតិបត្តិ

ដំណោះស្រាយលំហាត់ទី១៦

ប្រធាន៖ ចូរដោះស្រាយលីមីតខាងក្រោមឲ្យបានត្រឹមត្រូវ៖

ក. $\lim\limits_{n\to+\infty}\left(\frac{1+n}{n}\right)^{n+2020}$

បំបែកប្រភាគនៅគោល ៖ $\frac{1+n}{n}=\frac{1}{n}+\frac{n}{n}=1+\frac{1}{n}$ ។
លីមីតក្លាយជា ៖

$$\lim_{n\to+\infty}\left(1+\frac{1}{n}\right)^{n+2020}$$

យើងអាចបំបែកស្វ័យគុណជាផលគុណ ៖

$$=\lim_{n\to+\infty}\left[\left(1+\frac{1}{n}\right)^{n}\cdot\left(1+\frac{1}{n}\right)^{2020}\right]$$

$$=\lim_{n\to+\infty}\left(1+\frac{1}{n}\right)^{n}\cdot\lim_{n\to+\infty}\left(1+\frac{1}{n}\right)^{2020}$$

តាមរូបមន្តគ្រឹះ $\lim\limits_{n\to\infty}\left(1+\frac{1}{n}\right)^{n}=e$ ។
ចំណែកតួទីពីរ ជំនួស $n\to\infty$ គេបាន $(1+0)^{2020}=1^{2020}=1$ ។
ដូចនេះ លីមីតស្មើនឹង $e\cdot 1=e$ ។

ខ. $\lim\limits_{x\to+\infty}\left(\frac{2x+3}{2x+1}\right)^{x+1}$

បំបែកភាគយកដើម្បីឲ្យដូចភាគបែង ៖ $2x+3=(2x+1)+2$ ។

$$\begin{aligned}\frac{2x+3}{2x+1}&=\frac{2x+1+2}{2x+1}\\&=1+\frac{2}{2x+1}\end{aligned}$$

ជំនួសចូលលីមីត គេបានរាងមិនកំណត់ 1^{∞} ៖

$$\lim_{x\to+\infty}\left(1+\frac{2}{2x+1}\right)^{x+1}$$

ដើម្បី ប្រើ រូបមន្ត $\lim\limits_{u\to0}(1+u)^{1/u}=e$ យើង រៀបចំស្វ័យគុណឲ្យទៅជាចម្រាសនៃ $\frac{2}{2x+1}$ គឺ $\frac{2x+1}{2}$ ៖

$$=\lim_{x\to+\infty}\left[\left(1+\frac{2}{2x+1}\right)^{\frac{2x+1}{2}}\right]^{\frac{2}{2x+1}\cdot(x+1)}$$

តួក្នុងវង់ក្រចកជ្រុងមានលីមីតស្មើ e ។ ឥឡូវគណនាលីមីតនៃស្វ័យគុណ ៖

$$\begin{aligned}\lim_{x\to+\infty}\frac{2(x+1)}{2x+1}&=\lim_{x\to+\infty}\frac{2x+2}{2x+1}\\&=\frac{2}{2}\\&=1\end{aligned}$$

ដូចនេះ លីមីតស្មើនឹង $e^{1}=e$ ។

គ. $\lim\limits_{x\to+\infty}\left(1+\frac{1}{x}\right)^{\frac{x+1}{x}}$

បំបែកស្វ័យគុណ ៖ $\frac{x+1}{x}=1+\frac{1}{x}$ ។

$$\lim_{x\to+\infty}\left(1+\frac{1}{x}\right)^{1+\frac{1}{x}}$$

ជំនួស $x\to+\infty$ ចូល ៖
មូលដ្ឋានខិតជិត $1+0=1$។
ស្វ័យគុណខិតជិត $1+0=1$។
វាជារាងកំណត់ 1^1 មិនមែន 1^∞ ទេ។
ដូចនេះ លីមីតស្មើនឹង $1^1=1$ ។

ឃ. $\lim\limits_{x\to\infty}\left(1+\frac{1}{x}\right)^{3x}$

រាងមិនកំណត់ 1^∞ ។
យើងបំបែកស្វ័យគុណ $3x$ ជា $x\cdot 3$ ៖

$$\lim_{x\to\infty}\left[\left(1+\frac{1}{x}\right)^{x}\right]^{3}$$

ដោយ $\lim\limits_{x\to\infty}\left(1+\frac{1}{x}\right)^{x}=e$ នោះលីមីតស្មើនឹង e^3 ។

ង. $\lim\limits_{x\to\infty}\left(1+\frac{1}{x^2}\right)^{3x-4}$

រាងមិនកំណត់ 1^∞ ។ រៀបចំស្វ័យគុណឲ្យទៅជាច្រាស នៃ $\frac{1}{x^2}$ គឺ x^2 ៖

$$\lim_{x\to\infty}\left[\left(1+\frac{1}{x^2}\right)^{x^2}\right]^{\frac{1}{x^2}\cdot(3x-4)}$$

$$=\lim_{x\to\infty}\left[\left(1+\frac{1}{x^2}\right)^{x^2}\right]^{\frac{3x-4}{x^2}}$$

គណនាលីមីតស្វ័យគុណ ៖ $\lim\limits_{x\to\infty}\frac{3x-4}{x^2}=0$ ព្រោះដឺក្រេភាគបែងធំជាងភាគយក។
ដូចនេះ លីមីតសរុបស្មើនឹង $e^0=1$ ។

ច. $\lim\limits_{x\to\infty}\left(1+\frac{1}{5x}\right)^{2x+6}$

រាងមិនកំណត់ 1^∞ ។ រៀបចំស្វ័យគុណឲ្យមាន $5x$ ៖

$$\lim_{x\to\infty}\left[\left(1+\frac{1}{5x}\right)^{5x}\right]^{\frac{1}{5x}\cdot(2x+6)}$$

គណនាលីមីតស្វ័យគុណ ៖ $\lim\limits_{x\to\infty}\frac{2x+6}{5x}=\frac{2}{5}$ ។
ដូចនេះ លីមីតស្មើនឹង $e^{\frac{2}{5}}$ ។

ឆ. $\lim\limits_{x\to\infty}\left(1+\frac{7}{3x}\right)^{x-1}$

រាងមិនកំណត់ 1^∞ ។ រៀបចំស្វ័យគុណឲ្យមាន $\frac{3x}{7}$ ៖

$$\lim_{x\to\infty}\left[\left(1+\frac{7}{3x}\right)^{\frac{3x}{7}}\right]^{\frac{7}{3x}\cdot(x-1)}$$

គណនាលីមីតស្វ័យគុណ ៖ $\lim\limits_{x\to\infty}\frac{7(x-1)}{3x}=\lim\limits_{x\to\infty}\frac{7x-7}{3x}=\frac{7}{3}$ ។
ដូចនេះ លីមីតស្មើនឹង $e^{\frac{7}{3}}$ ។

ជ. $\lim\limits_{x\to\infty}\left(1-\frac{1}{3x}\right)^{x}$

រាងមិនកំណត់ 1^∞ ។ រៀបចំស្វ័យគុណឲ្យមាន $-3x$ ៖

$$\lim_{x\to\infty}\left[\left(1+\frac{-1}{3x}\right)^{-3x}\right]^{\frac{1}{-3x}\cdot x}$$

គណនាលីមីតស្វ័យគុណ ៖ $\lim\limits_{x\to\infty}\frac{x}{-3x}=-\frac{1}{3}$ ។
ដូចនេះ លីមីតស្មើនឹង $e^{-\frac{1}{3}}$ ។

ឈ. $\lim_{x\to\infty}\left(1-\frac{5}{x}\right)^x$

រាងមិនកំណត់ 1^∞ ។ រៀបចំស្វ័យគុណឲ្យមាន $-\frac{x}{5}$ ៖

$$\lim_{x\to\infty}\left[\left(1+\frac{-5}{x}\right)^{-\frac{x}{5}}\right]^{-\frac{5}{x}\cdot x}$$

គណនាលីមីតស្វ័យគុណ ៖ $\lim_{x\to\infty}\frac{-5x}{x}=-5$ ។

ដូចនេះ លីមីតស្មើនឹង e^{-5} ។

ញ. $\lim_{x\to\infty}\left(\frac{x-1}{x+1}\right)^x$

បំបែកភាគយក ៖ $\frac{x-1}{x+1}=\frac{(x+1)-2}{x+1}=1-\frac{2}{x+1}$ ។

រាងមិនកំណត់ 1^∞ ។ រៀបចំស្វ័យគុណឲ្យមាន $-\frac{x+1}{2}$ ៖

$$\lim_{x\to\infty}\left[\left(1+\frac{-2}{x+1}\right)^{-\frac{x+1}{2}}\right]^{\frac{-2}{x+1}\cdot x}$$

គណនាលីមីតស្វ័យគុណ ៖ $\lim_{x\to\infty}\frac{-2x}{x+1}=-2$ ។

ដូចនេះ លីមីតស្មើនឹង e^{-2} ។

ដ. $\lim_{x\to\infty}\left(\frac{x+6}{x+5}\right)^x$

បំបែកភាគយក ៖ $\frac{x+6}{x+5}=\frac{(x+5)+1}{x+5}=1+\frac{1}{x+5}$ ។

រាងមិនកំណត់ 1^∞ ។ រៀបចំស្វ័យគុណឲ្យមាន $x+5$ ៖

$$\lim_{x\to\infty}\left[\left(1+\frac{1}{x+5}\right)^{x+5}\right]^{\frac{1}{x+5}\cdot x}$$

គណនាលីមីតស្វ័យគុណ ៖ $\lim_{x\to\infty}\frac{x}{x+5}=1$ ។

ដូចនេះ លីមីតស្មើនឹង $e^1=e$ ។

ឋ. $\lim_{x\to\infty}\left(\frac{x-3}{x}\right)^{\frac{x}{2}}$

បំបែកប្រភាគ ៖ $\frac{x-3}{x}=1-\frac{3}{x}$ ។

រាងមិនកំណត់ 1^∞ ។ រៀបចំស្វ័យគុណឲ្យមាន $-\frac{x}{3}$ ៖

$$\lim_{x\to\infty}\left[\left(1+\frac{-3}{x}\right)^{-\frac{x}{3}}\right]^{-\frac{3}{x}\cdot\frac{x}{2}}$$

គណនាលីមីតស្វ័យគុណ ៖ $\lim_{x\to\infty}\frac{-3x}{2x}=-\frac{3}{2}$ ។

ដូចនេះ លីមីតស្មើនឹង $e^{-\frac{3}{2}}$ ។

ឌ. $\lim_{x\to\infty}\left(\frac{7x+10}{1+7x}\right)^{\frac{x}{3}}$

បំបែកភាគយក ៖ $\frac{7x+10}{7x+1}=\frac{(7x+1)+9}{7x+1}=1+\frac{9}{7x+1}$ ។

រាងមិនកំណត់ 1^∞ ។ រៀបចំស្វ័យគុណ ៖

$$\lim_{x\to\infty}\left[\left(1+\frac{9}{7x+1}\right)^{\frac{7x+1}{9}}\right]^{\frac{9}{7x+1}\cdot\frac{x}{3}}$$

គណនាលីមីតស្វ័យគុណ ៖ $\lim_{x\to\infty}\frac{9x}{3(7x+1)}=\lim_{x\to\infty}\frac{3x}{7x+1}=\frac{3}{7}$ ។

ដូចនេះ លីមីតស្មើនឹង $e^{\frac{3}{7}}$ ។

ឍ. $\lim_{x\to\infty}\left(\frac{2x-1}{2x+1}\right)^x$

បំបែកភាគយក ៖ $\frac{2x-1}{2x+1}=\frac{(2x+1)-2}{2x+1}=1-\frac{2}{2x+1}$ ។

រៀបចំស្វ័យគុណ ៖

$$\lim_{x\to\infty}\left[\left(1+\frac{-2}{2x+1}\right)^{-\frac{2x+1}{2}}\right]^{\frac{-2}{2x+1}\cdot x}$$

គណនាលីមីតស្វ័យគុណ ៖ $\lim_{x\to\infty}\frac{-2x}{2x+1}=-1$ ។

ដូចនេះ លីមីតស្មើនឹង e^{-1} ។

ណ. $\lim_{x\to\infty}\left(\frac{6+4x}{2+4x}\right)^{3-2x}$
បំបែកភាគយក ៖ $\frac{4x+6}{4x+2}=\frac{(4x+2)+4}{4x+2}=1+\frac{4}{4x+2}$ ។
រៀបចំស្វ័យគុណ ៖

$$\lim_{x\to\infty}\left[\left(1+\frac{4}{4x+2}\right)^{\frac{4x+2}{4}}\right]^{\frac{4}{4x+2}\cdot(3-2x)}$$

គណនាលីមីតស្វ័យគុណ ៖ $\lim_{x\to\infty}\frac{12-8x}{4x+2}=\frac{-8}{4}=-2$ ។
ដូចនេះ លីមីតស្មើនឹង e^{-2} ។

ត. $\lim_{x\to\infty}\left(\frac{x+5}{x+4}\right)^{2x-1}$
បំបែកភាគយក ៖ $1+\frac{1}{x+4}$ ។
រៀបចំស្វ័យគុណ ៖

$$\lim_{x\to\infty}\left[\left(1+\frac{1}{x+4}\right)^{x+4}\right]^{\frac{1}{x+4}\cdot(2x-1)}$$

គណនាលីមីតស្វ័យគុណ ៖ $\lim_{x\to\infty}\frac{2x-1}{x+4}=2$ ។
ដូចនេះ លីមីតស្មើនឹង e^{2} ។

ថ. $\lim_{x\to\infty}\left(\frac{2x+5}{2x}\right)^{3x-7}$
បំបែកប្រភាគ ៖ $1+\frac{5}{2x}$ ។
រៀបចំស្វ័យគុណ ៖

$$\lim_{x\to\infty}\left[\left(1+\frac{5}{2x}\right)^{\frac{2x}{5}}\right]^{\frac{5}{2x}\cdot(3x-7)}$$

គណនាលីមីតស្វ័យគុណ ៖ $\lim_{x\to\infty}\frac{15x-35}{2x}=\frac{15}{2}$ ។
ដូចនេះ លីមីតស្មើនឹង $e^{\frac{15}{2}}$ ។

ទ. $\lim_{x\to\infty}\left(\frac{2x+1}{2x-3}\right)^{3x}$
បំបែកភាគយក ៖ $1+\frac{4}{2x-3}$ ។
រៀបចំស្វ័យគុណ ៖

$$\lim_{x\to\infty}\left[\left(1+\frac{4}{2x-3}\right)^{\frac{2x-3}{4}}\right]^{\frac{4}{2x-3}\cdot 3x}$$

គណនាលីមីតស្វ័យគុណ ៖ $\lim_{x\to\infty}\frac{12x}{2x-3}=6$ ។
ដូចនេះ លីមីតស្មើនឹង e^{6} ។

ធ. $\lim_{x\to\infty}\left(\frac{x-1}{x+3}\right)^{5-4x}$
បំបែកភាគយក ៖ $1-\frac{4}{x+3}$ ។
រៀបចំស្វ័យគុណ ៖

$$\lim_{x\to\infty}\left[\left(1+\frac{-4}{x+3}\right)^{-\frac{x+3}{4}}\right]^{\frac{-4}{x+3}\cdot(5-4x)}$$

គណនាលីមីតស្វ័យគុណ ៖ $\lim_{x\to\infty}\frac{-20+16x}{x+3}=16$ ។
ដូចនេះ លីមីតស្មើនឹង e^{16} ។

ន. $\lim_{x\to\infty}\left(\frac{x}{x+1}\right)^{x}$
បំបែកភាគយក ៖ $1-\frac{1}{x+1}$ ។
រៀបចំស្វ័យគុណ ៖

$$\lim_{x\to\infty}\left[\left(1+\frac{-1}{x+1}\right)^{-(x+1)}\right]^{\frac{-1}{x+1}\cdot x}$$

គណនាលីមីតស្វ័យគុណ ៖ $\lim_{x\to\infty}\frac{-x}{x+1}=-1$ ។
ដូចនេះ លីមីតស្មើនឹង e^{-1} ។

ប. $\lim_{x\to\infty}\left(\frac{3x-2}{3x+1}\right)^{2x}$

បំបែកភាគយក ៖ $1-\frac{3}{3x+1}$ ។

រៀបចំស្វ័យគុណ ៖

$$\lim_{x\to\infty}\left[\left(1+\frac{-3}{3x+1}\right)^{-\frac{3x+1}{3}}\right]^{\frac{-3}{3x+1}\cdot 2x}$$

គណនាលីមីតស្វ័យគុណ ៖ $\lim_{x\to\infty}\frac{-6x}{3x+1}=-2$ ។

ដូចនេះ លីមីតស្មើនឹង e^{-2} ។

ផ. $\lim_{x\to\infty}\left(\frac{2x-5}{2x-2}\right)^{4x^2}$

បំបែកភាគយក ៖ $1-\frac{3}{2x-2}$ ។

រៀបចំស្វ័យគុណ ៖

$$\lim_{x\to\infty}\left[\left(1+\frac{-3}{2x-2}\right)^{-\frac{2x-2}{3}}\right]^{\frac{-3}{2x-2}\cdot 4x^2}$$

គណនាលីមីតស្វ័យគុណ ៖ $\lim_{x\to\infty}\frac{-12x^2}{2x-2}=-\infty$ (ព្រោះដឺក្រេភាគយកធំជាងភាគបែង)។

ដូចនេះ លីមីតស្មើនឹង $e^{-\infty}=0$ ។

ព. $\lim_{x\to\infty}\left(\frac{3x+6}{3x-1}\right)^{x^2}$

បំបែកភាគយក ៖ $1+\frac{7}{3x-1}$ ។

រៀបចំស្វ័យគុណ ៖

$$\lim_{x\to\infty}\left[\left(1+\frac{7}{3x-1}\right)^{\frac{3x-1}{7}}\right]^{\frac{7}{3x-1}\cdot x^2}$$

គណនាលីមីតស្វ័យគុណ ៖ $\lim_{x\to\infty}\frac{7x^2}{3x-1}=+\infty$ ។

ដូចនេះ លីមីតស្មើនឹង $e^{+\infty}=+\infty$ ។

ភ. $\lim_{x\to\infty}\left(\frac{x^2+2x+1}{x^2+3}\right)^{x}$

បំបែកភាគយក ៖ $\frac{(x^2+3)+2x-2}{x^2+3}=1+\frac{2x-2}{x^2+3}$ ។

រៀបចំស្វ័យគុណ ៖

$$\lim_{x\to\infty}\left[\left(1+\frac{2x-2}{x^2+3}\right)^{\frac{x^2+3}{2x-2}}\right]^{\frac{2x-2}{x^2+3}\cdot x}$$

គណនាលីមីតស្វ័យគុណ ៖ $\lim_{x\to\infty}\frac{2x^2-2x}{x^2+3}=2$ ។

ដូចនេះ លីមីតស្មើនឹង e^{2} ។

ម. $\lim_{x\to 0}(1+2x)^{\frac{1}{x}}$

រាងមិនកំណត់ 1^{∞} ពេល $x\to 0$ ។

រៀបចំស្វ័យគុណ ៖

$$\lim_{x\to 0}\left[(1+2x)^{\frac{1}{2x}}\right]^{2}$$

ដោយ $\lim_{u\to 0}(1+u)^{1/u}=e$ នោះលីមីតស្មើនឹង e^{2} ។

យ. $\lim_{x\to 0}(1+5x)^{\frac{1}{8x}}$

រៀបចំស្វ័យគុណ ៖

$$\lim_{x\to 0}\left[(1+5x)^{\frac{1}{5x}}\right]^{\frac{5x}{8x}}$$

គណនាលីមីតស្វ័យគុណ ៖ $\lim_{x\to 0}\frac{5x}{8x}=\frac{5}{8}$ ។

ដូចនេះ លីមីតស្មើនឹង $e^{\frac{5}{8}}$ ។

រ. $\lim\limits_{x\to\infty}\ln\left(\frac{3x+1}{3x-5}\right)^{2-x}$

គណនាលីមីតនៃកន្សោមក្នុងលោការីតសិន ៖

$$\lim_{x\to\infty}\left(\frac{3x+1}{3x-5}\right)^{2-x}$$

បំបែក ៖ $\frac{(3x-5)+6}{3x-5}=1+\frac{6}{3x-5}$ ។

$$\lim_{x\to\infty}\left[\left(1+\frac{6}{3x-5}\right)^{\frac{3x-5}{6}}\right]^{\frac{6}{3x-5}\cdot(2-x)}$$

លីមីតស្វ័យគុណ ៖ $\lim\limits_{x\to\infty}\frac{12-6x}{3x-5}=-2$ ។

ដូចនេះកន្សោមក្នុងលោការីតមានលីមីតស្មើ e^{-2} ។

នាំឲ្យលីមីតនៃលោការីតគឺ $\ln(e^{-2})=-2$ ។

វ. $\lim\limits_{x\to\infty}x[\ln x-\ln(x+2)]$

ធ្វើដូចខាងលើ ៖

$$=\lim_{x\to\infty}\ln\left(\frac{x}{x+2}\right)^{x}$$

$$=\ln\left[\lim_{x\to\infty}\left(1-\frac{2}{x+2}\right)^{x}\right]$$

គណនាលីមីតខាងក្នុង ៖

$$\lim_{x\to\infty}\left[\left(1+\frac{-2}{x+2}\right)^{-\frac{x+2}{2}}\right]^{\frac{-2}{x+2}\cdot x}=e^{\lim\frac{-2x}{x+2}}$$

$$=e^{-2}$$

ដូចនេះ លីមីតសរុបគឺ $\ln(e^{-2})=-2$ ។

ល. $\lim\limits_{x\to\infty}x[\ln(x+3)-\ln x]$

ប្រើ លក្ខណៈ លោការីត $\ln A-\ln B=\ln(A/B)$ និង $n\ln A=\ln(A^n)$ ៖

$$=\lim_{x\to\infty}\ln\left(\frac{x+3}{x}\right)^{x}$$

$$=\ln\left[\lim_{x\to\infty}\left(1+\frac{3}{x}\right)^{x}\right]$$

លីមីតខាងក្នុងគឺ ៖

$$\lim_{x\to\infty}\left[\left(1+\frac{3}{x}\right)^{\frac{x}{3}}\right]^{3}$$

$$=e^{3}$$

ដូចនេះ លីមីតសរុបគឺ $\ln(e^{3})=3$ ។

ស. $\lim\limits_{x\to\infty}(x+1)[\ln(x+1)-\ln x]$

ធ្វើដូចខាងលើ ៖

$$=\lim_{x\to\infty}\ln\left(\frac{x+1}{x}\right)^{x+1}$$

$$=\ln\left[\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^{x+1}\right]$$

លីមីតខាងក្នុងគឺ ៖

$$\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^{x}\cdot\left(1+\frac{1}{x}\right)^{1}=e\cdot 1$$

$$=e$$

ដូចនេះ លីមីតសរុបគឺ $\ln(e)=1$ ។

ដំណោះស្រាយលំហាត់ទី១៧

ប្រធានៈ ចូរដោះស្រាយលីមីតខាងក្រោមឲ្យបានត្រឹមត្រូវ៖

ក. $\lim\limits_{x\to 0^+}(1+2x)^{\cot x}$
កាលណា $x\to 0^+$ នោះ $1+2x\to 1$ និង $\cot x\to +\infty$ នាំឲ្យមានរាងមិនកំណត់ 1^∞ ។
យើងប្រើរូបមន្ត $\lim\limits_{u\to 0}(1+u)^{1/u}=e$ ៖

$$\begin{aligned}&\lim_{x\to 0^+}(1+2x)^{\cot x}\\&=\lim_{x\to 0^+}\left[(1+2x)^{\frac{1}{2x}}\right]^{2x\cot x}\end{aligned}$$

ដោយ $\lim\limits_{x\to 0}(1+2x)^{\frac{1}{2x}}=e$ ។ យើង គណនា លីមីត ស្វ័យគុណបន្ត ៖

$$\begin{aligned}\lim_{x\to 0^+}2x\cot x&=\lim_{x\to 0^+}\frac{2x}{\tan x}\\&=\lim_{x\to 0^+}2\left(\frac{x}{\tan x}\right)\\&=2(1)\\&=2\end{aligned}$$

ដូចនេះ លីមីតស្មើនឹង e^2 ។

ខ. $\lim\limits_{x\to 0}(1+\sin x)^{\frac{1}{x}}$
កាលណា $x\to 0$ នោះ $1+\sin x\to 1$ និង $\frac{1}{x}\to\infty$ ។ នេះជារាងមិនកំណត់ 1^∞ ។
យើងរៀបចំស្វ័យគុណឲ្យមាន $\frac{1}{\sin x}$ ៖

$$\begin{aligned}&\lim_{x\to 0}(1+\sin x)^{\frac{1}{x}}\\&=\lim_{x\to 0}\left[(1+\sin x)^{\frac{1}{\sin x}}\right]^{\frac{\sin x}{x}}\end{aligned}$$

តួក្នុងវង់ក្រចកជ្រុងមានលីមីតស្មើ e (ព្រោះ $\sin x\to 0$) ។
លីមីតស្វ័យគុណគឺ $\lim\limits_{x\to 0}\frac{\sin x}{x}=1$ ។
ដូចនេះ លីមីតស្មើនឹង $e^1=e$ ។

គ. $\lim\limits_{x\to 0}(2-e^x)^{\frac{1}{x}}$
កាលណា $x\to 0$ នោះ $2-e^0=2-1=1$ និង $\frac{1}{x}\to\infty$ ។ រាងមិនកំណត់ 1^∞ ។
យើងបំបែកមូលដ្ឋាន ៖ $2-e^x=1+(1-e^x)$ ។

$$\begin{aligned}&\lim_{x\to 0}(1+(1-e^x))^{\frac{1}{x}}\\&=\lim_{x\to 0}\left[(1+(1-e^x))^{\frac{1}{1-e^x}}\right]^{\frac{1-e^x}{x}}\end{aligned}$$

តួក្នុងវង់ក្រចកជ្រុងមានលីមីតស្មើ e ។
លីមីតស្វ័យគុណគឺ ៖

$$\begin{aligned}\lim_{x\to 0}\frac{1-e^x}{x}&=-\lim_{x\to 0}\frac{e^x-1}{x}\\&=-1\end{aligned}$$

ដូចនេះ លីមីតស្មើនឹង $e^{-1}=\frac{1}{e}$ ។

ឃ. $\lim\limits_{x\to 0}(1+\sin x)^{\cot x}$
ជារាងមិនកំណត់ 1^∞ ព្រោះ $\cot 0\to\infty$ ។
រៀបចំស្វ័យគុណ ៖

$$=\lim_{x\to 0}\left[(1+\sin x)^{\frac{1}{\sin x}}\right]^{\sin x\cdot\cot x}$$

ដោយ $\sin x\cdot\cot x=\sin x\cdot\frac{\cos x}{\sin x}=\cos x$ ។
គណនាលីមីតស្វ័យគុណ ៖ $\lim\limits_{x\to 0}\cos x=\cos 0=1$ ។
ដូចនេះ លីមីតស្មើនឹង $e^1=e$ ។

ង. $\lim_{x\to 0}(\cos x)^{\frac{1}{\sin x}}$

ជារាងមិនកំណត់ 1^∞ ។

បំបែកមូលដ្ឋាន ៖ $\cos x = 1 + (\cos x - 1)$ ។

$$= \lim_{x\to 0}\left[(1+(\cos x-1))^{\frac{1}{\cos x-1}}\right]^{\frac{\cos x-1}{\sin x}}$$

គណនាលីមីតស្វ័យគុណ ៖

$$\begin{aligned}\lim_{x\to 0}\frac{\cos x-1}{\sin x} &= \lim_{x\to 0}\frac{-2\sin^2(x/2)}{2\sin(x/2)\cos(x/2)}\\ &= \lim_{x\to 0}\frac{-\sin(x/2)}{\cos(x/2)}\\ &= -\tan 0\\ &= 0\end{aligned}$$

ដូចនេះ លីមីតស្មើនឹង $e^0 = 1$ ។

ច. $\lim_{x\to 0}(1+3\tan^2 x)^{\cot^2 x}$

ជារាងមិនកំណត់ 1^∞ ។

រៀបចំស្វ័យគុណ ៖

$$= \lim_{x\to 0}\left[(1+3\tan^2 x)^{\frac{1}{3\tan^2 x}}\right]^{3\tan^2 x\cdot\cot^2 x}$$

ដោយ $\tan^2 x\cdot\cot^2 x = 1$ ។ នោះលីមីតស្វ័យគុណគឺ $3(1) = 3$ ។

ដូចនេះ លីមីតស្មើនឹង e^3 ។

ឆ. $\lim_{x\to 0}\sqrt[\sin x]{1+x}$

សរសេរ៉ាឌីកាល់ជាស្វ័យគុណប្រភាគ ៖ $\sqrt[\sin x]{1+x} = (1+x)^{\frac{1}{\sin x}}$ ។

ជារាងមិនកំណត់ 1^∞ ។

រៀបចំស្វ័យគុណ ៖

$$= \lim_{x\to 0}\left[(1+x)^{\frac{1}{x}}\right]^{\frac{x}{\sin x}}$$

តួក្នុងវង់ក្រចកជ្រុងមានលីមីតស្មើ e ។

លីមីតស្វ័យគុណ ៖ $\lim_{x\to 0}\frac{x}{\sin x} = 1$ ។

ដូចនេះ លីមីតស្មើនឹង $e^1 = e$ ។

ជ. $\lim_{x\to 0}\left(\frac{2x+3}{2x+1}\right)^{\frac{1}{\sin 3x}}$

ជំនួស $x = 0$ ៖ មូលដ្ឋានខិតជិត $\frac{3}{1} = 3$ ។

ចំណែកស្វ័យគុណ $\frac{1}{\sin 3x} \to \infty$ ។

នេះមិនមែនជារាងមិនកំណត់ 1^∞ ទេ តែជារាង 3^∞ ។

- បើ $x \to 0^+$ នោះ $\sin 3x \to 0^+$ នាំឲ្យស្វ័យគុណទៅ $+\infty$ ។ ដូច្នេះ $3^{+\infty} = +\infty$ ។
- បើ $x \to 0^-$ នោះ $\sin 3x \to 0^-$ នាំឲ្យស្វ័យគុណទៅ $-\infty$ ។ ដូច្នេះ $3^{-\infty} = 0$ ។

ដោយសារលីមីតឆ្វេង និងស្តាំមិនស្មើគ្នា ដូចនេះលីមីតនេះ **គ្មានតម្លៃ** ទេ ។

ឈ. $\lim\limits_{x\to 0}(x+2^x)^{\frac{1}{x}}$

ជំនួស $x=0$ ៖ ជារាងមិនកំណត់ 1^∞ ។

បំបែកមូលដ្ឋាន ៖ $x+2^x=1+(x+2^x-1)$ ។

$$=\lim_{x\to 0}\left[(1+(x+2^x-1))^{\frac{1}{x+2^x-1}}\right]^{\frac{x+2^x-1}{x}}$$

គណនាលីមីតស្វ័យគុណ ៖

$$\begin{aligned}\lim_{x\to 0}\frac{x+2^x-1}{x} &= \lim_{x\to 0}\left(\frac{x}{x}+\frac{2^x-1}{x}\right)\\ &= 1+\ln 2\end{aligned}$$

ដូចនេះ លីមីតសរុបគឺ $e^{1+\ln 2}=e^1\cdot e^{\ln 2}=e\cdot 2=2e$ ។

ញ. $\lim\limits_{x\to 0}(\cos x+\sin x)^{\frac{1}{x}}$

ជារាងមិនកំណត់ 1^∞ ។

បំបែកមូលដ្ឋាន ៖ $\cos x+\sin x=1+(\cos x-1+\sin x)$ ។

$$=\lim_{x\to 0}\left[(1+(\cos x-1+\sin x))^{\frac{1}{\cos x-1+\sin x}}\right]^{\frac{\cos x-1+\sin x}{x}}$$

គណនាលីមីតស្វ័យគុណ ៖

$$\begin{aligned}\lim_{x\to 0}\left(\frac{\cos x-1}{x}+\frac{\sin x}{x}\right) &= 0+1\\ &= 1\end{aligned}$$

ដូចនេះ លីមីតស្មើនឹង $e^1=e$ ។

ដ. $\lim\limits_{x\to 0}(1+\tan^2\sqrt{x})^{\frac{1}{2x}}$

ជារាងមិនកំណត់ 1^∞ ។

រៀបចំស្វ័យគុណ ៖

$$=\lim_{x\to 0}\left[(1+\tan^2\sqrt{x})^{\frac{1}{\tan^2\sqrt{x}}}\right]^{\frac{\tan^2\sqrt{x}}{2x}}$$

គណនាលីមីតស្វ័យគុណ ៖

$$\begin{aligned}\lim_{x\to 0}\frac{\tan^2\sqrt{x}}{2x} &= \frac{1}{2}\lim_{x\to 0}\left(\frac{\tan\sqrt{x}}{\sqrt{x}}\right)^2\\ &= \frac{1}{2}(1)^2\\ &= \frac{1}{2}\end{aligned}$$

ដូចនេះ លីមីតស្មើនឹង $e^{1/2}=\sqrt{e}$ ។

ឋ. $\lim\limits_{x\to 0}\left(\frac{x^2-x+1}{x^2+x+1}\right)^{\frac{1}{\sin x}}$

ជារាងមិនកំណត់ 1^∞ ។

បំបែកមូលដ្ឋាន ៖ $\frac{x^2-x+1}{x^2+x+1}=\frac{x^2+x+1-2x}{x^2+x+1}=1+\frac{-2x}{x^2+x+1}$ ។

រៀបចំស្វ័យគុណ ៖

$$=\lim_{x\to 0}\left[\left(1+\frac{-2x}{x^2+x+1}\right)^{\frac{x^2+x+1}{-2x}}\right]^{\frac{-2x}{x^2+x+1}\cdot\frac{1}{\sin x}}$$

គណនាលីមីតស្វ័យគុណ ៖

$$\begin{aligned}\lim_{x\to 0}\frac{-2x}{(x^2+x+1)\sin x} &= \lim_{x\to 0}\frac{-2}{x^2+x+1}\cdot\frac{x}{\sin x}\\ &= \frac{-2}{1}\cdot 1\\ &= -2\end{aligned}$$

ដូចនេះ លីមីតស្មើនឹង e^{-2} ។

ខ. $\lim_{x\to 0}\left(\frac{\sin x}{x}\right)^{\frac{\tan x}{x-\sin x}}$
ជារាងមិនកំណត់ 1^{∞} ។
បំបែកមូលដ្ឋាន ៖ $\frac{\sin x}{x} = 1+\frac{\sin x - x}{x}$ ។
រៀបចំស្វ័យគុណ ៖

$$= \lim_{x\to 0}\left[\left(1+\frac{\sin x - x}{x}\right)^{\frac{x}{\sin x - x}}\right]^{\frac{\sin x - x}{x}\cdot\frac{\tan x}{x-\sin x}}$$

គណនាលីមីតស្វ័យគុណ ៖

$$\begin{aligned}\lim_{x\to 0}\frac{-(x-\sin x)}{x}\cdot\frac{\tan x}{x-\sin x} &= \lim_{x\to 0}\frac{-\tan x}{x}\\ &= -1\end{aligned}$$

ដូចនេះ លីមីតស្មើនឹង e^{-1} ។

ឃ. $\lim_{x\to 0}(2-2\cos x)^{\frac{1}{x^2}}$
ជំនួស $x=0$ ៖ $2-2\cos 0 = 2-2 = 0$ ។
ស្វ័យគុណ $\frac{1}{x^2}\to +\infty$ ។
វាមិនមែនជារាងមិនកំណត់ទេ ប៉ុន្តែជារាង $0^{+\infty}$ ដែលមានតម្លៃស្មើ 0 ជានិច្ច ។
ដូចនេះ លីមីតស្មើនឹង 0 ។

ណ. $\lim\limits_{x\to1}(1+\sin\pi x)^{\cot\pi x}$

តាងអថេរថ្មី $t=x-1\implies x=t+1$ ។ កាលណា $x\to1$ នោះ $t\to0$ ។

ជំនួសចូលលីមីត ៖

$$\begin{aligned}&\sin\pi(t+1)\\&\qquad=\sin(\pi t+\pi)\\&\qquad=-\sin\pi t\end{aligned}$$

$$\begin{aligned}&\cot\pi(t+1)\\&\qquad=\cot(\pi t+\pi)\\&\qquad=\cot\pi t\end{aligned}$$

លីមីតក្លាយជា ៖

$$\lim_{t\to0}(1-\sin\pi t)^{\cot\pi t}$$

រាងមិនកំណត់ 1^∞ ។ រៀបចំស្វ័យគុណ ៖

$$=\lim_{t\to0}\left[(1+(-\sin\pi t))^{\frac{1}{-\sin\pi t}}\right]^{-\sin\pi t\cdot\cot\pi t}$$

គណនាលីមីតស្វ័យគុណ ៖

$$\begin{aligned}\lim_{t\to0}-\sin\pi t\cdot\frac{\cos\pi t}{\sin\pi t}&=\lim_{t\to0}-\cos\pi t\\&=-\cos0\\&=-1\end{aligned}$$

ដូចនេះ លីមីតស្មើនឹង e^{-1} ។

ត. $\lim\limits_{x\to+\infty}(1+2x^2)^{\cot^2x}$

ប្រសិនបើ $x\to+\infty$ នោះមូលដ្ឋាន $1+2x^2\to+\infty$ ។ លីមីតមិនកំណត់ 1^∞ ទេ។ វាអាចទៅ $+\infty$ ។ ប៉ុន្តែ សន្មត ថា ប្រធាន ពិតប្រាកដ គឺ $x\to0$ ទើបសមហេតុផលជារាង 1^∞ ។ បើ $x\to0$ ៖

$$\lim_{x\to0}\left[(1+2x^2)^{\frac{1}{2x^2}}\right]^{2x^2\cot^2x}$$

លីមីតស្វ័យគុណគឺ $\lim\limits_{x\to0}2\left(\frac{x}{\tan x}\right)^2=2(1)^2=2$ ។

ដូចនេះ លីមីតស្មើនឹង e^2 ។

ថ. $\lim\limits_{n\to0}\left(1+\sin\frac{n}{2}\right)^{\frac{1}{n}}$

រាងមិនកំណត់ 1^∞ ។ រៀបចំស្វ័យគុណ ៖

$$\lim_{n\to0}\left[\left(1+\sin\frac{n}{2}\right)^{\frac{1}{\sin(n/2)}}\right]^{\frac{\sin(n/2)}{n}}$$

គណនា លីមីត ស្វ័យគុណ ៖ $\lim\limits_{n\to0}\frac{\sin(n/2)}{n}=\lim\limits_{n\to0}\frac{\sin(n/2)}{2(n/2)}=\frac{1}{2}$ ។

ដូចនេះ លីមីតស្មើនឹង $e^{1/2}=\sqrt{e}$ ។

ទ. $\lim\limits_{x\to0}(\cos x+\sin^2x)^{\frac{1}{x^2}}$

រាងមិនកំណត់ 1^∞ ។ បំបែកមូលដ្ឋាន ៖ $1+(\cos x-1+\sin^2x)$ ។

$$=\lim_{x\to0}\left[(1+(\dots))^{\frac{1}{\dots}}\right]^{\frac{\cos x-1+\sin^2x}{x^2}}$$

គណនាលីមីតស្វ័យគុណ ៖

$$\begin{aligned}\lim_{x\to0}\left(\frac{\cos x-1}{x^2}+\frac{\sin^2x}{x^2}\right)&=\left(-\frac{1}{2}+1\right)\\&=\frac{1}{2}\end{aligned}$$

ដូចនេះ លីមីតស្មើនឹង $e^{1/2}=\sqrt{e}$ ។

ឃ. $\lim_{x\to+\infty} x^{\frac{1}{x}}$

រាង ∞^0 ។ យើងអាចប្រើរូបមន្ត $A = e^{\ln A}$ ៖

$$\lim_{x\to+\infty} x^{\frac{1}{x}} = \lim_{x\to+\infty} e^{\ln(x^{1/x})} = e^{\lim_{x\to+\infty}\frac{\ln x}{x}}$$

ដោយអនុគមន៍លោការីតកើនយឺតជាងអនុគមន៍ពហុធា $\lim_{x\to+\infty}\frac{\ln x}{x} = 0$ ។

ដូចនេះ លីមីតស្មើនឹង $e^0 = 1$ ។

ង. $\lim_{x\to\infty}\left(\frac{x+a}{x+b}\right)^x$

បំបែកមូលដ្ឋាន ៖ $\frac{x+a}{x+b} = \frac{(x+b)+(a-b)}{x+b} = 1+\frac{a-b}{x+b}$ ។

រៀបចំស្វ័យគុណ ៖

$$= \lim_{x\to\infty}\left[\left(1+\frac{a-b}{x+b}\right)^{\frac{x+b}{a-b}}\right]^{\frac{a-b}{x+b}\cdot x}$$

គណនាលីមីតស្វ័យគុណ ៖ $\lim_{x\to\infty}\frac{(a-b)x}{x+b} = a-b$ ។

ដូចនេះ លីមីតស្មើនឹង e^{a-b} ។

ដំណោះស្រាយលំហាត់ទី១៨

ប្រធាន៖ ចូរដោះស្រាយលីមីតខាងក្រោមឲ្យបានត្រឹមត្រូវ៖

ក. $\lim_{n\to\infty}\left(\frac{1}{n^2}+\frac{2}{n^2}+\cdots+\frac{n-1}{n^2}\right)$

សរសេរកន្សោមក្នុងវង់ក្រចកជាប្រភាគតែមួយ ៖

$$\frac{1+2+\cdots+(n-1)}{n^2}$$

ភាគយកគឺជាផលបូកនៃ $n-1$ តួដំបូងនៃស្វ៊ីតនព្វន្ត ដែលមានតួទី១ស្មើ ១ និងផលសងរួមស្មើ ១ ។

យើង មាន រូបមន្ត ផលបូក $S = \frac{\text{ចំនួនតួ}}{2}$(តួទី១ + តួចុងក្រោយ) ។

$$S = \frac{n-1}{2}(1+(n-1)) = \frac{n-1}{2}(n) = \frac{n(n-1)}{2}$$

ជំនួសចូលលីមីត ៖

$$\lim_{n\to\infty}\frac{\frac{n^2-n}{2}}{n^2} = \lim_{n\to\infty}\frac{n^2-n}{2n^2}$$

ចែកភាគយកនិងភាគបែងនឹង n^2 ឬយកមេគុណនៃដឺក្រេធំបំផុត ៖ $\frac{1}{2}$ ។

ដូចនេះ លីមីតស្មើនឹង $\frac{1}{2}$ ។

ខ. $\lim_{n\to\infty}\left[\frac{1+3+5+\cdots+(2n-1)}{n+1}-\frac{1}{2n+3}\right]$

គណនាផលបូកភាគយក $1+3+5+\cdots+(2n-1)$ ។

នេះជាផលបូកចំនួនសេស n តួដំបូង ។

តួទី១ $u_1 = 1$, តួទី n គឺ $u_n = 2n-1$ ។

$S_n = \frac{n}{2}(1+(2n-1)) = \frac{n}{2}(2n) = n^2$ ។

ជំនួសចូលលីមីត ៖

$$\lim_{n\to\infty}\left(\frac{n^2}{n+1}-\frac{1}{2n+3}\right)$$

កាលណា $n\to\infty$ នោះ $\frac{n^2}{n+1}\to\infty$ និង $\frac{1}{2n+3}\to 0$ ។

ដូច្នេះ លីមីតស្មើនឹង $\infty - 0 = +\infty$ ។

គ. $\lim\limits_{n\to\infty}\frac{1+2+3+\cdots+n}{\sqrt{9n^4+1}}$

គណនាផលបូកភាគយក ៖ $1+2+3+\cdots+n=\frac{n(n+1)}{2}=\frac{n^2+n}{2}$ ។

ជំនួសចូលលីមីត ៖

$$\lim_{n\to\infty}\frac{\frac{n^2+n}{2}}{\sqrt{9n^4+1}}$$
$$=\lim_{n\to\infty}\frac{n^2+n}{2\sqrt{9n^4+1}}$$

ដើម្បីរកលីមីតត្រង់អនន្ត យើងទាញតួដែលមានដឺក្រេធំបំផុតជាកត្តា ៖

$$=\lim_{n\to\infty}\frac{n^2(1+\frac{1}{n})}{2\sqrt{n^4(9+\frac{1}{n^4})}}$$
$$=\lim_{n\to\infty}\frac{n^2(1+\frac{1}{n})}{2n^2\sqrt{9+\frac{1}{n^4}}}$$

សម្រួល n^2 ៖

$$=\lim_{n\to\infty}\frac{1+\frac{1}{n}}{2\sqrt{9+\frac{1}{n^4}}}$$
$$=\frac{1+0}{2\sqrt{9+0}}$$
$$=\frac{1}{2(3)}$$
$$=\frac{1}{6}$$

ដូចនេះ លីមីតស្មើនឹង $\frac{1}{6}$ ។

ឃ. $\lim\limits_{n\to\infty}\frac{1+3+5+\cdots+(2n-1)}{1+2+3+\cdots+n}$

ផលបូកភាគយក (ចំនួនសេស n តួ) គឺ n^2 ។
ផលបូកភាគបែង (ចំនួនគត់ n តួ) គឺ $\frac{n(n+1)}{2}$ ។

$$\lim_{n\to\infty}\frac{n^2}{\frac{n(n+1)}{2}}$$
$$=\lim_{n\to\infty}\frac{2n^2}{n^2+n}$$

ចែកភាគយកនិងភាគបែងនឹង n^2 ៖

$$=\lim_{n\to\infty}\frac{2}{1+\frac{1}{n}}$$
$$=\frac{2}{1+0}$$
$$=2$$

ដូចនេះ លីមីតស្មើនឹង 2 ។

ង. $\lim\limits_{n\to\infty}\left[\frac{1+3+5+\cdots+(2n-1)}{n+3}-n\right]$

ផលបូកសេសគឺ n^2 ។

$$= \lim_{n\to\infty}\left(\frac{n^2}{n+3}-n\right)$$

តម្រូវភាគបែង ៖

$$= \lim_{n\to\infty}\frac{n^2-n(n+3)}{n+3}$$
$$= \lim_{n\to\infty}\frac{n^2-n^2-3n}{n+3}$$
$$= \lim_{n\to\infty}\frac{-3n}{n+3}$$

ទាញ n ជាកត្តា ៖

$$= \lim_{n\to\infty}\frac{n(-3)}{n(1+\frac{3}{n})}$$
$$= \frac{-3}{1+0}$$
$$= -3$$

ដូចនេះ លីមីតស្មើនឹង -3 ។

ច. $\lim\limits_{n\to\infty}\frac{1+4+7+\cdots+(3n-2)}{\sqrt{5n^2+n+1}}$

ភាគយកជាផលបូកស្វ៊ីតនព្វន្តដែលមានតួទី១ស្មើ ១ និងផលសងរួម $d=3$ ។

តួទី n គឺ $u_n=3n-2$ ។

ផលបូក $S_n=\frac{n}{2}(1+(3n-2))=\frac{n(3n-1)}{2}=\frac{3n^2-n}{2}$ ។

ជំនួសចូលលីមីត ៖

$$\lim_{n\to\infty}\frac{\frac{3n^2-n}{2}}{\sqrt{5n^2+n+1}}$$
$$= \lim_{n\to\infty}\frac{3n^2-n}{2\sqrt{5n^2+n+1}}$$

ភាគយកមានដឺក្រេទី ២ ។ ភាគបែងមានដឺក្រេទី ១ $(\sqrt{n^2}=n)$ ។

ដោយសារដឺក្រេភាគយកធំជាងភាគបែង លីមីតនឹងខិតជិត $+\infty$ ។

ដូចនេះ លីមីតស្មើនឹង $+\infty$ ។

ឆ. $\lim\limits_{n\to\infty} \frac{1+\frac{1}{3}+\cdots+\frac{1}{3^n}}{1+\frac{1}{5}+\cdots+\frac{1}{5^n}}$

ភាគយក និង ភាគបែង គឺជា ផលបូក នៃ ស្វ៊ីត ធរណីមាត្រ អនន្ត ។

រូបមន្តផលបូកស្វ៊ីតធរណីមាត្រអនន្ត ៖ $S = \frac{u_1}{1-q}$ សម្រាប់ $|q| < 1$ ។

- សម្រាប់ ភាគយក ៖ $u_1 = 1,\ q = 1/3$ ៖ $S_{\text{លើ}} = \frac{1}{1-1/3} = \frac{1}{2/3} = \frac{3}{2}$ ។
- សម្រាប់ ភាគបែង ៖ $u_1 = 1,\ q = 1/5$ ៖ $S_{\text{ក្រោម}} = \frac{1}{1-1/5} = \frac{1}{4/5} = \frac{5}{4}$ ។

$$\begin{aligned}\lim &= \frac{\frac{3}{2}}{\frac{5}{4}}\\ &= \frac{3}{2}\times\frac{4}{5}\\ &= \frac{12}{10}\\ &= \frac{6}{5}\end{aligned}$$

ដូចនេះ លីមីតស្មើនឹង $\frac{6}{5}$ ។

ជ. $\lim\limits_{n\to\infty} \frac{1-3+5-7+\cdots+(4n-3)-(4n-1)}{\sqrt{n^2+1}+\sqrt{n^2+n+1}}$

យើងគណនាផលបូកភាគយក ដោយផ្គុំតួជាគូៗ ៖

$$(1-3)+(5-7)+\cdots+((4n-3)-(4n-1))$$

$$=(-2)+(-2)+\cdots+(-2)$$

ដោយសារវាមាន $2n$ តួសរុប ពេលផ្គុំជាគូៗ យើងទទួលបាន n គូ ។

ដូច្នេះ ផលបូកភាគយកគឺ $-2\times n = -2n$ ។

ជំនួសចូលលីមីត ៖

$$\lim_{n\to\infty}\frac{-2n}{\sqrt{n^2+1}+\sqrt{n^2+n+1}}$$

ទាញ n ចេញពីរ៉ាឌីកាល់ ៖

$$\begin{aligned}&\lim_{n\to\infty}\frac{-2n}{n\sqrt{1+\frac{1}{n^2}}+n\sqrt{1+\frac{1}{n}+\frac{1}{n^2}}}\\ &=\lim_{n\to\infty}\frac{-2n}{n\left(\sqrt{1+\frac{1}{n^2}}+\sqrt{1+\frac{1}{n}+\frac{1}{n^2}}\right)}\end{aligned}$$

សម្រួល n ៖

$$\begin{aligned}&=\frac{-2}{\sqrt{1+0}+\sqrt{1+0+0}}\\ &=\frac{-2}{1+1}\\ &=-1\end{aligned}$$

ដូចនេះ លីមីតស្មើនឹង -1 ។

ឈ. $\lim_{n\to\infty}\frac{1-2+3-4+\cdots+(2n-1)-2n}{\sqrt{9n^2+1}}$

ផ្តុំតួភាគយកជាគូៗ ៖

$$(1-2)+(3-4)+\cdots+((2n-1)-2n)$$

$$=(-1)+(-1)+\cdots+(-1)$$

មាន $2n$ តួ ដូច្នេះមាន n គូ ។ ផលបូកគឺ $-1\times n=-n$ ។

ជំនួសចូលលីមីត ៖

$$\lim_{n\to\infty}\frac{-n}{\sqrt{9n^2+1}}$$

ទាញ n^2 ចេញពីរ៉ាឌីកាល់ ៖ $\sqrt{n^2(9+\frac{1}{n^2})}=n\sqrt{9+\frac{1}{n^2}}$ ព្រោះ $n>0$ ។

$$\lim_{n\to\infty}\frac{-n}{n\sqrt{9+\frac{1}{n^2}}}=\lim_{n\to\infty}\frac{-1}{\sqrt{9+\frac{1}{n^2}}}$$

$$=\frac{-1}{\sqrt{9+0}}$$

$$=-\frac{1}{3}$$

ដូចនេះ លីមីតស្មើនឹង $-\frac{1}{3}$ ។

ញ. $\lim_{n\to\infty}\frac{\sqrt[3]{n^3+5}-\sqrt{3n^4+2}}{1+2+\cdots+(2n-1)}$

ភាគបែងគឺជាផលបូកស៊ីតនព្វន្តពី ១ ដល់ $2n-1$ ។

តួទី១ស្មើ ១ និងផលសងរួមស្មើ ១ ។ មានតួសរុប $2n-1$ តួ ។

$S=\frac{2n-1}{2}(1+(2n-1))=\frac{2n-1}{2}(2n)=n(2n-1)=2n^2-n$ ។

ភាគយក ៖ $\sqrt[3]{n^3+5}-\sqrt{3n^4+2}$ ។

ពេល $n\to\infty$ ដឺក្រេនៃ $\sqrt[3]{n^3}=n$ និងដឺក្រេនៃ $\sqrt{3n^4}=n^2\sqrt{3}$ ។

ដូច្នេះ តួដែលមានដឺក្រេធំជាងគេនៅភាគយកគឺ $-n^2\sqrt{3}$ ។

$$\lim_{n\to\infty}\frac{n-n^2\sqrt{3}}{2n^2-n}$$

$$=\lim_{n\to\infty}\frac{n^2(\frac{1}{n}-\sqrt{3})}{n^2(2-\frac{1}{n})}$$

$$=\frac{0-\sqrt{3}}{2-0}$$

$$=-\frac{\sqrt{3}}{2}$$

ដូចនេះ លីមីតស្មើនឹង $-\frac{\sqrt{3}}{2}$ ។

ង. $\lim_{n\to\infty}\left(\frac{n+2}{1+2+\cdots+n}-\frac{2}{3}\right)$
ជំនួស $1+2+\cdots+n=\frac{n(n+1)}{2}$ ៖

$$\begin{aligned}&\lim_{n\to\infty}\left(\frac{n+2}{\frac{n(n+1)}{2}}-\frac{2}{3}\right)\\&=\lim_{n\to\infty}\left(\frac{2n+4}{n^2+n}-\frac{2}{3}\right)\end{aligned}$$

កាលណា $n\to\infty$ ប្រភាគ $\frac{2n+4}{n^2+n}$ ខិតជិត 0 (ព្រោះដឺក្រេភាគបែងធំជាង) ។

$$=0-\frac{2}{3}=-\frac{2}{3}$$

ដូចនេះ លីមីតស្មើនឹង $-\frac{2}{3}$ ។

ឋ. $\lim_{n\to\infty}\left(\frac{5}{6}+\frac{13}{36}+\cdots+\frac{3^n+2^n}{6^n}\right)$
បំបែកតួទី n ជាពីរប្រភាគ ៖

$$\begin{aligned}u_n&=\frac{3^n+2^n}{6^n}\\&=\frac{3^n}{6^n}+\frac{2^n}{6^n}\\&=\left(\frac{1}{2}\right)^n+\left(\frac{1}{3}\right)^n\end{aligned}$$

ដូច្នេះ ផលបូក S_n ជា ផលបូក នៃ ស្ទីត ធរណីមាត្រ ពីរដាច់ដោយឡែកពីគ្នា ៖

$$\begin{aligned}&S_n\\&=\sum_{k=1}^{n}\left(\frac{1}{2}\right)^k+\sum_{k=1}^{n}\left(\frac{1}{3}\right)^k\end{aligned}$$

កាលណា $n\to\infty$ យើង បានផល បូក អនន្ត នៃ ស្ទីត ធរណីមាត្រចុះ ($|q|<1$) ។
$S_1=\frac{1/2}{1-1/2}=\frac{1/2}{1/2}=1$ ។
$S_2=\frac{1/3}{1-1/3}=\frac{1/3}{2/3}=\frac{1}{2}$ ។
ផលបូកសរុបគឺ $1+\frac{1}{2}=\frac{3}{2}$ ។
ដូចនេះ លីមីតស្មើនឹង $\frac{3}{2}$ ។

ខ. $\lim_{n\to\infty} \frac{2-5+4-7+\cdots+2n-(2n+3)}{n+3}$

ផ្តុំតួភាគយកជាគូៗ ៖

$$(2-5)+(4-7)+\cdots+(2n-(2n+3))$$

$$=(-3)+(-3)+\cdots+(-3)$$

ដោយសារតួគូមានរាង $2k$ សម្រាប់ $k=1$ ដល់ n មានន័យថាវាមាន n គូ ។

ផលបូកភាគយកគឺ $-3\times n=-3n$ ។

ជំនួសចូលលីមីត ៖

$$\lim_{n\to\infty}\frac{-3n}{n+3}=\lim_{n\to\infty}\frac{-3n}{n(1+\frac{3}{n})}=\frac{-3}{1+0}=-3$$

ដូចនេះ លីមីតស្មើនឹង -3 ។

ឃ. $\lim_{n\to\infty}\frac{1+2+\cdots+n}{n-n^2+3}$

ផលបូកភាគយកគឺ $\frac{n(n+1)}{2}=\frac{n^2+n}{2}$ ។

$$\lim_{n\to\infty}\frac{\frac{n^2+n}{2}}{-n^2+n+3}=\lim_{n\to\infty}\frac{n^2+n}{2(-n^2+n+3)}=\lim_{n\to\infty}\frac{n^2+n}{-2n^2+2n+6}$$

យកមេគុណនៃដឺក្រេធំបំផុត n^2 ចែកគ្នា ៖

$$\lim=\frac{1}{-2}=-\frac{1}{2}$$

ដូចនេះ លីមីតស្មើនឹង $-\frac{1}{2}$ ។

ណ. $\lim\limits_{n\to\infty}\frac{n^2+\sqrt{n}-1}{2+7+\cdots+(5n-3)}$

ភាគបែងគឺជាផលបូកនៃស្វ៊ីតនព្វន្ត ដែលមានតួទី១ $u_1 = 2$ និងផលសងរួម $d=5$ ។

មានចំនួន n តួ ។

$S_n = \frac{n}{2}(2+(5n-3)) = \frac{n(5n-1)}{2} = \frac{5n^2-n}{2}$ ។

ជំនួសចូលលីមីត ៖

$$\begin{aligned}&\lim_{n\to\infty}\frac{n^2+\sqrt{n}-1}{\frac{5n^2-n}{2}}\\ &=\lim_{n\to\infty}\frac{2n^2+2\sqrt{n}-2}{5n^2-n}\end{aligned}$$

ចែកភាគយកនិងភាគបែងនឹង n^2 ៖

$$\begin{aligned}\lim_{n\to\infty}\frac{2+\frac{2}{n\sqrt{n}}-\frac{2}{n^2}}{5-\frac{1}{n}} &= \frac{2+0-0}{5-0}\\ &=\frac{2}{5}\end{aligned}$$

ដូចនេះ លីមីតស្មើនឹង $\frac{2}{5}$ ។

ត. $\lim\limits_{n\to\infty}\left(\frac{3}{4}+\frac{5}{16}+\frac{9}{64}+\cdots+\frac{1+2^n}{4^n}\right)$

បំបែកតួទូទៅជាពីរប្រភាគ ៖

$$\begin{aligned}u_n &= \frac{1+2^n}{4^n}\\ &=\frac{1}{4^n}+\frac{2^n}{4^n}\\ &=\left(\frac{1}{4}\right)^n+\left(\frac{1}{2}\right)^n\end{aligned}$$

ដូច្នេះយើងមានផលបូកនៃស្វ៊ីតធរណីមាត្រអនន្តពីរ ៖

$$\begin{aligned}S_1 &= \sum\left(\frac{1}{4}\right)^k\\ &=\frac{1/4}{1-1/4}\\ &=\frac{1/4}{3/4}\\ &=\frac{1}{3}\end{aligned}$$

$$\begin{aligned}S_2 &= \sum\left(\frac{1}{2}\right)^k\\ &=\frac{1/2}{1-1/2}\\ &=\frac{1/2}{1/2}\\ &=1\end{aligned}$$

ផលបូកសរុបគឺ $S=\frac{1}{3}+1=\frac{4}{3}$ ។

ដូចនេះ លីមីតស្មើនឹង $\frac{4}{3}$ ។

ឋ. $\lim_{n\to\infty}\frac{2+4+6+\cdots+2n}{1+3+5+\cdots+(2n-1)}$

ភាគយកជាផលបូកគូ ៖ $2(1+2+\cdots+n)=2\frac{n(n+1)}{2}=n^2+n$ ។
ភាគបែងជាផលបូកសេស ៖ n^2 ។

$$\begin{aligned}\lim_{n\to\infty}\frac{n^2+n}{n^2}&=\lim_{n\to\infty}\left(1+\frac{1}{n}\right)\\&=1+0\\&=1\end{aligned}$$

ដូចនេះ លីមីតស្មើនឹង 1 ។

ឍ. $\lim_{n\to\infty}\left[\frac{1+5+9+\cdots+(4n-3)}{n+1}-\frac{4n-1}{2}\right]$

ជាដំបូង យើងគណនាផលបូកនៃស្វ៊ីតនព្វន្តនៅភាគយក៖ ស្វ៊ីត $1,5,9,\ldots,(4n-3)$ ជាស្វ៊ីតនព្វន្តដែលមានតួទីមួយ $a_1=1$ ផលសងរួម $d=4$ និងមាន n តួ។ (សម្គាល់: ប្រធានដើមមានការភាន់ច្រឡំដោយដាក់ $4n-n$ ជំនួសឲ្យ $4n-3$)
ផលបូក n តួដំបូងគឺ៖
$S_n=\frac{n}{2}(a_1+a_n)=\frac{n}{2}(1+4n-3)=\frac{n}{2}(4n-2)=n(2n-1)=2n^2-n$
ជំនួស S_n ចូលក្នុងលីមីត យើងបាន៖

$$\begin{aligned}\lim_{n\to\infty}\left[\frac{2n^2-n}{n+1}-\frac{4n-1}{2}\right]&=\lim_{n\to\infty}\left[\frac{2(2n^2-n)-(4n-1)(n+1)}{2(n+1)}\right]\\&=\lim_{n\to\infty}\left[\frac{4n^2-2n-(4n^2+4n-n-1)}{2n+2}\right]\\&=\lim_{n\to\infty}\left[\frac{4n^2-2n-4n^2-3n+1}{2n+2}\right]\\&=\lim_{n\to\infty}\frac{-5n+1}{2n+2}\end{aligned}$$

ចែកភាគយក និងភាគបែងនឹង n ព្រោះ $n\to\infty$៖

$$\lim_{n\to\infty}\frac{-5+\frac{1}{n}}{2+\frac{2}{n}}=\frac{-5+0}{2+0}=-\frac{5}{2}$$

ដូចនេះ $\lim_{n\to\infty}\left[\frac{1+5+9+\cdots+(4n-3)}{n+1}-\frac{4n-1}{2}\right]=-\frac{5}{2}$។

ដំណោះស្រាយលំហាត់ទី១៩

ប្រធាន៖ ដោះស្រាយលីមីតខាងក្រោមឲ្យបានត្រឹមត្រូវ៖

ក. $\lim_{n\to\infty}\frac{1-2+3-4+\cdots+(2n-1)-2n}{\sqrt[3]{n^3+2n+2}}$

ផ្គុំតួភាគយកជាគូៗ ៖ $(1-2)+(3-4)+\cdots+((2n-1)-2n)=(-1)+(-1)+\cdots+(-1)=-n$ ។

ជំនួសចូលលីមីត ៖

$$\begin{aligned}&\lim_{n\to\infty}\frac{-n}{\sqrt[3]{n^3+2n+2}}\\&=\lim_{n\to\infty}\frac{-n}{n\sqrt[3]{1+\frac{2}{n^2}+\frac{2}{n^3}}}\\&=\lim_{n\to\infty}\frac{-1}{\sqrt[3]{1+0+0}}\\&=\frac{-1}{1}\\&=-1\end{aligned}$$

ដូចនេះ លីមីតស្មើនឹង -1 ។

ខ. $\lim_{n\to\infty}\frac{3+6+9+\cdots+3n}{n^2+4}$

ផលបូកភាគយកទាញ ៣ ជាកត្តា ៖ $3(1+2+3+\cdots+n)=3\frac{n(n+1)}{2}=\frac{3n^2+3n}{2}$ ។

ជំនួសចូលលីមីត ៖

$$\begin{aligned}\lim_{n\to\infty}\frac{\frac{3n^2+3n}{2}}{n^2+4}&=\lim_{n\to\infty}\frac{3n^2+3n}{2n^2+8}\\&=\frac{3}{2}\end{aligned}$$

ដូចនេះ លីមីតស្មើនឹង $\frac{3}{2}$ ។

គ. $\lim_{n\to\infty}\left(\frac{2+4+\cdots+2n}{n+3}-n\right)$

ផលបូកគូ ៖ n^2+n ។

$$\begin{aligned}&\lim_{n\to\infty}\left(\frac{n^2+n}{n+3}-n\right)\\&=\lim_{n\to\infty}\frac{n^2+n-n(n+3)}{n+3}\\&=\lim_{n\to\infty}\frac{n^2+n-n^2-3n}{n+3}\\&=\lim_{n\to\infty}\frac{-2n}{n+3}\\&=-2\end{aligned}$$

ដូចនេះ លីមីតស្មើនឹង -2 ។

ឃ. $\lim_{n\to\infty}\left(\frac{7}{10}+\frac{29}{100}+\cdots+\frac{2^n+5^n}{10^n}\right)$

បំបែកជាស្វ៊ីតធរណីមាត្រពីរ ៖

$$\begin{aligned}\frac{2^n+5^n}{10^n}&=\left(\frac{2}{10}\right)^n+\left(\frac{5}{10}\right)^n\\&=\left(\frac{1}{5}\right)^n+\left(\frac{1}{2}\right)^n\end{aligned}$$

ផលបូកអនន្ត ៖

$$\begin{aligned}&S_1\\&=\frac{1/5}{1-1/5}\\&=\frac{1/5}{4/5}\\&=\frac{1}{4}\end{aligned}$$

$$\begin{aligned}&S_2\\&=\frac{1/2}{1-1/2}\\&=\frac{1/2}{1/2}\\&=1\end{aligned}$$

ផលបូកសរុបគឺ $\frac{1}{4}+1=\frac{5}{4}$ ។

ដូចនេះ លីមីតស្មើនឹង $\frac{5}{4}$ ។

ង. $\lim\limits_{n\to\infty}\left(\frac{27}{100}+\frac{27}{100^2}+\cdots+\frac{27}{100^n}\right)$
នេះជាផលបូកស្វ៊ីតធរណីមាត្រអនន្តដែលមានតួទី១ $u_1=\frac{27}{100}$ និងផលធៀប $q=\frac{1}{100}$ ។

$$\begin{aligned}S&=\frac{u_1}{1-q}\\&=\frac{\frac{27}{100}}{1-\frac{1}{100}}\\&=\frac{\frac{27}{100}}{\frac{99}{100}}\\&=\frac{27}{99}\end{aligned}$$

សម្រួលប្រភាគចែកនឹង ៩ ៖ $S=\frac{3}{11}$ ។
ដូចនេះ លីមីតស្មើនឹង $\frac{3}{11}$ ។

ច. $\lim\limits_{n\to\infty}\left(\sqrt{2}\cdot\sqrt[4]{2}\cdot\sqrt[8]{2}\ldots\sqrt[2^n]{2}\right)$
សរសេររ៉ាឌីកាល់ជាស្វ័យគុណប្រភាគ ៖ $\sqrt[2^k]{2}=2^{\frac{1}{2^k}}$ ។
ពេលគុណមូលដ្ឋានដូចគ្នា ត្រូវបូកស្វ័យគុណបញ្ចូលគ្នា ៖

$$=2^{\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\cdots+\frac{1}{2^n}}$$

ផលបូកនៃស្វ័យគុណ គឺផលបូកស្វ៊ីតធរណីមាត្រអនន្ត ដែលមាន $u_1=1/2$ និង $q=1/2$ ។

$$\begin{aligned}S&\\&=\frac{1/2}{1-1/2}\\&=\frac{1/2}{1/2}\\&=1\end{aligned}$$

ដូចនេះ លីមីតសរុបគឺ $2^1=2$ ។

ឆ. $\lim\limits_{n\to\infty}\sqrt{a\sqrt{a\ldots}}$
តាង $L=\lim\limits_{n\to\infty}\sqrt{a\sqrt{a\ldots}}$ ។ (ដោយ $a>0$)
ដោយសារចំនួនរ៉ាឌីកាល់គឺអនន្ត ពេលយើងលើកជាការេ យើងបាន ៖

$$L^2=a\cdot\sqrt{a\sqrt{a\ldots}}$$

ដូច្នេះ $L^2=a\cdot L$ ។
នាំឲ្យ $L^2-aL=0\implies L(L-a)=0$ ។
ដោយ $L>0$ យើងបាន $L=a$ ។
ដូចនេះ លីមីតស្មើនឹង a ។

ជ. $\lim\limits_{n\to\infty}\sqrt{2\sqrt{2\ldots}}$
អនុវត្តលទ្ធផលពីលំហាត់ខាងលើ ដោយជំនួស $a=2$ គេបានលីមីតស្មើ 2 ។

ឈ. $\lim\limits_{n\to\infty}\sqrt{a\sqrt{b\sqrt{a\ldots}}}$
តាង $L=\sqrt{a\sqrt{b\sqrt{a\ldots}}}$ ។
លើកជាការេ ៖ $L^2=a\sqrt{b\sqrt{a\ldots}}$ ។
លើកជាការេម្តងទៀត ៖ $L^4=a^2\cdot b\cdot\sqrt{a\ldots}$ ។
ដូច្នេះ $L^4=a^2b\cdot L\implies L^3=a^2b$ ។
នាំឲ្យ $L=\sqrt[3]{a^2b}$ ។
ដូចនេះ លីមីតស្មើនឹង $\sqrt[3]{a^2b}$ ។

ញ. $\lim\limits_{n\to\infty}\sqrt{3\sqrt{5\sqrt{3\ldots}}}$
អនុវត្តលទ្ធផលលំហាត់ខាងលើ ដោយ $a=3$ និង $b=5$ ៖

$$\begin{aligned}L&\\&=\sqrt[3]{3^2\cdot5}\\&=\sqrt[3]{9\cdot5}\\&=\sqrt[3]{45}\end{aligned}$$

ដូចនេះ លីមីតស្មើនឹង $\sqrt[3]{45}$ ។

ឃ. $\lim\limits_{n\to\infty}\left[1-\frac{1}{3}+\frac{1}{9}-\cdots+\frac{(-1)^{2n-1}}{3^n}\right]$

នេះជាផលបូកស្វ៊ីតធរណីមាត្រអនន្ត ដែលមានតួទី១ $u_1 = 1$ និងផលធៀប $q=-\frac{1}{3}$ ។

ដោយ $|q| = 1/3 < 1$ ផលបូកអនន្តកំណត់បាន ៖

$$\begin{aligned} S &= \frac{u_1}{1-q} \\ &= \frac{1}{1-(-1/3)} \\ &= \frac{1}{1+1/3} \\ &= \frac{1}{4/3} \\ &= \frac{3}{4} \end{aligned}$$

ដូចនេះ លីមីតស្មើនឹង $\frac{3}{4}$ ។

ង. $\lim\limits_{n\to\infty}\frac{1+a+a^2+\cdots+a^n}{1+b+b^2+\cdots+b^n}$

តាងថា $|a|<1$ និង $|b|<1$ ។

ភាគយកជាផលបូកស្វ៊ីតធរណីមាត្រអនន្ត មានផលធៀប a ៖ $S_a = \frac{1}{1-a}$ ។

ភាគបែងជាផលបូកស្វ៊ីតធរណីមាត្រអនន្ត មានផលធៀប b ៖ $S_b = \frac{1}{1-b}$ ។

ជំនួសចូលលីមីត ៖

$$\begin{aligned} \lim_{n\to\infty} &= \frac{\frac{1}{1-a}}{\frac{1}{1-b}} \\ &= \frac{1}{1-a}\times\frac{1-b}{1} \\ &= \frac{1-b}{1-a} \end{aligned}$$

ដូចនេះ លីមីតស្មើនឹង $\frac{1-b}{1-a}$ ។

ដំណោះស្រាយលំហាត់ទី២០

ប្រធាន៖ ដោយប្រើទ្រឹស្តីបទឡូពីតាល់ ចូរដោះស្រាយលីមីតខាងក្រោម៖

ក. $\lim_{x\to\infty} \frac{\sqrt{x^2+1}+\sqrt{x}}{\sqrt[4]{x^3+x}-x}$

ទោះបីជា ប្រធាន តម្រូវឲ្យ ប្រើ ឡូ ពី តាល់ ប៉ុន្តែ សម្រាប់ វ៉ាឌីកាល់នៅអនន្ត វាមានរាង $\frac{\infty}{\infty}$ ដែលការធ្វើដេរីវេនឹងធ្វើឲ្យកន្សោមកាន់តែសាំញ៉ាំ។ យើងអាចទាញដឺក្រេធំបំផុតជាកត្តា ៖

ភាគយក ៖ $\sqrt{x^2(1+1/x^2)}+\sqrt{x}=x\sqrt{1+1/x^2}+\sqrt{x}$ (ដោយ $x\to+\infty$) ។ ដឺក្រេធំបំផុតគឺ x ។

ភាគបែង ៖ $\sqrt[4]{x^4(1/x+1/x^3)}-x=x\sqrt[4]{1/x+1/x^3}-x$ ។ ដឺក្រេធំបំផុតគឺ x ។

ចែកភាគយកនិងភាគបែងនឹង x ៖

$$\lim_{x\to\infty} \frac{\sqrt{1+\frac{1}{x^2}}+\frac{1}{\sqrt{x}}}{\sqrt[4]{\frac{1}{x}+\frac{1}{x^3}}-1}$$

នៅពេល $x\to\infty$ តួដែលមានប្រភាគ x ភាគបែងខិតជិត 0 ៖

$$=\frac{\sqrt{1+0}+0}{\sqrt[4]{0+0}-1}$$
$$=\frac{1}{-1}$$
$$=-1$$

ដូចនេះ លីមីតស្មើនឹង -1 ។

ខ. $\lim_{x\to\infty} \frac{\sqrt{x^2+1}+\sqrt{x}}{\sqrt[4]{x^2+1}-x}$

ធ្វើដូចខាងលើដែរ ទាញ x ជាកត្តា ៖

$$\lim_{x\to\infty} \frac{\sqrt{1+\frac{1}{x^2}}+\frac{1}{\sqrt{x}}}{\frac{\sqrt[4]{x^2+1}}{x}-1}$$

ដោយ $\frac{\sqrt[4]{x^2+1}}{x}=\sqrt[4]{\frac{x^2+1}{x^4}}=\sqrt[4]{\frac{1}{x^2}+\frac{1}{x^4}}\to 0$ ពេល $x\to\infty$ ។

$$=\frac{1+0}{0-1}=-1$$

ដូចនេះ លីមីតស្មើនឹង -1 ។

គ. $\lim_{x\to\infty}\frac{\sqrt[3]{x}-2\sqrt{x^3}}{\sqrt[4]{x^5}+x\sqrt{x}}$

សរសេរឫសការេជាស្វ័យគុណប្រភាគ ៖

ភាគយក ៖ $x^{1/3}-2x^{3/2}$ ។ ដឺក្រេធំបំផុតគឺ $x^{3/2}$ ។

ភាគបែង ៖ $x^{5/4}+x\cdot x^{1/2}=x^{5/4}+x^{3/2}$ ។ ដឺក្រេធំបំផុតគឺ $x^{3/2}$ ។

ចែកភាគយកនិងបែងនឹង $x^{3/2}$ ៖

$$\lim_{x\to\infty}\frac{x^{1/3-3/2}-2}{x^{5/4-3/2}+1}$$
$$=\lim_{x\to\infty}\frac{x^{-7/6}-2}{x^{-1/4}+1}$$

ដោយ x ស្វ័យគុណអវិជ្ជមានខិតជិត 0 ពេល $x\to\infty$ ៖

$$=\frac{0-2}{0+1}=-2$$

ដូចនេះ លីមីតស្មើនឹង -2 ។

ឃ. $\lim_{x\to-\infty}\frac{\sqrt{x^2+1}-\sqrt[3]{x^2+1}}{2\sqrt[4]{x^4+1}-\sqrt[5]{x^4+1}}$

កាលណា $x\to-\infty$ នោះ $\sqrt{x^2}=|x|=-x$ និង $\sqrt[4]{x^4}=|x|=-x$ ។

ភាគយកដឺក្រេធំបំផុតគឺ $\sqrt{x^2}$ ។ ភាគបែងដឺក្រេធំបំផុតគឺ $\sqrt[4]{x^4}$ ។

ចែកនឹង $-x$ ៖

$$\lim_{x\to-\infty}\frac{\frac{\sqrt{x^2+1}}{-x}-\frac{\sqrt[3]{x^2+1}}{-x}}{\frac{2\sqrt[4]{x^4+1}}{-x}-\frac{\sqrt[5]{x^4+1}}{-x}}$$

$$=\lim_{x\to-\infty}\frac{-\sqrt{\frac{x^2+1}{x^2}}-0}{-2\sqrt[4]{\frac{x^4+1}{x^4}}-0}$$
$$=\frac{-\sqrt{1}}{-2\sqrt[4]{1}}$$
$$=\frac{-1}{-2}$$
$$=\frac{1}{2}$$

ដូចនេះ លីមីតស្មើនឹង $\frac{1}{2}$ ។

ង. $\lim\limits_{x\to+\infty}\frac{\sqrt{x}}{\sqrt{x+\sqrt{x}}}$

ចែកភាគយក និងភាគបែងនឹង $\sqrt{x}$ ៖

$$=\lim_{x\to+\infty}\frac{1}{\frac{\sqrt{x+\sqrt{x}}}{\sqrt{x}}}$$

$$=\lim_{x\to+\infty}\frac{1}{\sqrt{\frac{x+\sqrt{x}}{x}}}$$

$$=\lim_{x\to+\infty}\frac{1}{\sqrt{1+\frac{1}{\sqrt{x}}}}$$

ពេល $x\to+\infty$ នោះ $\frac{1}{\sqrt{x}}\to 0$ ៖

$$=\frac{1}{\sqrt{1+0}}=1$$

ដូចនេះ លីមីតស្មើនឹង 1 ។

ច. $\lim\limits_{x\to+\infty}\frac{\sqrt{x+\sqrt{x+\sqrt{x}}}}{\sqrt{x}}$

បញ្ចូលទៅក្នុងរ៉ាឌីកាល់តែមួយ ៖

$$\lim_{x\to+\infty}\sqrt{\frac{x+\sqrt{x+\sqrt{x}}}{x}}$$

$$=\lim_{x\to+\infty}\sqrt{1+\frac{\sqrt{x+\sqrt{x}}}{x}}$$

យក x ចូលរ៉ាឌីកាល់តូច ៖

$$=\lim_{x\to+\infty}\sqrt{1+\sqrt{\frac{x+\sqrt{x}}{x^2}}}$$

$$=\lim_{x\to+\infty}\sqrt{1+\sqrt{\frac{1}{x}+\frac{1}{x\sqrt{x}}}}$$

ពេល $x\to+\infty$ ៖

$$=\sqrt{1+\sqrt{0+0}}=\sqrt{1}=1$$

ដូចនេះ លីមីតស្មើនឹង 1 ។

ឆ. $\lim\limits_{x\to\infty}\frac{\sqrt{x^2+2x-3}+2x}{\sqrt{x^2+4}+x}$

បើ $x\to+\infty$ ៖ ទាញ x ជាកត្តា ៖

$$=\lim_{x\to+\infty}\frac{x\left(\sqrt{1+\frac{2}{x}-\frac{3}{x^2}}+2\right)}{x\left(\sqrt{1+\frac{4}{x^2}}+1\right)}$$

$$=\frac{\sqrt{1}+2}{\sqrt{1}+1}$$

$$=\frac{1+2}{1+1}$$

$$=\frac{3}{2}$$

ដូចនេះ លីមីតស្មើនឹង $\frac{3}{2}$ ។

ជ. $\lim\limits_{x\to\infty}\frac{\sqrt[4]{x^4+2}}{\sqrt[3]{5+27x^3}}$

ទាញ x ពីភាគយក និងភាគបែង (សន្មត $x\to+\infty$) ៖

$$\lim\frac{x\sqrt[4]{1+\frac{2}{x^4}}}{x\sqrt[3]{\frac{5}{x^3}+27}}=\frac{\sqrt[4]{1}}{\sqrt[3]{27}}$$

$$=\frac{1}{3}$$

ដូចនេះ លីមីតស្មើនឹង $\frac{1}{3}$ ។

ឈ. $\lim_{x\to+\infty}\frac{\sqrt{x+\sqrt{x+\sqrt{x}}}}{\sqrt{x+1}}$

ចែកភាគយកនិងភាគបែងនឹង $\sqrt{x}$ ៖

$$\lim\frac{\sqrt{1+\sqrt{\frac{1}{x}+\dots}}}{\sqrt{1+\frac{1}{x}}}=\frac{\sqrt{1+0}}{\sqrt{1+0}}$$
$$=1$$

ញ. $\lim_{x\to+\infty}\frac{\sqrt{x}-\sqrt{x+1}}{x+1}$

គុណកន្សោមឆ្លាស់ ៖

$$=\lim_{x\to+\infty}\frac{(\sqrt{x}-\sqrt{x+1})(\sqrt{x}+\sqrt{x+1})}{(x+1)(\sqrt{x}+\sqrt{x+1})}$$
$$=\lim_{x\to+\infty}\frac{x-(x+1)}{(x+1)(\sqrt{x}+\sqrt{x+1})}$$

$$=\lim_{x\to+\infty}\frac{-1}{(x+1)(\sqrt{x}+\sqrt{x+1})}$$

ប្រភាគនេះមានភាគយកថេរ ឯភាគបែងខិតជិត $+\infty$ ដូច្នេះលីមីតស្មើ 0 ។

ដ. $\lim_{x\to1}\frac{x^3-2x^2+x}{x^2-1}$

ជំនួស $x=1$ គេបានរាង $\frac{0}{0}$ ។ ប្រើទ្រឹស្តីបទឡូពីតាល់ ៖

ដេរីវេភាគយក ៖ $\frac{d}{dx}(x^3-2x^2+x)=3x^2-4x+1$

ដេរីវេភាគបែង ៖ $\frac{d}{dx}(x^2-1)=2x$

$$\lim_{x\to1}\frac{3x^2-4x+1}{2x}=\frac{3(1)^2-4(1)+1}{2(1)}$$
$$=\frac{3-4+1}{2}$$
$$=\frac{0}{2}$$
$$=0$$

ដូចនេះ លីមីតស្មើនឹង 0 ។

ឋ. $\lim_{x\to3}\frac{x^3-9x}{x^4-3x^3-x+3}$

ជំនួស $x=3$ បានរាង $\frac{0}{0}$ ។ ប្រើឡូពីតាល់ ៖

ដេរីវេភាគយក ៖ $\frac{d}{dx}(x^3-9x)=3x^2-9$

ដេរីវេភាគបែង ៖ $\frac{d}{dx}(x^4-3x^3-x+3)=4x^3-9x^2-1$

ជំនួស $x=3$ ៖

ភាគយក ៖ $3(9)-9=27-9=18$

ភាគបែង ៖ $4(27)-9(9)-1=108-81-1=26$

$$\lim_{x\to3}\frac{3x^2-9}{4x^3-9x^2-1}=\frac{18}{26}$$
$$=\frac{9}{13}$$

ដូចនេះ លីមីតស្មើនឹង $\frac{9}{13}$ ។

ឌ. $\lim_{x\to1}\frac{\sqrt[3]{8x}-2x}{\sqrt[4]{x}-x}$

ជំនួស $x=1$ បានរាង $\frac{0}{0}$ ។ ប្រើឡូពីតាល់ ៖

$\sqrt[3]{8x}=2x^{1/3}$ នាំឲ្យដេរីវេគឺ $2\cdot\frac{1}{3}x^{-2/3}=\frac{2}{3}x^{-2/3}$ ។

ភាគយកដេរីវេបាន ៖ $\frac{2}{3}x^{-2/3}-2$ ។

ភាគបែង $\sqrt[4]{x}=x^{1/4}$ ដេរីវេបាន ៖ $\frac{1}{4}x^{-3/4}-1$ ។

ជំនួស $x=1$ ចំពោះកន្សោមថ្មី ៖

$$\begin{aligned}\lim_{x\to1}\frac{\frac{2}{3}x^{-2/3}-2}{\frac{1}{4}x^{-3/4}-1}&=\frac{\frac{2}{3}(1)-2}{\frac{1}{4}(1)-1}\\&=\frac{\frac{2}{3}-\frac{6}{3}}{\frac{1}{4}-\frac{4}{4}}\\&=\frac{-4/3}{-3/4}\\&=\frac{16}{9}\end{aligned}$$

ដូចនេះ លីមីតស្មើនឹង $\frac{16}{9}$ ។

ណ. $\lim_{x\to0}\frac{5^x-1}{x}$

រាង $\frac{0}{0}$ ។ ប្រើឡូពីតាល់ ៖

ដេរីវេភាគយក ៖ $\frac{d}{dx}(5^x-1)=5^x\ln5$ ។

ដេរីវេភាគបែង ៖ $\frac{d}{dx}(x)=1$ ។

$$\begin{aligned}\lim_{x\to0}\frac{5^x\ln5}{1}&=5^0\ln5\\&=1\cdot\ln5\\&=\ln5\end{aligned}$$

ដូចនេះ លីមីតស្មើនឹង $\ln5$ ។

ថ. $\lim_{x\to0}\frac{e^x-e^{-x}}{x}$

រាង $\frac{0}{0}$ ។ ប្រើឡូពីតាល់ ៖

ដេរីវេភាគយក ៖ $e^x-e^{-x}(-1)=e^x+e^{-x}$ ។

ដេរីវេភាគបែង ៖ 1 ។

$$\begin{aligned}&\lim_{x\to0}\frac{e^x+e^{-x}}{1}\\&=e^0+e^0\\&=1+1\\&=2\end{aligned}$$

ដូចនេះ លីមីតស្មើនឹង 2 ។

ឍ. $\lim_{x\to1}\frac{\sqrt{x+1}-\sqrt{2}}{x^2-1}$

រាង $\frac{0}{0}$ ។ ប្រើឡូពីតាល់ ៖

ដេរីវេភាគយក ៖ $\frac{d}{dx}(\sqrt{x+1})=\frac{1}{2\sqrt{x+1}}$ ។

ដេរីវេភាគបែង ៖ $\frac{d}{dx}(x^2-1)=2x$ ។

$$\begin{aligned}\lim_{x\to1}\frac{\frac{1}{2\sqrt{x+1}}}{2x}&=\frac{\frac{1}{2\sqrt{2}}}{2}\\&=\frac{1}{4\sqrt{2}}\\&=\frac{\sqrt{2}}{8}\end{aligned}$$

ដូចនេះ លីមីតស្មើនឹង $\frac{\sqrt{2}}{8}$ ។

ត. $\lim_{x\to1}\frac{3x^3-3}{3^x-3}$

រាង $\frac{0}{0}$ ។ ប្រើឡូពីតាល់ ៖

ដេរីវេភាគយក ៖ $9x^2$ ។

ដេរីវេភាគបែង ៖ $3^x\ln3$ ។

$$\begin{aligned}\lim_{x\to1}\frac{9x^2}{3^x\ln3}&=\frac{9(1)^2}{3^1\ln3}\\&=\frac{9}{3\ln3}\\&=\frac{3}{\ln3}\end{aligned}$$

ដូចនេះ លីមីតស្មើនឹង $\frac{3}{\ln3}$ ។

ទ. $\lim_{x\to0}\frac{e^x-1}{x^2}$

រាង $\frac{0}{0}$ ។ ប្រើឡូពីតាល់ ៖

ដេរីវេភាគយក ៖ e^x ។

ដេរីវេភាគបែង ៖ $2x$ ។

$$\begin{aligned}&\lim_{x\to0}\frac{e^x}{2x}\\&=\frac{1}{0}\end{aligned}$$

នេះលែងជារាងមិនកំណត់ហើយ វាស្មើនឹង ∞ (គ្មានលីមីតកំណត់) ។

ឆ. $\lim_{x\to\frac{\pi}{3}} \frac{1-\cos x}{\pi-3x}$
ជំនួស $x=\pi/3$ ៖ $1-\cos(\pi/3)=1-1/2=1/2$ និង $\pi-3(\pi/3)=0$ ។
រាងគឺ $\frac{1/2}{0}$ ដូច្នេះលីមីតស្មើ ∞ ។ មិនចាំបាច់ប្រើឡូពីតាល់ទេ។

ជ. $\lim_{x\to 0} \frac{\sin x}{e^x-1}$
រាង $\frac{0}{0}$ ។ ប្រើឡូពីតាល់ ៖
ដេរីវេភាគយក ៖ $\cos x$ ។
ដេរីវេភាគបែង ៖ e^x ។

$$\lim_{x\to 0}\frac{\cos x}{e^x}=\frac{\cos 0}{e^0}=\frac{1}{1}=1$$

ដូចនេះ លីមីតស្មើនឹង 1 ។

ដំណោះស្រាយលំហាត់ទី២១

ប្រធាន៖ ដោយប្រើទ្រឹស្តីបទឡូពីតាល់ ចូរដោះស្រាយលីមីតខាងក្រោម៖

ក. $\lim_{x\to 0} \frac{\arctan 3x}{\arcsin 2x}$
ជំនួស $x=0$ បានរាង $\frac{0}{0}$ ។ អនុវត្តវិធានឡូពីតាល់ដោយធ្វើដេរីវេភាគយក និងភាគបែង ៖
ដេរីវេភាគយក ៖ $\frac{d}{dx}(\arctan 3x)=\frac{(3x)'}{1+(3x)^2}=\frac{3}{1+9x^2}$ ។
ដេរីវេភាគបែង ៖ $\frac{d}{dx}(\arcsin 2x)=\frac{(2x)'}{\sqrt{1-(2x)^2}}=\frac{2}{\sqrt{1-4x^2}}$ ។
ជំនួសចូលលីមីត ៖

$$\lim_{x\to 0}\frac{\frac{3}{1+9x^2}}{\frac{2}{\sqrt{1-4x^2}}}=\frac{\frac{3}{1}}{\frac{2}{1}}=\frac{3}{2}$$

ដូចនេះ លីមីតស្មើនឹង $\frac{3}{2}$ ។

ខ. $\lim_{x\to 0} \frac{e^x-e^{-x}}{2x-\sin x}$
ជំនួស $x=0$ បានរាង $\frac{0}{0}$ ។ ប្រើឡូពីតាល់ ៖
ដេរីវេភាគយក ៖ $e^x-e^{-x}(-1)=e^x+e^{-x}$ ។
ដេរីវេភាគបែង ៖ $2-\cos x$ ។
ជំនួស $x=0$ ៖

$$\lim_{x\to 0}\frac{e^x+e^{-x}}{2-\cos x}=\frac{1+1}{2-1}=\frac{2}{1}=2$$

ដូចនេះ លីមីតស្មើនឹង 2 ។

គ. $\lim_{x\to 0} \frac{x^3+\pi x}{\sin 3x}$
រាង $\frac{0}{0}$ ។ ប្រើឡូពីតាល់ ៖
ដេរីវេភាគយក ៖ $3x^2+\pi$ ។
ដេរីវេភាគបែង ៖ $3\cos 3x$ ។
ជំនួស $x=0$ ៖

$$\lim_{x\to 0}\frac{3x^2+\pi}{3\cos 3x}=\frac{0+\pi}{3(1)}=\frac{\pi}{3}$$

ដូចនេះ លីមីតស្មើនឹង $\frac{\pi}{3}$ ។

ឃ. $\lim_{x\to 0} \frac{\ln(1+4x)}{3^x-1}$
រាង $\frac{0}{0}$ ។ ប្រើឡូពីតាល់ ៖
ដេរីវេភាគយក ៖ $\frac{4}{1+4x}$ ។
ដេរីវេភាគបែង ៖ $3^x\ln 3$ ។
ជំនួស $x=0$ ៖

$$\lim_{x\to 0}\frac{\frac{4}{1+4x}}{3^x\ln 3}=\frac{4}{1\cdot\ln 3}=\frac{4}{\ln 3}$$

ដូចនេះ លីមីតស្មើនឹង $\frac{4}{\ln 3}$ ។

ង. $\lim_{x\to 0}\frac{\sin 4x}{\ln(1+\sin x)}$

រាង $\frac{0}{0}$ ។ ប្រើឡូពីតាល់ ៖

ដេរីវេភាគយក ៖ $4\cos 4x$ ។

ដេរីវេភាគបែង ៖ $\frac{\cos x}{1+\sin x}$ ។

ជំនួស $x=0$ ៖

$$\lim_{x\to 0}\frac{4\cos 4x}{\frac{\cos x}{1+\sin x}}=\frac{4(1)}{\frac{1}{1+0}}=4$$

ដូចនេះ លីមីតស្មើនឹង 4 ។

ច. $\lim_{x\to 0}\frac{3\ln(1-2x)}{2\arctan 3x}$

រាង $\frac{0}{0}$ ។ ប្រើឡូពីតាល់ ៖

ដេរីវេភាគយក ៖ $3\cdot\frac{-2}{1-2x}=\frac{-6}{1-2x}$ ។

ដេរីវេភាគបែង ៖ $2\cdot\frac{3}{1+9x^2}=\frac{6}{1+9x^2}$ ។

ជំនួស $x=0$ ៖

$$\lim_{x\to 0}\frac{\frac{-6}{1-2x}}{\frac{6}{1+9x^2}}=\frac{-6/1}{6/1}=-1$$

ដូចនេះ លីមីតស្មើនឹង -1 ។

ឆ. $\lim_{x\to 0}\frac{\arctan x+x^2}{2^{3x}-3^{2x}}$

រាង $\frac{0}{0}$ ។ ប្រើឡូពីតាល់ ៖

ដេរីវេភាគយក ៖ $\frac{1}{1+x^2}+2x$ ។

ដេរីវេ ភាគបែង ៖ $2^{3x}\ln 2\cdot(3)-3^{2x}\ln 3\cdot(2)=3\cdot 2^{3x}\ln 2-2\cdot 3^{2x}\ln 3$ ។

ជំនួស $x=0$ ៖

$$\lim_{x\to 0}\frac{\frac{1}{1+0}+0}{3\cdot 1\cdot\ln 2-2\cdot 1\cdot\ln 3}=\frac{1}{3\ln 2-2\ln 3}$$

ដោយ $3\ln 2-2\ln 3=\ln(2^3)-\ln(3^2)=\ln(8)-\ln(9)=\ln(8/9)$ ។

ដូចនេះ លីមីតស្មើនឹង $\frac{1}{\ln(8/9)}$ ។

ជ. $\lim_{x\to\frac{\pi}{2}}\frac{\arctan(x-\frac{\pi}{2})}{\pi-2x}$

រាង $\frac{0}{0}$ ។ ប្រើឡូពីតាល់ ៖

ដេរីវេភាគយក ៖ $\frac{1}{1+(x-\pi/2)^2}$ ។

ដេរីវេភាគបែង ៖ -2 ។

ជំនួស $x=\pi/2$ ៖

$$\lim_{x\to\frac{\pi}{2}}\frac{\frac{1}{1+0}}{-2}=-\frac{1}{2}$$

ដូចនេះ លីមីតស្មើនឹង $-\frac{1}{2}$ ។

ឈ. $\lim_{x\to 0}\frac{e^{3x}-e^{-2x}}{2\arcsin x-\sin x}$

រាង $\frac{0}{0}$ ។ ប្រើឡូពីតាល់ ៖

ដេរីវេភាគយក ៖ $3e^{3x}-e^{-2x}(-2)=3e^{3x}+2e^{-2x}$ ។

ដេរីវេភាគបែង ៖ $\frac{2}{\sqrt{1-x^2}}-\cos x$ ។

ជំនួស $x=0$ ៖

$$\lim_{x\to 0}\frac{3(1)+2(1)}{\frac{2}{1}-1}=\frac{5}{2-1}=5$$

ដូចនេះ លីមីតស្មើនឹង 5 ។

ញ. $\lim_{x\to 0}\frac{\ln(\cos 3x)}{\arctan 4x}$

រាង $\frac{0}{0}$ ។ ប្រើឡូពីតាល់ ៖

ដេរីវេភាគយក ៖ $\frac{(\cos 3x)'}{\cos 3x}=\frac{-3\sin 3x}{\cos 3x}=-3\tan 3x$ ។

ដេរីវេភាគបែង ៖ $\frac{4}{1+16x^2}$ ។

ជំនួស $x=0$ ៖

$$\lim_{x\to 0}\frac{-3\tan 0}{\frac{4}{1+0}}=\frac{0}{4}=0$$

ដូចនេះ លីមីតស្មើនឹង 0 ។

ដ. $\lim\limits_{x\to 0}\frac{(1+x)^2-(1+2x)}{x^2+4x^3}$

ជំនួស $x=0$ បានរាង $\frac{0}{0}$ ។

យើងអាចពន្លាតភាគយកមុននឹងធ្វើដេរីវេ ៖

$(1+x)^2-(1+2x)=1+2x+x^2-1-2x=x^2$ ។

លីមីតក្លាយជា ៖

$$\begin{aligned}\lim_{x\to 0}\frac{x^2}{x^2+4x^3}&=\lim_{x\to 0}\frac{x^2}{x^2(1+4x)}\\&=\lim_{x\to 0}\frac{1}{1+4x}\\&=\frac{1}{1}\\&=1\end{aligned}$$

ដូចនេះ លីមីតស្មើនឹង 1 ។

ឋ. $\lim\limits_{x\to -1}\frac{(x^2+3x+2)^2}{x^3-3x-2}$

ជំនួស $x=-1$ បានរាង $\frac{0}{0}$ ។ បំបែកជាកត្តាសិន ៖

$x^2+3x+2=(x+1)(x+2)$ ។ ភាគយកគឺ $(x+1)^2(x+2)^2$ ។

ភាគបែង x^3-3x-2 ចែកដាច់នឹង $x+1$ (ឬសាកឫស គឺ -1) ៖ $x^3-3x-2=(x+1)^2(x-2)$ ។

លីមីតក្លាយជា ៖

$$\begin{aligned}\lim_{x\to -1}\frac{(x+1)^2(x+2)^2}{(x+1)^2(x-2)}&=\lim_{x\to -1}\frac{(x+2)^2}{x-2}\\&=\frac{(-1+2)^2}{-1-2}\\&=\frac{1}{-3}\\&=-\frac{1}{3}\end{aligned}$$

ដូចនេះ លីមីតស្មើនឹង $-\frac{1}{3}$ ។

ឌ. $\lim\limits_{x\to 0}\frac{1-\cos^4 x}{4x^2}$

រាង $\frac{0}{0}$ ។ ប្រើឡូពីតាល់ ៖

ដេរីវេភាគយក ៖ $-4\cos^3 x(-\sin x)=4\cos^3 x\sin x$ ។

ដេរីវេភាគបែង ៖ $8x$ ។

$$\begin{aligned}\lim_{x\to 0}\frac{4\cos^3 x\sin x}{8x}&=\lim_{x\to 0}\frac{1}{2}\cos^3 x\frac{\sin x}{x}\\&=\frac{1}{2}(1)^3(1)\\&=\frac{1}{2}\end{aligned}$$

ដូចនេះ លីមីតស្មើនឹង $\frac{1}{2}$ ។

ឍ. $\lim\limits_{x\to 0}\frac{1-\cos x}{2x\sin x}$

រាង $\frac{0}{0}$ ។ ប្រើឡូពីតាល់ ៖

ដេរីវេភាគយក ៖ $\sin x$ ។

ដេរីវេ ភាគបែង ៖ $2\sin x+2x\cos x$ (តាមវិធាន ដេរីវេ ផលគុណ) ។

$$\lim_{x\to 0}\frac{\sin x}{2\sin x+2x\cos x}$$

នៅតែមានរាង $\frac{0}{0}$ ។ ប្រើឡូពីតាល់ម្តងទៀត ៖

ដេរីវេភាគយកម្តងទៀត ៖ $\cos x$ ។

ដេរីវេភាគបែងម្តងទៀត ៖ $2\cos x+2(\cos x-x\sin x)=4\cos x-2x\sin x$ ។

ជំនួស $x=0$ ៖

$$\begin{aligned}\lim_{x\to 0}\frac{\cos x}{4\cos x-2x\sin x}&=\frac{1}{4(1)-0}\\&=\frac{1}{4}\end{aligned}$$

ដូចនេះ លីមីតស្មើនឹង $\frac{1}{4}$ ។

ណ. $\lim_{x\to 0}\frac{x-\sin x}{e^x-e^{-x}-2x}$

រាង $\frac{0}{0}$ ។ ប្រើឡូពីតាល់ ៖

ដេរីវេទី១ ៖

$$\frac{1-\cos x}{e^x+e^{-x}-2}\to\frac{0}{0}$$

ដេរីវេទី២ ៖

$$\frac{\sin x}{e^x-e^{-x}}\to\frac{0}{0}$$

ដេរីវេទី៣ ៖

$$\frac{\cos x}{e^x+e^{-x}}$$

ជំនួស $x=0$ ៖

$$\begin{aligned}&\frac{\cos 0}{e^0+e^{-0}}\\&=\frac{1}{1+1}\\&=\frac{1}{2}\end{aligned}$$

ដូចនេះ លីមីតស្មើនឹង $\frac{1}{2}$ ។

ត. $\lim_{x\to 0}\frac{e^{3x}-3x-1}{\sin^2 x}$

រាង $\frac{0}{0}$ ។ ប្រើឡូពីតាល់ ៖

ដេរីវេទី១ ៖

$$\begin{aligned}&\frac{3e^{3x}-3}{2\sin x\cos x}\\&=\frac{3e^{3x}-3}{\sin 2x}\to\frac{0}{0}\end{aligned}$$

ដេរីវេទី២ ៖

$$\frac{9e^{3x}}{2\cos 2x}$$

ជំនួស $x=0$ ៖

$$\frac{9(1)}{2(1)}=\frac{9}{2}$$

ដូចនេះ លីមីតស្មើនឹង $\frac{9}{2}$ ។

ថ. $\lim_{x\to 0}\frac{x^3}{x-\arctan x}$

រាង $\frac{0}{0}$ ។ ប្រើឡូពីតាល់ ៖

ដេរីវេទី១ ៖

$$\begin{aligned}\frac{3x^2}{1-\frac{1}{1+x^2}}&=\frac{3x^2}{\frac{1+x^2-1}{1+x^2}}\\&=\frac{3x^2}{\frac{x^2}{1+x^2}}\\&=3(1+x^2)\end{aligned}$$

ជំនួស $x=0$ ៖ $3(1+0)=3$ ។

ដូចនេះ លីមីតស្មើនឹង 3 ។

ទ. $\lim_{x\to 1}\frac{\sin(\ln x)}{\ln x}$

តាង $u=\ln x$ ។ កាលណា $x\to 1$ នោះ $u\to 0$។

$$\lim_{x\to 1}\frac{\sin(\ln x)}{\ln x}=\lim_{u\to 0}\frac{\sin u}{u}=1$$

ដូចនេះ $\lim_{x\to 1}\frac{\sin(\ln x)}{\ln x}=1$។

ធ. $\lim\limits_{x\to-\infty}\frac{\tan(e^x)}{e^x}$

តាង $u=e^x$ ។ កាលណា $x\to-\infty$ នោះ $u\to 0$។

$$\lim_{x\to-\infty}\frac{\tan(e^x)}{e^x}=\lim_{u\to0}\frac{\tan u}{u}=1$$

ដូចនេះ $\lim\limits_{x\to-\infty}\frac{\tan(e^x)}{e^x}=1$។

ន. $\lim\limits_{x\to0}\frac{\sin(x^2)}{x}$

គុណភាគយក និងភាគបែងនឹង x៖

$$\lim_{x\to0}\frac{\sin(x^2)}{x}=\lim_{x\to0}\left(\frac{\sin(x^2)}{x^2}\cdot x\right)=1\cdot0=0$$

ដូចនេះ $\lim\limits_{x\to0}\frac{\sin(x^2)}{x}=0$។

ប. $\lim\limits_{x\to0}\frac{\sin(\sin^2 2x)}{x^2}$

បំបែកកន្សោមដោយប្រើរូបមន្តលីមីតត្រីកោណមាត្រ៖

$$\lim_{x\to0}\frac{\sin(\sin^2 2x)}{x^2}=\lim_{x\to0}\left[\frac{\sin(\sin^2 2x)}{\sin^2 2x}\cdot\frac{\sin^2 2x}{(2x)^2}\cdot4\right]$$
$$=1\cdot1^2\cdot4=4$$

ដូចនេះ $\lim\limits_{x\to0}\frac{\sin(\sin^2 2x)}{x^2}=4$។

ផ. $\lim\limits_{x\to0}\frac{1-\cos x}{x^2}$

ប្រើរូបមន្ត $1-\cos x=2\sin^2\left(\frac{x}{2}\right)$៖

$$\lim_{x\to0}\frac{1-\cos x}{x^2}=\lim_{x\to0}\frac{2\sin^2\left(\frac{x}{2}\right)}{x^2}=\lim_{x\to0}2\left(\frac{\sin\left(\frac{x}{2}\right)}{\frac{x}{2}}\cdot\frac{1}{2}\right)^2=2\cdot\left(1\cdot\right.$$

ដូចនេះ $\lim\limits_{x\to0}\frac{1-\cos x}{x^2}=\frac{1}{2}$។

ព. $\lim\limits_{x\to1}\frac{\cos\left(\frac{\pi x}{2}\right)}{1-x}$

តាង $u=1-x\Rightarrow x=1-u$ ។ កាលណា $x\to1$ នោះ $u\to0$។

$$\lim_{x\to1}\frac{\cos\left(\frac{\pi x}{2}\right)}{1-x}=\lim_{u\to0}\frac{\cos\left(\frac{\pi(1-u)}{2}\right)}{u}$$
$$=\lim_{u\to0}\frac{\cos\left(\frac{\pi}{2}-\frac{\pi u}{2}\right)}{u}$$
$$=\lim_{u\to0}\frac{\sin\left(\frac{\pi u}{2}\right)}{u}=\lim_{u\to0}\left[\frac{\sin\left(\frac{\pi u}{2}\right)}{\frac{\pi u}{2}}\cdot\frac{\pi}{2}\right]=1\cdot\frac{\pi}{2}=\frac{\pi}{2}$$

ដូចនេះ $\lim\limits_{x\to1}\frac{\cos\left(\frac{\pi x}{2}\right)}{1-x}=\frac{\pi}{2}$។

ភ. $\lim\limits_{x\to0}\frac{(1-\cos2x)\sin5x}{x^2\sin3x}$

ប្រើរូបមន្ត $1-\cos2x=2\sin^2x$៖

$$\lim_{x\to0}\frac{(1-\cos2x)\sin5x}{x^2\sin3x}=\lim_{x\to0}\frac{2\sin^2x\cdot\sin5x}{x^2\sin3x}$$
$$=\lim_{x\to0}2\left(\frac{\sin x}{x}\right)^2\cdot\left(\frac{\sin5x}{5x}\right)\cdot5\cdot\left(\frac{3x}{\sin3x}\right.$$
$$=2\cdot(1)^2\cdot1\cdot5\cdot1\cdot\frac{1}{3}=\frac{10}{3}$$

ដូចនេះ $\lim\limits_{x\to0}\frac{(1-\cos2x)\sin5x}{x^2\sin3x}=\frac{10}{3}$។

ម. $\lim\limits_{x\to0}\frac{\sin4x}{1-\sqrt{1-x}}$

គុណ ភាគយក និង ភាគបែង នឹង កន្សោម ឆ្លាស់ $1+\sqrt{1-x}$៖

$$\begin{aligned}\lim_{x\to0}\frac{\sin4x}{1-\sqrt{1-x}}&=\lim_{x\to0}\frac{\sin4x(1+\sqrt{1-x})}{(1-\sqrt{1-x})(1+\sqrt{1-x})}\\&=\lim_{x\to0}\frac{\sin4x(1+\sqrt{1-x})}{1-(1-x)}\\&=\lim_{x\to0}\frac{\sin4x(1+\sqrt{1-x})}{x}\\&=\lim_{x\to0}\left(\frac{\sin4x}{4x}\cdot4\cdot(1+\sqrt{1-x})\right)\\&=1\cdot4\cdot(1+\sqrt{1-0})=4\cdot2=8\end{aligned}$$

ដូចនេះ $\lim\limits_{x\to0}\frac{\sin4x}{1-\sqrt{1-x}}=8$។

រ. $\lim\limits_{x\to0}\frac{\cos x+4\tan x}{2-x-2x^4}$

ជំនួស $x=0$ ចូលក្នុងកន្សោម៖

$$\lim_{x\to0}\frac{\cos x+4\tan x}{2-x-2x^4}=\frac{\cos(0)+4\tan(0)}{2-0-2(0)^4}=\frac{1+4(0)}{2}=\frac{1}{2}$$

ដូចនេះ $\lim\limits_{x\to0}\frac{\cos x+4\tan x}{2-x-2x^4}=\frac{1}{2}$។

យ. $\lim\limits_{x\to0}\frac{\sin x}{\sqrt{x^2}}$

ដោយសារ $\sqrt{x^2}=|x|$ យើងត្រូវគណនាលីមីតឆ្វេង និងស្តាំ៖

លីមីតខាងស្តាំ $(x\to0^+)$ នោះ $x>0\Rightarrow|x|=x$៖

$$\lim_{x\to0^+}\frac{\sin x}{|x|}=\lim_{x\to0^+}\frac{\sin x}{x}=1$$

លីមីតខាងឆ្វេង $(x\to0^-)$ នោះ $x<0\Rightarrow|x|=-x$៖

$$\lim_{x\to0^-}\frac{\sin x}{|x|}=\lim_{x\to0^-}\frac{\sin x}{-x}=-1$$

ដោយសារលីមីតឆ្វេងមិនស្មើនឹងលីមីតស្តាំ។

ដូចនេះ $\lim\limits_{x\to0}\frac{\sin x}{\sqrt{x^2}}$ គ្មានលីមីតទេ។

ល. $\lim\limits_{x\to0}\frac{\sqrt{x+4}-2}{\sin5x}$

គុណភាគយក និងភាគបែងនឹងកន្សោមឆ្លាស់ $\sqrt{x+4}+2$៖

$$\begin{aligned}\lim_{x\to0}\frac{\sqrt{x+4}-2}{\sin5x}&=\lim_{x\to0}\frac{(\sqrt{x+4}-2)(\sqrt{x+4}+2)}{\sin5x(\sqrt{x+4}+2)}\\&=\lim_{x\to0}\frac{x+4-4}{\sin5x(\sqrt{x+4}+2)}\\&=\lim_{x\to0}\frac{x}{\sin5x(\sqrt{x+4}+2)}\\&=\lim_{x\to0}\left[\frac{5x}{\sin5x}\cdot\frac{1}{5(\sqrt{x+4}+2)}\right]\\&=1\cdot\frac{1}{5(\sqrt{0+4}+2)}=\frac{1}{5(2+2)}=\frac{1}{20}\end{aligned}$$

ដូចនេះ $\lim\limits_{x\to0}\frac{\sqrt{x+4}-2}{\sin5x}=\frac{1}{20}$។

ឃ. $\lim\limits_{x\to1}\frac{\sin(1-x)}{\sqrt{x}-1}$

គុណភាគយក និងភាគបែងនឹងកន្សោមឆ្លាស់ $\sqrt{x}+1$៖

$$\begin{aligned}\lim_{x\to1}\frac{\sin(1-x)}{\sqrt{x}-1}&=\lim_{x\to1}\frac{\sin(1-x)(\sqrt{x}+1)}{(\sqrt{x}-1)(\sqrt{x}+1)}\\&=\lim_{x\to1}\frac{\sin(1-x)(\sqrt{x}+1)}{x-1}\\&=\lim_{x\to1}\left[\frac{\sin(1-x)}{-(1-x)}\cdot(\sqrt{x}+1)\right]\\&=-1\cdot(\sqrt{1}+1)=-2\end{aligned}$$

ដូចនេះ $\lim\limits_{x\to1}\frac{\sin(1-x)}{\sqrt{x}-1}=-2$។

ស. $\lim\limits_{a\to\pi}\frac{\sin a}{1-\frac{a^2}{\pi^2}}$

តាង $u=\pi-a\Rightarrow a=\pi-u$ ។ កាលណា $a\to\pi$ នោះ $u\to0$។

$$\begin{aligned}\lim_{a\to\pi}\frac{\sin a}{1-\frac{a^2}{\pi^2}}&=\lim_{u\to0}\frac{\sin(\pi-u)}{1-\frac{(\pi-u)^2}{\pi^2}}\\&=\lim_{u\to0}\frac{\sin u}{\frac{\pi^2-(\pi^2-2\pi u+u^2)}{\pi^2}}\\&=\lim_{u\to0}\frac{\sin u}{\frac{2\pi u-u^2}{\pi^2}}\\&=\lim_{u\to0}\left[\frac{\sin u}{u}\cdot\frac{\pi^2}{2\pi-u}\right]\\&=1\cdot\frac{\pi^2}{2\pi-0}=\frac{\pi}{2}\end{aligned}$$

ដូចនេះ $\lim\limits_{a\to\pi}\frac{\sin a}{1-\frac{a^2}{\pi^2}}=\frac{\pi}{2}$។

ដំណោះស្រាយលំហាត់ប្រឡងថ្នាក់ជាតិ

ដំណោះស្រាយលំហាត់ទី២២

ប្រធាន៖ គណនាលីមីត $\lim\limits_{x\to 0}\frac{2-\sqrt{x+4}}{x}$។

ចម្លើយ៖ យើងមានលីមីតមានរាងមិនកំណត់ $\frac{0}{0}$

$$\begin{aligned}
\lim_{x\to 0}\frac{2-\sqrt{x+4}}{x} &= \lim_{x\to 0}\frac{(2-\sqrt{x+4})(2+\sqrt{x+4})}{x(2+\sqrt{x+4})}\\
&= \lim_{x\to 0}\frac{4-(x+4)}{x(2+\sqrt{x+4})}\\
&= \lim_{x\to 0}\frac{-x}{x(2+\sqrt{x+4})}\\
&= \lim_{x\to 0}\frac{-1}{2+\sqrt{x+4}}\\
&= \frac{-1}{2+\sqrt{4}} = -\frac{1}{4}
\end{aligned}$$

ដូចនេះ $\lim\limits_{x\to 0}\frac{2-\sqrt{x+4}}{x} = -\frac{1}{4}$។

ដំណោះស្រាយលំហាត់ទី២៣

ប្រធាន៖ fជាអនុគមន៍កំណត់ដោយ $f(x)=\begin{cases} x-2+\dfrac{e}{\ln x} & \text{បើ} x>0 \\ m; & \text{បើ} x=0 \end{cases}$ ។ គណនា $\lim\limits_{x\to 0^+} f(x)$។

កំណត់តម្លៃmដើម្បីអោយអនុគមន៍fជាប់ខាងស្ដាំត្រង់$x=0$។

ចម្លើយ៖

- **គណនាលីមីតខាងស្ដាំត្រង់** 0៖

$$\lim_{x\to 0^+} f(x)=\lim_{x\to 0^+}\left(x-2+\frac{e}{\ln x}\right)$$

ដោយ $\lim\limits_{x\to 0^+}\ln x=-\infty$ នាំឲ្យ $\lim\limits_{x\to 0^+}\dfrac{e}{\ln x}=0$។ ដូចនេះ $\lim\limits_{x\to 0^+} f(x)=0-2+0=-2$។

- **កំណត់តម្លៃ** m៖ ដើម្បីឲ្យអនុគមន៍ f ជាប់ខាងស្ដាំត្រង់ $x=0$ លុះត្រាតែ $\lim\limits_{x\to 0^+} f(x)=f(0)$។ ដោយ $f(0)=m$ និង $\lim\limits_{x\to 0^+} f(x)=-2$ យើងបាន $m=-2$។

ដំណោះស្រាយលំហាត់ទី២៤

ប្រធាន៖ គណនាលីមីត៖

ក. $\lim_{x\to 0}\frac{\sin x+e^x-1}{x^2+x}$

ខ. $\lim_{x\to +\infty}\frac{\ln x+2}{x+1}$

ចម្លើយ៖

ក. គណនា $\lim_{x\to 0}\frac{\sin x+e^x-1}{x^2+x}$ (រាងមិនកំណត់ $\frac{0}{0}$)

$$\lim_{x\to 0}\frac{\sin x+e^x-1}{x(x+1)}=\lim_{x\to 0}\frac{\frac{\sin x}{x}+\frac{e^x-1}{x}}{x+1}$$
$$=\frac{1+1}{0+1}=2$$

ដូចនេះ $\lim_{x\to 0}\frac{\sin x+e^x-1}{x^2+x}=2$។

ខ. គណនា $\lim_{x\to +\infty}\frac{\ln x+2}{x+1}$ (រាងមិនកំណត់ $\frac{\infty}{\infty}$)

$$\lim_{x\to +\infty}\frac{\ln x+2}{x+1}=\lim_{x\to +\infty}\frac{x\left(\frac{\ln x}{x}+\frac{2}{x}\right)}{x\left(1+\frac{1}{x}\right)}=\lim_{x\to +\infty}\frac{\frac{\ln x}{x}+\frac{2}{x}}{1+\frac{1}{x}}$$
$$=\frac{0+0}{1+0}=0$$

ដូចនេះ $\lim_{x\to +\infty}\frac{\ln x+2}{x+1}=0$។

ដំណោះស្រាយលំហាត់ទី២៥

ប្រធាន៖

ក. គណនាលីមីត៖ $\lim\limits_{x\to+\infty}\dfrac{e^x\ln x+1}{x^2}$។

ខ. គណនាដេរីវេនៃ $y=e^x\ln(x+1), x>-1$។ ទាញរកព្រីមីទីវនៃ $f(x)=e^x\ln(x+1)+\dfrac{e^x+1}{x+1}$ ដោយដឹងថាព្រីមីទីវមានតម្លៃស្មើ1ចំពោះ $x=0$។

ចម្លើយ៖

ក. **គណនាលីមីត៖**

$$\lim_{x\to+\infty}\frac{e^x\ln x+1}{x^2}=\lim_{x\to+\infty}\left(\frac{e^x}{x^2}\ln x+\frac{1}{x^2}\right)$$

ដោយ $\lim\limits_{x\to+\infty}\dfrac{e^x}{x^2}=+\infty$ និង $\lim\limits_{x\to+\infty}\ln x=+\infty$ នាំឲ្យលីមីតស្មើ $+\infty$។ ដូចនេះ $\lim\limits_{x\to+\infty}\dfrac{e^x\ln x+1}{x^2}=+\infty$។

ខ. **គណនាដេរីវេ និង ទាញរកព្រីមីទីវ៖** យើងមាន $y=e^x\ln(x+1)$

$$\begin{aligned}y'&=(e^x)'\ln(x+1)+e^x(\ln(x+1))'\\&=e^x\ln(x+1)+e^x\frac{1}{x+1}\\&=e^x\ln(x+1)+\frac{e^x}{x+1}\end{aligned}$$

យើងមាន $f(x)=e^x\ln(x+1)+\dfrac{e^x+1}{x+1}=e^x\ln(x+1)+\dfrac{e^x}{x+1}+\dfrac{1}{x+1}=y'+\dfrac{1}{x+1}$។ ដូចនេះព្រីមីទីវ $F(x)=y+\ln|x+1|+C=e^x\ln(x+1)+\ln(x+1)+C$ ដោយ $x>-1$។ ដោយ $F(0)=1\implies e^0\ln(1)+\ln(1)+C=1\implies C=1$។ ដូចនេះ $F(x)=e^x\ln(x+1)+\ln(x+1)+1$។

ដំណោះស្រាយលំហាត់ទី២៦

ប្រធាន៖ គណនាលីមីត៖

ក. $\lim_{x\to\frac{\pi}{3}} \dfrac{\sqrt{2\cos x}-1}{2\cos 2x+1}$

ខ. $\lim_{x\to 0} \dfrac{\frac{1}{(x+a)^2}-\frac{1}{a^2}}{x}$

ចម្លើយ៖

ក. គណនាលីមីត $\lim_{x\to\frac{\pi}{3}} \dfrac{\sqrt{2\cos x}-1}{2\cos 2x+1}$ (រាង $\frac{0}{0}$) តាង $u=x-\frac{\pi}{3} \implies x=u+\frac{\pi}{3}$ កាលណា $x\to\frac{\pi}{3}$ នោះ $u\to 0$។

$$\lim_{u\to 0} \frac{\sqrt{2\cos(u+\frac{\pi}{3})}-1}{2\cos(2u+\frac{2\pi}{3})+1} = \lim_{u\to 0} \frac{\sqrt{\cos u-\sqrt{3}\sin u}-1}{2(-\frac{1}{2}\cos 2u-\frac{\sqrt{3}}{2}\sin 2u)+1}$$
$$= \lim_{u\to 0} \frac{\sqrt{\cos u-\sqrt{3}\sin u}-1}{1-\cos 2u-\sqrt{3}\sin 2u}$$

ដោយប្រើរូបមន្ត $1-\cos 2u = 2\sin^2 u$ តែដោយស្មុគស្មាញ យើងអាចប្រើទ្រឹស្តីបទឡូពីតាល់៖

$$L = \lim_{x\to\frac{\pi}{3}} \frac{\frac{-2\sin x}{2\sqrt{2\cos x}}}{-4\sin 2x} = \lim_{x\to\frac{\pi}{3}} \frac{\sin x}{4\sin 2x\sqrt{2\cos x}}$$
$$= \frac{\frac{\sqrt{3}}{2}}{4(\frac{\sqrt{3}}{2})\sqrt{2(1/2)}} = \frac{1}{4}$$

ដូចនេះ លីមីតស្មើ $\frac{1}{4}$។

ខ. គណនាលីមីត $\lim_{x\to 0} \dfrac{\frac{1}{(x+a)^2}-\frac{1}{a^2}}{x}$ (រាង $\frac{0}{0}$)

$$L = \lim_{x\to 0} \frac{a^2-(x+a)^2}{x(x+a)^2a^2}$$
$$= \lim_{x\to 0} \frac{-x(2a+x)}{x(x+a)^2a^2} = \lim_{x\to 0} \frac{-(2a+x)}{(x+a)^2a^2}$$
$$= \frac{-2a}{a^4} = -\frac{2}{a^3}$$

ដូចនេះ លីមីតស្មើ $-\frac{2}{a^3}$។

ដំណោះស្រាយលំហាត់ទី២៧

ប្រធាន៖ គណនាលីមីត៖

ក. $\lim_{x\to\frac{\pi}{4}} \dfrac{2\sin\left(x-\frac{\pi}{4}\right)}{\frac{\pi}{4}-x}$

ខ. $\lim_{x\to 0} \dfrac{2-\sin 5x}{\sqrt{5}-\sqrt{x+5}}$

គ. $\lim_{x\to 0} \dfrac{1-\cos^2 3x}{-2x^2}$

ឃ. $\lim_{x\to 0} \dfrac{x^2+x}{|x|}$

ចម្លើយ៖

ក. គណនា $\lim_{x\to\frac{\pi}{4}} \dfrac{2\sin\left(x-\frac{\pi}{4}\right)}{\frac{\pi}{4}-x}$ តាង $u=x-\dfrac{\pi}{4}$ នោះ $u\to 0$។ លីមីតទៅជា៖ $\lim_{u\to 0}\dfrac{2\sin u}{-u} = -2\lim_{u\to 0}\dfrac{\sin u}{u} = -2(1) = -2$។

ខ. គណនា $\lim_{x\to 0} \dfrac{2-\sin 5x}{\sqrt{5}-\sqrt{x+5}}$ ត្រង់ $x=0$ កន្សោមភាគយក $2-\sin 0 = 2$ ឯភាគបែង $\sqrt{5}-\sqrt{5}=0$។ លីមីតមានរាង $\dfrac{2}{0}=\infty$។ ដោយលីមីតឆ្វេងស្តាំមានសញ្ញាខុសគ្នា ដូចនេះលីមីតគ្មានតម្លៃ។

គ. គណនា $\lim_{x\to 0} \dfrac{1-\cos^2 3x}{-2x^2} = \lim_{x\to 0}\dfrac{\sin^2 3x}{-2x^2} = -\dfrac{1}{2}\lim_{x\to 0}\left(\dfrac{\sin 3x}{3x}\times 3\right)^2 = -\dfrac{9}{2}$។

ឃ. គណនា $\lim_{x\to 0} \dfrac{x^2+x}{|x|}$ បើ $x\to 0^+$ លីមីតស្មើ 1។ បើ $x\to 0^-$ លីមីតស្មើ -1។ លីមីតគ្មានតម្លៃ។

ដំណោះស្រាយលំហាត់ទី២៨

ប្រធាន៖ គណនាលីមីត៖

ក. $\lim\limits_{x\to 1}\dfrac{x^3-x^2+x-1}{x-1}$

ខ. $\lim\limits_{x\to \frac{\pi}{2}}\dfrac{\sin^2 x-1}{1+\sin x}$

គ. $\lim\limits_{x\to +\infty}(\sqrt{x^2+x}-x)$

ឃ. $\lim\limits_{x\to 0}\dfrac{(e^{-x}+e^x)\sin^2 x}{2x^2}$

ង. $\lim\limits_{x\to +\infty}\left[\ln(x+2)-\ln x-\dfrac{2}{x+2}+\dfrac{1}{4}\right]$

ចម្លើយ៖

ក. $\lim\limits_{x\to 1}\dfrac{x^2(x-1)+(x-1)}{x-1}=\lim\limits_{x\to 1}\dfrac{(x-1)(x^2+1)}{x-1}=\lim\limits_{x\to 1}(x^2+1)=2$ ។

ខ. $\lim\limits_{x\to \frac{\pi}{2}}\dfrac{(\sin x-1)(\sin x+1)}{1+\sin x}=\lim\limits_{x\to \frac{\pi}{2}}(\sin x-1)=1-1=0$ ។

គ. $\lim\limits_{x\to +\infty}(\sqrt{x^2+x}-x)=\lim\limits_{x\to +\infty}\dfrac{x^2+x-x^2}{\sqrt{x^2+x}+x}=\lim\limits_{x\to +\infty}\dfrac{x}{x\sqrt{1+1/x}+x}=\lim\limits_{x\to +\infty}\dfrac{1}{\sqrt{1+1/x}+1}=\dfrac{1}{2}$ ។

ឃ. $\lim\limits_{x\to 0}\dfrac{e^{-x}+e^x}{2}\times\left(\dfrac{\sin x}{x}\right)^2=\dfrac{1+1}{2}\times(1)^2=1$ ។

ង. $\lim\limits_{x\to +\infty}\left[\ln\left(\dfrac{x+2}{x}\right)-\dfrac{2}{x+2}+\dfrac{1}{4}\right]=\ln(1)-0+\dfrac{1}{4}=\dfrac{1}{4}$ ។

ដំណោះស្រាយលំហាត់ទី២៩

ប្រធាន៖ គណនាលីមីត៖

ក. $\lim\limits_{x\to 2}\dfrac{x^3-8}{\sqrt{x+2}-2}$

ខ. $\lim\limits_{x\to 0}\dfrac{\cos x-1}{\sin^2 x}$

គ. $\lim\limits_{x\to 0}\dfrac{2\sin 3x}{x}$

ចម្លើយ៖

ក. $\lim\limits_{x\to 2}\dfrac{(x-2)(x^2+2x+4)(\sqrt{x+2}+2)}{x+2-4}=\lim\limits_{x\to 2}(x^2+2x+4)(\sqrt{x+2}+2)=(12)(4)=48$ ។

ខ. $\lim\limits_{x\to 0}\dfrac{-(1-\cos x)}{1-\cos^2 x}=\lim\limits_{x\to 0}\dfrac{-1}{1+\cos x}=-\dfrac{1}{2}$ ។

គ. $\lim\limits_{x\to 0}\dfrac{2\sin 3x}{3x}\times 3=2(1)\times 3=6$ ។

ដំណោះស្រាយលំហាត់ទី៣០

ប្រធាន៖ គណនាលីមីត៖

ក. $\lim\limits_{x\to 1}\dfrac{1-x^2}{x^2+2-3x}$

ខ. $\lim\limits_{x\to 3}\dfrac{\sqrt{x+6}-3}{x^3-27}$

គ. $\lim\limits_{x\to 0}\dfrac{5\sin 5x}{x}$

ចម្លើយ៖

ក. $\lim\limits_{x\to 1}\dfrac{(1-x)(1+x)}{(x-1)(x-2)}=\lim\limits_{x\to 1}\dfrac{-(x-1)(x+1)}{(x-1)(x-2)}=\lim\limits_{x\to 1}\dfrac{-(x+1)}{x-2}=\dfrac{-2}{-1}=2$ ។

ខ. $\lim\limits_{x\to 3}\dfrac{x+6-9}{(x-3)(x^2+3x+9)(\sqrt{x+6}+3)}=\lim\limits_{x\to 3}\dfrac{x-3}{(x-3)(x^2+3x+9)(\sqrt{x+6}+3)}=\dfrac{1}{27(6)}=\dfrac{1}{162}$ ។

គ. $\lim\limits_{x\to 0}\dfrac{5\sin 5x}{5x}\times 5=25$ ។

ដំណោះស្រាយលំហាត់ទី៣១

ប្រធាន៖ គណនាលីមីត៖

ក. $\lim\limits_{x\to 1}\dfrac{1-x^2}{x^3-x^2+x-1}$ ខ. $\lim\limits_{x\to 0}\dfrac{\sin 3x}{-x}$ គ. $\lim\limits_{x\to 0}\dfrac{\sqrt{x+2}-\sqrt{2-x}}{\sin x}$

ចម្លើយ៖

ក. $\lim\limits_{x\to 1}\dfrac{(1-x)(1+x)}{x^2(x-1)+(x-1)}=\lim\limits_{x\to 1}\dfrac{-(x-1)(x+1)}{(x-1)(x^2+1)}=\lim\limits_{x\to 1}\dfrac{-(x+1)}{x^2+1}=\dfrac{-2}{2}=-1$ ។

ខ. $\lim\limits_{x\to 0}\dfrac{\sin 3x}{-x}=-3\lim\limits_{x\to 0}\dfrac{\sin 3x}{3x}=-3(1)=-3$ ។

គ. $\lim\limits_{x\to 0}\dfrac{x+2-2+x}{\sin x(\sqrt{x+2}+\sqrt{2-x})}=\lim\limits_{x\to 0}\dfrac{2x}{\sin x(\sqrt{x+2}+\sqrt{2-x})}=\lim\limits_{x\to 0}\dfrac{2}{\frac{\sin x}{x}(\sqrt{x+2}+\sqrt{2-x})}=\dfrac{2}{1(2\sqrt{2})}=\dfrac{\sqrt{2}}{2}$ ។

ដំណោះស្រាយលំហាត់ទី៣២

ប្រធាន៖ គណនាលីមីត៖

ក. $\lim\limits_{x\to 1}\dfrac{x^2(x-2)+x^2+x-1}{1-x}$ ខ. $\lim\limits_{x\to 0}\dfrac{-2x}{\sin 3x}$ គ. $\lim\limits_{x\to\frac{\pi}{3}}\dfrac{\sin x-\sqrt{3}\cos x}{2(\pi-3x)}$

ចម្លើយ៖

ក. ភាគយក $x^3-2x^2+x^2+x-1=x^3-x^2+x-1=x^2(x-1)+(x-1)=(x-1)(x^2+1)$។ លីមីត ស្មើ $\lim\limits_{x\to 1}\dfrac{(x-1)(x^2+1)}{-(x-1)}=\lim\limits_{x\to 1}-(x^2+1)=-2$។

ខ. $\lim\limits_{x\to 0}\dfrac{-2x}{\sin 3x}=-\dfrac{2}{3}\lim\limits_{x\to 0}\dfrac{3x}{\sin 3x}=-\dfrac{2}{3}(1)=-\dfrac{2}{3}$។

គ. តាង $u=x-\dfrac{\pi}{3}$ នោះ $x=u+\dfrac{\pi}{3}$។ កាលណា $x\to\dfrac{\pi}{3}$ នោះ $u\to 0$។ ភាគបែង $2(\pi-3x)=2(-3)(x-\frac{\pi}{3})=-6u$។ ភាគយក $\sin x-\sqrt{3}\cos x=2(\frac{1}{2}\sin x-\frac{\sqrt{3}}{2}\cos x)=2\sin(x-\frac{\pi}{3})=2\sin u$។ លីមីតស្មើ $\lim\limits_{u\to 0}\dfrac{2\sin u}{-6u}=-\dfrac{1}{3}\lim\limits_{u\to 0}\dfrac{\sin u}{u}=-\dfrac{1}{3}$។

ដំណោះស្រាយលំហាត់ទី៣៣

ប្រធាន៖ គណនាលីមីត

ក. $\lim\limits_{x\to+\infty}\left[\sqrt{x^2+2x+3}-(x+1)\right]$

ខ. $\lim\limits_{x\to 0}\frac{-2x\sin x}{1-\cos^2 x}$

គ. $\lim\limits_{x\to-\frac{\pi}{2}}\frac{1+\sin x}{\sin^4 x-1}$

ចម្លើយ៖

ក. $\lim\limits_{x\to+\infty}\frac{x^2+2x+3-(x+1)^2}{\sqrt{x^2+2x+3}+(x+1)}=\lim\limits_{x\to+\infty}\frac{x^2+2x+3-x^2-2x-1}{\sqrt{x^2(1+2/x+3/x^2)}+x+1}=\lim\limits_{x\to+\infty}\frac{2}{x\sqrt{1+0}+x}=\lim\limits_{x\to+\infty}\frac{2}{2x}=0$ ។

ខ. $\lim\limits_{x\to 0}\frac{-2x\sin x}{\sin^2 x}=\lim\limits_{x\to 0}\frac{-2x}{\sin x}=-2(1)=-2$ ។

គ. $\lim\limits_{x\to-\frac{\pi}{2}}\frac{1+\sin x}{(\sin^2 x-1)(\sin^2 x+1)}=\lim\limits_{x\to-\frac{\pi}{2}}\frac{1+\sin x}{(\sin x-1)(\sin x+1)(\sin^2 x+1)}=\lim\limits_{x\to-\frac{\pi}{2}}\frac{1}{(\sin x-1)(\sin^2 x+1)}=$
$\frac{1}{(-1-1)((-1)^2+1)}=\frac{1}{-4}=-\frac{1}{4}$ ។

ដំណោះស្រាយលំហាត់ទី៣៤

ប្រធាន៖ គណនាលីមីត

ក. $\lim\limits_{x\to 3}\sqrt{3x^2-11}$

ខ. $\lim\limits_{x\to \frac{\pi}{3}}\dfrac{\sqrt{3}\sin\left(x-\frac{\pi}{3}\right)}{\left(\frac{\pi}{3}-x\right)}$

គ. $\lim\limits_{x\to +\infty}(2x-7-11\ln x)$

ចម្លើយ៖

ក. $\lim\limits_{x\to 3}\sqrt{3x^2-11}=\sqrt{3(3)^2-11}=\sqrt{27-11}=\sqrt{16}=4$។

ខ. តាង $u=x-\frac{\pi}{3}$ នោះ $\frac{\pi}{3}-x=-u$។ $\lim\limits_{u\to 0}\dfrac{\sqrt{3}\sin u}{-u}=-\sqrt{3}(1)=-\sqrt{3}$។

គ. $\lim\limits_{x\to +\infty}x(2-\dfrac{7}{x}-11\dfrac{\ln x}{x})=(+\infty)(2-0-0)=+\infty$។

ដំណោះស្រាយលំហាត់ទី៣៥

ប្រធាន៖ គណនាលីមីត៖

ក. $\lim\limits_{x\to 0}\dfrac{e^x+1}{2e^x}$ ខ. $\lim\limits_{x\to 1}\dfrac{\sqrt{x+3}-2}{\sqrt{x}-1}$ គ. $\lim\limits_{x\to \frac{\pi}{3}}\dfrac{\sqrt{3}\left(\frac{\pi}{3}-x\right)}{\sin\left(x-\frac{\pi}{3}\right)}$

ចម្លើយ៖

ក. $\lim\limits_{x\to 0}\dfrac{e^x+1}{2e^x}=\dfrac{e^0+1}{2e^0}=\dfrac{1+1}{2(1)}=\dfrac{2}{2}=1$។

ខ. $\lim\limits_{x\to 1}\dfrac{(\sqrt{x+3}-2)(\sqrt{x+3}+2)(\sqrt{x}+1)}{(\sqrt{x}-1)(\sqrt{x}+1)(\sqrt{x+3}+2)}=\lim\limits_{x\to 1}\dfrac{(x+3-4)(\sqrt{x}+1)}{(x-1)(\sqrt{x+3}+2)}=\lim\limits_{x\to 1}\dfrac{(x-1)(\sqrt{x}+1)}{(x-1)(\sqrt{x+3}+2)}=$
$\lim\limits_{x\to 1}\dfrac{\sqrt{x}+1}{\sqrt{x+3}+2}=\dfrac{1+1}{\sqrt{4}+2}=\dfrac{2}{4}=\dfrac{1}{2}$។

គ. តាង $u=x-\frac{\pi}{3}$ នោះ $\frac{\pi}{3}-x=-u$។ $\lim\limits_{u\to 0}\dfrac{\sqrt{3}(-u)}{\sin u}=-\sqrt{3}\lim\limits_{u\to 0}\dfrac{u}{\sin u}=-\sqrt{3}(1)=-\sqrt{3}$។

ដំណោះស្រាយលំហាត់ទី៣៦

ប្រធាន៖ គណនាលីមីត

ក. $\lim_{x\to+\infty}\left[x\ln\left(1+\frac{1}{x}\right)\right]$ គេដឹងថា $\lim_{x\to 0}\frac{\ln(1+x)}{x}=1$

ខ. $\lim_{x\to-\frac{\pi}{2}}\frac{1-\sin^2 x}{2+2\sin x}$

គ. $\lim_{x\to 3}\frac{x^3-27}{27(\sqrt{x+6}-3)}$

ចម្លើយ៖

ក. តាង $u=\frac{1}{x}$ កាលណា $x\to+\infty$ នោះ $u\to 0$។ លីមីតក្លាយជា $\lim_{u\to 0}\frac{1}{u}\ln(1+u)=\lim_{u\to 0}\frac{\ln(1+u)}{u}=1$។

ខ. $\lim_{x\to-\frac{\pi}{2}}\frac{(1-\sin x)(1+\sin x)}{2(1+\sin x)}=\lim_{x\to-\frac{\pi}{2}}\frac{1-\sin x}{2}=\frac{1-(-1)}{2}=\frac{2}{2}=1$។

គ. $\lim_{x\to 3}\frac{(x-3)(x^2+3x+9)(\sqrt{x+6}+3)}{27(x+6-9)}=\lim_{x\to 3}\frac{(x-3)(x^2+3x+9)(\sqrt{x+6}+3)}{27(x-3)}=$
$\lim_{x\to 3}\frac{(x^2+3x+9)(\sqrt{x+6}+3)}{27}=\frac{(9+9+9)(3+3)}{27}=\frac{27\times 6}{27}=6$។

ដំណោះស្រាយលំហាត់ទី៣៧

ប្រធាន៖ គណនាលីមីត៖

ក. $\lim\limits_{x\to 0}(2e^{3x}-xe^x+2e^x+4)$

ខ. រកតម្លៃ a បើ $\lim\limits_{x\to+\infty}\frac{8x^2-x+1}{ax^2+8}=1$

គ. $\lim\limits_{x\to-\infty}\left(x+\sqrt{x^2+9}\right)$

ឃ. $\lim\limits_{x\to 0}\frac{\sin x-\sin x\cos x}{x^3}$

ចម្លើយ៖

ក. $\lim\limits_{x\to 0}(2e^{3x}-xe^x+2e^x+4)=2(1)-0(1)+2(1)+4=2+0+2+4=8$។

ខ. តាមលក្ខណៈលីមីតអនន្តនៃអនុគមន៍សនិទាន ដោយដឹក្រេភាគយកស្មើភាគបែង $\lim\limits_{x\to+\infty}\frac{8x^2-x+1}{ax^2+8}=\frac{8}{a}$។ ដោយ $\frac{8}{a}=1\implies a=8$។

គ. $\lim\limits_{x\to-\infty}(x+\sqrt{x^2+9})=\lim\limits_{x\to-\infty}\frac{x^2-(x^2+9)}{x-\sqrt{x^2+9}}=\lim\limits_{x\to-\infty}\frac{-9}{x-|x|\sqrt{1+9/x^2}}=\lim\limits_{x\to-\infty}\frac{-9}{x+x\sqrt{1+9/x^2}}=\lim\limits_{x\to-\infty}\frac{-9}{x(1+\sqrt{1+0})}=0$។

ឃ. $\lim\limits_{x\to 0}\frac{\sin x(1-\cos x)}{x^3}=\lim\limits_{x\to 0}\frac{\sin x}{x}\times\frac{1-\cos x}{x^2}=1\times\frac{1}{2}=\frac{1}{2}$។

ដំណោះស្រាយលំហាត់ទី៣៨

ប្រធាន៖ គណនាលីមីត៖

ក. $\lim_{x\to 1}(\sqrt{x^2-1}+3x)$

ខ. $\lim_{x\to 0}\frac{e^{4x}-1}{e^x-1}$

គ. $\lim_{x\to 2}\frac{2-\sqrt{x+2}}{x^3-8}$

ឃ. $\lim_{x\to\frac{\pi}{3}}\frac{x-\frac{\pi}{3}}{\sin x-\sqrt{3}\cos x}$

ចម្លើយ៖

ក. $\lim_{x\to 1}(\sqrt{x^2-1}+3x)=\sqrt{1^2-1}+3(1)=0+3=3$។

ខ. $\lim_{x\to 0}\frac{e^{4x}-1}{e^x-1}=\lim_{x\to 0}\frac{e^{4x}-1}{4x}\times\frac{4x}{e^x-1}=\lim_{x\to 0}\frac{e^{4x}-1}{4x}\times\frac{x}{e^x-1}\times 4=1\times 1\times 4=4$។

គ. $\lim_{x\to 2}\frac{(2-\sqrt{x+2})(2+\sqrt{x+2})}{(x-2)(x^2+2x+4)(2+\sqrt{x+2})}=\lim_{x\to 2}\frac{4-(x+2)}{(x-2)(x^2+2x+4)(2+\sqrt{x+2})}=\lim_{x\to 2}\frac{-(x-2)}{(x-2)(x^2+2x+4)(2+\sqrt{x+2})}=\lim_{x\to 2}\frac{-1}{(x^2+2x+4)(2+\sqrt{x+2})}=\frac{-1}{12(4)}=-\frac{1}{48}$។

ឃ. តាង $u=x-\frac{\pi}{3}$ នោះ $x=u+\frac{\pi}{3}$។ ភាគបែង $\sin x-\sqrt{3}\cos x=2(\frac{1}{2}\sin x-\frac{\sqrt{3}}{2}\cos x)=2\sin(x-\frac{\pi}{3})=2\sin u$។ លីមីតក្លាយជា $\lim_{u\to 0}\frac{u}{2\sin u}=\frac{1}{2}\lim_{u\to 0}\frac{u}{\sin u}=\frac{1}{2}(1)=\frac{1}{2}$។

ដំណោះស្រាយលំហាត់ទី៣៩

ប្រធាន៖ គណនា៖

ក. $\lim_{x\to 3}\sqrt{\dfrac{3x^2+9}{x^2-5}}$

ខ. $\lim_{x\to+\infty}\dfrac{1+3x^2}{\sqrt{x^4+1}}$

គ. $\lim_{x\to 1}\dfrac{\sin(2x-2)}{1-x}$

ឃ. $\lim_{x\to e}\dfrac{2(\ln^2 x+1)-4\ln x}{4(1-\ln x)}$

ចម្លើយ៖

ក. $\lim_{x\to 3}\sqrt{\dfrac{3x^2+9}{x^2-5}}=\sqrt{\dfrac{3(3)^2+9}{3^2-5}}=\sqrt{\dfrac{27+9}{9-5}}=\sqrt{\dfrac{36}{4}}=\sqrt{9}=3$។

ខ. $\lim_{x\to+\infty}\dfrac{1+3x^2}{\sqrt{x^4+1}}=\lim_{x\to+\infty}\dfrac{x^2\left(\frac{1}{x^2}+3\right)}{x^2\sqrt{\frac{1}{x^4}+1}}=\lim_{x\to+\infty}\dfrac{\frac{1}{x^2}+3}{\sqrt{\frac{1}{x^4}+1}}=\dfrac{0+3}{\sqrt{0+1}}=3$។

គ. តាង $u=1-x$ នោះ $x=1-u$។ កាលណា $x\to 1$ នោះ $u\to 0$។ លីមីតក្លាយជា $\lim_{u\to 0}\dfrac{\sin(-2u)}{u}=\lim_{u\to 0}\dfrac{-\sin(2u)}{u}=$ $-2\lim_{u\to 0}\dfrac{\sin(2u)}{2u}=-2\times 1=-2$។

ឃ. $\lim_{x\to e}\dfrac{2(\ln^2 x+1)-4\ln x}{4(1-\ln x)}=\lim_{x\to e}\dfrac{2(\ln^2 x-2\ln x+1)}{-4(\ln x-1)}=\lim_{x\to e}\dfrac{2(\ln x-1)^2}{-4(\ln x-1)}=\lim_{x\to e}\dfrac{\ln x-1}{-2}=\dfrac{1-1}{-2}=0$។

ឯកសារយោង

1. ក្រសួងអប់រំ យុវជន និងកីឡា, «សៀវភៅពុម្ពគណិតវិទ្យា ថ្នាក់ទី១២ (កម្រិតមូលដ្ឋាន និងកម្រិតខ្ពស់)»។
2. ក្រសួងអប់រំ យុវជន និងកីឡា, «វិញ្ញាសាប្រឡងសញ្ញាបត្រមធ្យមសិក្សាទុតិយភូមិ មុខវិជ្ជាគណិតវិទ្យា ចាប់ពីឆ្នាំ២០០២ ដល់បច្ចុប្បន្ន»។
3. ឯកសារស្រាវជ្រាវ និងលំហាត់បន្ថែមទាក់ទងនឹងប្រធានបទ «លីមីតនៃអនុគមន៍» សម្រាប់កម្រិតវិទ្យាល័យ។